5.1 Verifying Trig Identities

Step 1: Choose the more complicated side, and work with only that side

Step 2: Apply fundamental identities (p.~~586~~) 580

Fundamental Trig Identities

Reciprocal Identities:

$\sin x = \frac{1}{\csc x}$ $\cos x = \frac{1}{\sec x}$ $\tan x = \frac{1}{\cot x}$

$\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities:

$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities:

$\sin^2 x + \cos^2 x = 1$ $1 + \tan^2 x = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Even-Odd Identities:

$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$

$\csc(-x) = -\csc x$ $\sec(-x) = \sec x$ $\cot(-x) = -\cot x$

Step 3:

A. Re-write in terms of sin and cos
B. Factor
C. Separate fractions $\left(\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}\right)$
D. Combine fractions
E. Introduce expressions we need

Examples:

1. $\cos x(\csc x) = \cot x$

$\cos x\left(\frac{1}{\sin x}\right) =$

$\frac{\cos x}{\sin x}$

$\cot x = \cot x$

2. $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

$= \cos^2 x + \cos^2 x - 1$

$= (1 - \sin^2 x) + \cos^2 x - 1$

$\cos^2 x - \sin^2 x = \cos^2 x - \sin^2 x$

3. $\tan \Theta + \cot \Theta = \sec \Theta \csc \Theta$

$\tan\theta + \frac{1}{\tan\theta}$ common denom.

$\frac{\tan^2\theta}{\tan\theta} + \frac{1}{\tan\theta}$

$\frac{\tan^2\theta + 1}{\tan\theta}$

$\frac{\sec^2\theta}{\tan\theta}$

$\sec \cdot \frac{1}{\cancel{\cos}} \cdot \frac{\cancel{\cos}}{\sin}$

$\frac{\sec\theta}{\sin\theta}$

$\sec\theta\csc\theta = \sec\theta\csc\theta$

4. $\csc x \tan x = \sec x$

$\frac{1}{\cancel{\sin}}\left(\frac{\cancel{\sin}}{\cos}\right)$

$\frac{1}{\cos}$

$\sec x = \sec x$

5. $\sin x - \sin x \cos^2 x = \sin^3 x$

$\sin x - \sin x(1-\sin^2 x)$

$\sin x - \sin x + \sin^3 x$

$\sin^3 x = \sin^3 x$

6. $\frac{1+\cos\theta}{\sin\theta} = \csc\theta + \cot\theta$

$= \frac{1}{\sin} + \frac{\cos}{\sin\theta}$

$\frac{1+\cos\theta}{\sin\theta} = \frac{1+\cos\theta}{\sin\theta}$

7. $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\csc x$

$\sin x$ $1+\cos x$

$\frac{\sin^2 x}{\sin(1+\cos x)} + \frac{(1+\cos x)(1+\cos x)}{\sin(1+\cos x)}$

$\frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin(1+\cos x)}$ $* \sin^2 x + \cos^2 x = 1$

$\frac{1+1+2\cos x}{\sin x(1+\cos x)}$

$\frac{2(\cancel{1+\cos x})}{\sin x(\cancel{1+\cos x})} \rightarrow \frac{2}{\sin x}$

$2\csc x = 2\csc x$

8. $\frac{\cos x}{1+\sin x} \cdot \frac{(1-\sin x)}{(1-\sin x)} = \frac{1-\sin x}{\cos x}$ * multiply by a version of 1 when we need to introduce new expressions

$\frac{\cos x(1-\sin x)}{1-\sin^2 x}$

$\frac{\cancel{\cos x}(1-\sin x)}{\cos^{\cancel{2}} x}$

$\frac{1-\sin x}{\cos x} = \frac{1-\sin x}{\cos x}$

9. $\frac{\sec x+\csc(-x)}{\sec x \csc x} = \sin x - \cos x$

$\frac{\sec x - \csc x}{\sec x \cdot \csc x}$

$\frac{\sin x \frac{1}{\cos x} - \frac{1}{\sin x} \cdot \cos x}{\frac{1}{\cos \sin}}$

$\sin x - \cos x = \sin x - \cos x$

$\frac{\frac{\sin x - \cos x}{\cancel{\cos \sin}}}{\frac{1}{\cancel{\cos \sin}}}$

10. $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2 + 2\tan^2\Theta$

$\frac{1-\sin\theta}{1-\sin^2\theta} + \frac{1+\sin\theta}{1-\sin^2\theta}$

$\frac{2}{1-\sin^2\theta}$

$2 + 2\tan^2\theta = 2 + 2\tan^2\theta$

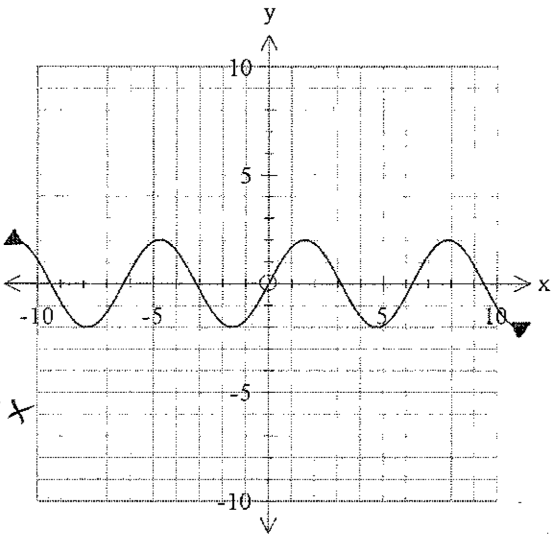
$2\left(\frac{1}{\cos^2}\right)$

$2(\sec^2\theta)$

$2(1+\tan^2\theta)$

(p. 595 #63, 65, 66)

