Trigonometry
Graph of a General Tangent Function

General Form

The general form of a tangent function is: $\boldsymbol{y = A\ \tan(Bx - C) + D}$.

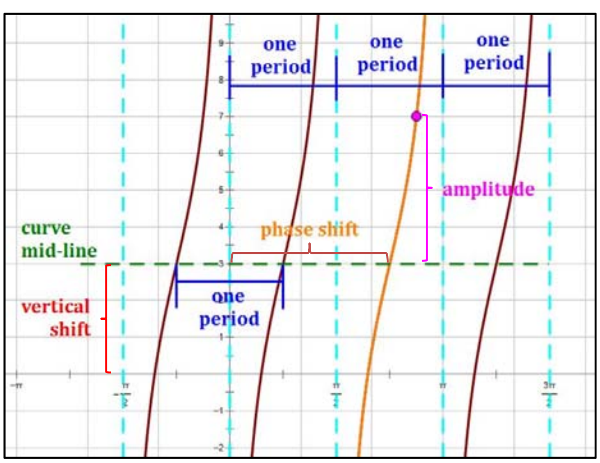
In this equation, we find several parameters of the function which will help us graph it. In particular:

- Amplitude: $\boldsymbol{Amp = |A|}$. The amplitude is the magnitude of the stretch or compression of the function from its parent function: $y = \tan x$.
- Period: $\boldsymbol{P = \frac{\pi}{B}}$. The period of a trigonometric function is the horizontal distance over which the curve travels before it begins to repeat itself (i.e., begins a new cycle). For a tangent or cotangent function, this is the horizontal distance between consecutive asymptotes (it is also the distance between x-intercepts). Note that π is the period of $y = \tan x$.
- Phase Shift: $\boldsymbol{PS = \frac{C}{B}}$. The phase shift is the distance of the horizontal translation of the function. Note that the value of $\boldsymbol{C}$ in the general form has a minus sign in front of it, just like $\boldsymbol{h}$ does in the vertex form of a quadratic equation: $\boldsymbol{y = (x - h)^2 + k}$. So,
  - o A minus sign in front of the $\boldsymbol{C}$ implies a translation to the right, and
  - o A plus sign in front of the $\boldsymbol{C}$ implies a implies a translation to the left.
- Vertical Shift: $\boldsymbol{VS = D}$. This is the distance of the vertical translation of the function. This is equivalent to $\boldsymbol{k}$ in the vertex form of a quadratic equation: $\boldsymbol{y = (x - h)^2 + k}$.

Example: $\boldsymbol{y = 4\tan\left(2x - \frac{3}{2}\pi\right) + 3}$

The midline has the equation $\mathrm{y = D}$. In this example, the midline is: $\mathrm{y = 3}$. One cycle, shifted to the right, is shown in orange below.

Note that, for the tangent curve, we typically graph half of the principal cycle at the point of the phase shift, and then fill in the other half of the cycle to the left (see next page).



For this example:

$\boldsymbol{A = 4;\ \ B = 2;\ \ C = \frac{3}{2}\pi;\ \ D = 3}$

Amplitude: $\boldsymbol{Amp = |A| = |4| = 4}$

Period: $\boldsymbol{P = \frac{\pi}{B} = \frac{\pi}{2} = \frac{1}{2}\pi}$

Phase Shift: $\boldsymbol{PS = \frac{C}{B} = \frac{\frac{3}{2}\pi}{2} = \frac{3}{4}\pi}$

Vertical Shift: $\boldsymbol{VS = D = 3}$

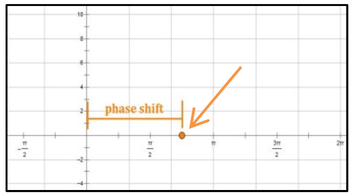
Trigonometry

Graphing a Tangent Function with No Vertical Shift: $y = A\tan(Bx - C)$

A cycle of **the tangent function** has two asymptotes and a zero point halfway in-between. It flows upward to the right if $A > 0$ and downward to the right if $A < 0$.

Example: $y = 4\tan\left(2x - \frac{3}{2}\pi\right)$.

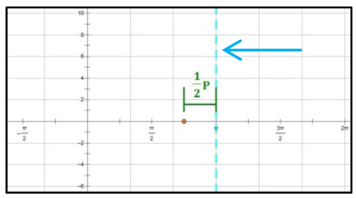
Step 1: Phase Shift: $PS = \frac{C}{B}$.
The first cycle begins at the "zero" point **PS units to the right of the Origin**.



$PS = \frac{C}{B} = \frac{\frac{3}{2}\pi}{2} = \frac{3}{4}\pi$.

The point is: $\left(\frac{3}{4}\pi, 0\right)$

Step 2: Period: $P = \frac{\pi}{B}$.
Place a vertical asymptote $\frac{1}{2}P$ units to the right of the beginning of the cycle.

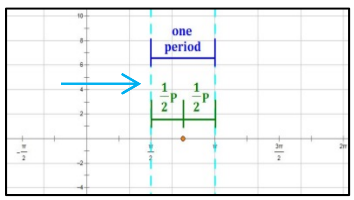


$P = \frac{\pi}{B} = \frac{\pi}{2} = \frac{1}{2}\pi$. $\frac{1}{2}P = \frac{1}{4}\pi$.

The right asymptote is at:

$x = \frac{3}{4}\pi + \frac{1}{4}\pi = \pi$

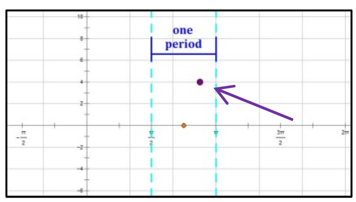
Step 3: Place a vertical asymptote $\frac{1}{2}P$ units to the left of the beginning of the cycle.



The left asymptote is at:

$x = \frac{3}{4}\pi - \frac{1}{4}\pi = \frac{1}{2}\pi$

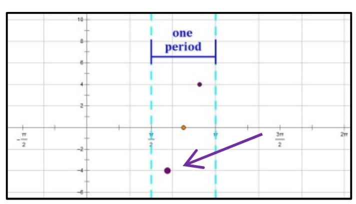
Step 4: The y-value of the point halfway between the zero point and the right asymptote is "A".



The point is:

$\left(\frac{\frac{3}{4}\pi + \pi}{2}, 4\right) = \left(\frac{7}{8}\pi, 4\right)$

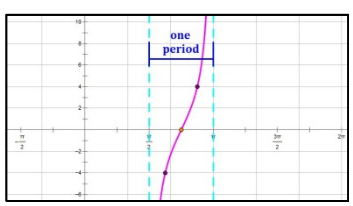
Step 5: The y-value of the point halfway between the left asymptote and the zero point is "$-A$".



The point is:

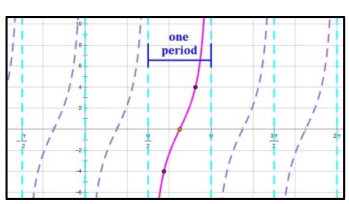
$\left(\frac{\frac{1}{2}\pi + \frac{3}{4}\pi}{2}, -4\right) = \left(\frac{5}{8}\pi, -4\right)$

Step 6: Draw a smooth curve through the three key points, approaching the asymptotes on each side.



This will produce the graph of one wave of the function.

Step 7: Duplicate the wave to the left and right as desired.



Note: If $D \neq 0$, all points on the curve are shifted vertically by D units.