CHECKING POLYNOMIAL OPERATIONS BY CASTING OUT NINES.

We are **assuming** that you *already* know how to check base 10 arithmetic by casting out nines. If not, then read the article that explains it. Then come back to this article.

Since the **coefficients** we use in writing polynomials are usually written in base 10, we can cast out nines to check various polynomial operations including addition, subtraction, multiplication, division and factoring. The method is essentially the same used for checking these operations in base 10 arithmetic. The main difference is that the coefficients can be negative and/or multiple digit. For example

$$2x^3+13x^2+\bar{5}x+\bar{1}\bar{1} = 2\ 13\ \bar{5}\ \bar{1}\bar{1} = 2x^3+13x^2-5x-11$$

Casting out nines on this polynomial produces $\bar{1}$ (or 8 if we add nine to the onebar) by simply adding all the digits 2,1,3,$\bar{5}$,$\bar{1}\bar{1}$.

In **base 10** we typically have coefficients that are all in the range of **zero to nine**. Hence the sum is positive and we then continue the process of casting out nines to arrive at a **single** digit in the range **zero to eight**.

From the example above we see however that casting out nines in the algebra of polynomials can be **easier** than in arithmetic base ten since often the positive and negative digits tend to **cancel each other out**.

A few examples should illustrate the process nicely.

Consider $P(x)= 3x^2-12x+8$ and $Q(x)= -8x^2-7x+17$

$$P(x) = 3x^2+\bar{1}\bar{2}x+8 = 3\ \bar{1}\bar{2}\ 8 \;\text{-->}\; \bar{1} \text{ or } 8$$

$$Q(x) = \bar{8}x^2+\bar{7}x+17 = \bar{8}\ \bar{7}\ 17 \;\text{-->}\; 2 \text{ or } \bar{7}$$

Casting out nines on P(x) and Q(x) we got $\bar{1}$ and **2** respectively.

Then consider

$$P(x)+Q(x) = \bar{5}x^2+\bar{1}\bar{9}x+25 = \bar{5}\ \bar{1}\bar{9}\ 25 \;\text{-->}\; 1 \text{ or } \bar{8}$$

$$P(x)-Q(x) = 11x^2+\bar{5}x+\bar{9} = 11\ \bar{5}\ \bar{9} \;\text{-->}\; \bar{3} \text{ or } 6$$

$$P(x)*Q(x) = \bar{2}\bar{4}\ 75\ 71\ \bar{2}\bar{6}\bar{0}\ 136 \;\text{-->}\; \bar{2} \text{ or } 7$$

Add 9 to any single barred digit to get the corresponding positive digit. Add ninebar to any single positive digit to get the corresponding barred digit. Either of these can be used to check the problem. So the sum, difference and product above can be checked using either of the two digits following the "-->".

Since the cast out digits for P(x) and Q(x) are $\bar{1}$ and 2, we obtain **$\bar{1}+2=1$, $\bar{1}-2=\bar{3}$, and $\bar{1}*2=\bar{2}$** for the sum, difference and product, respectively. These **agree** with the cast out digit obtained from the **results** of the operations on the polynomials, so the problems all check.

In essence to check these three operations we cast out nines on P(x) and Q(x) and then do the **same** operation on these cast out digits (CODs) that we do on the **original** problem. These results **should match** the CODs obtained from the **answers** to the original problems. If these do not match, then something is wrong with the problem, the process of casting out nines *or perhaps* both.

Casting out nines is not 100% foolproof, but then neither is any method of checking. If we are really terrible at doing our arithmetic and **we make two or more mistakes** then the CO9's check will work about **89%** of the time. **If we are highly accurate** in our arithmetic, then the check's accuracy is closer to **100%.**

With a bit of practice CO9s becomes a really **fast** and **accurate** way of checking polynomial operations.

We can check **factoring** by simply treating the factorization as a multiplication problem. For example if we factor

$6x^2-x-12 = 6\ \bar{1}\ \bar{1}\bar{2}$ into the factors **$(2\ \bar{3})(3\ 4) = (2x-3)(3x+4)$**

then we have the multiplication problem **$(2\ \bar{3})(3\ 4) = 6\ \bar{1}\ \bar{1}\bar{2}$**.

Casting out nines on these three we obtain **$\bar{1}*7 \stackrel{?}{=} 6+\bar{1}+\bar{1}\bar{2} = \bar{7}$** which is certainly true. So the factorization is correct.

Checking division is a little different. It follows the pattern of checking division in base 10 arithmetic which is: Multiply the divisor times the quotient and add the remainder. This should give the dividend. For example

8551/41 yields quotient **208** and remainder **23.** Usual check:

$41*208+23 \stackrel{?}{=} 8551$ which it certainly does. So it checks.

$5*1 + 5 \stackrel{?}{=} 8+5+5+1 = 19$ --> 1 (CO9s check)

5 + 5

10

1 So the 1's indicate that it checks by CO9s.

In **algebra** we check division **the same way** using the coefficients.

We cast out nines on the divisor, quotient and remainder. Do the **same** operations that we would do in the **original** check and CO9s on that result if necessary. This should then match the COD for the dividend.

Perhaps an *easy way to **remember*** this check is that if we do the very simple division **7/2** to get quotient **3** and remainder **1**, the check **2*3+1 = 7** is **both** the usual check **and** the CO9's check because the numbers are so small that the COD for each of the four numbers is the same as the original number itself.