11. $\dfrac{\cos x + \cot x \cdot \sin x}{\cot x} = ?$ $\boxed{2\sin x}$



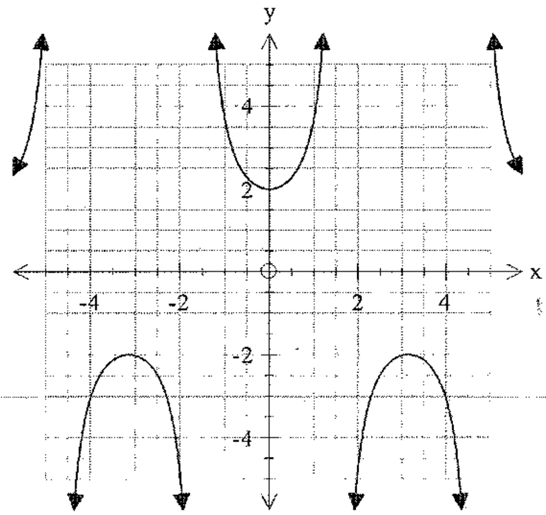
$$\frac{\cos * \frac{\cos}{\sin}\left(\frac{\sin}{1}\right)}{\frac{\cos}{\sin}}$$

$$\frac{2\cos x}{\frac{\cos x}{\sin x}}$$

$\rightarrow 2\cancel{\cos x} \cdot \dfrac{\sin x}{\cancel{\cos x}}$

$2\sin x = 2\sin x$

12. $\dfrac{1}{\sec x + \tan x} + \dfrac{1}{\sec x - \tan x} = ?$ $2\sec x$

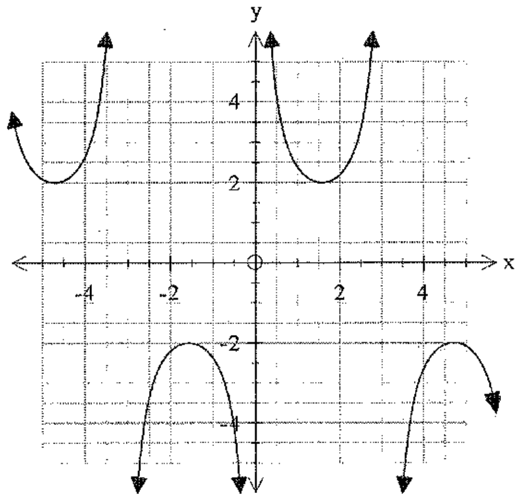


$$\frac{\sec - \cancel{\tan} + \sec + \cancel{\tan}}{\sec^2 - \tan^2}$$

$$\frac{2\sec x}{1 + \cancel{\tan^2} - \cancel{\tan^2}}$$

$2\sec x = 2\sec x$

13. $\dfrac{1+\cos x}{\sin x} + \dfrac{\sin x}{1+\cos x} = ?$ $2\csc x$



$1 + 2\cos x + \underbrace{(\cos^2 x + \sin^2 x)}_{=1}$

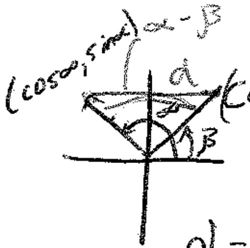
$$\frac{2 + 2\cos x}{\sin x(1+\cos x)}$$

$$\frac{2(1+\cancel{\cos x})}{\sin x(1+\cancel{\cos x})}$$

$2\left(\dfrac{1}{\sin x}\right)$

5.2 Day 1: Sum and Difference of Two Angles (foldable)

The cosine of the Difference of Two Angles:



$$\cos(\alpha - \beta) = \cos\alpha\,\cos\beta + \sin\alpha\,\sin\beta$$

* Apply distance formula

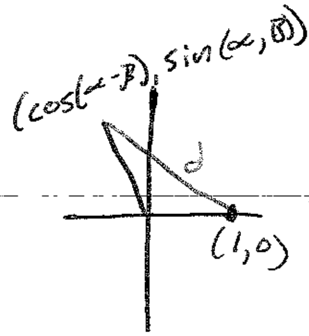
$$d = \sqrt{(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2}$$

$$d = \sqrt{(\cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta) + (\sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta)}$$

$$d = \sqrt{\underline{(\cos^2\alpha + \sin^2\alpha)} + \underline{(\sin^2\beta + \cos^2\beta)} - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta}$$

$$d = \sqrt{1 + 1 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta}$$

$$\boxed{d = \sqrt{2 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta}}$$



Now Let $(x_1, y_1) = (1, 0)$ & $(x_2, y_2) = (\cos(\alpha - \beta), \sin(\alpha - \beta))$

$$d = \sqrt{[\cos(\alpha - \beta) - 1]^2 + [\sin(\alpha - \beta) - 0]^2}$$

$$d = \sqrt{\underline{\cos^2(\alpha - \beta)} - 2\cos(\alpha - \beta) + 1 + \underline{\sin^2(\alpha - \beta)}}$$ Pythag Identity $= 1$

$$d = \sqrt{1 - 2\cos(\alpha - \beta) + 1}$$

$$d = \sqrt{2 - 2\cos(\alpha - \beta)}$$

* set these equal to each other.

$$\sqrt{2 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta}^{\,2} = \sqrt{2 - 2\cos(\alpha - \beta)}^{\,2}$$ * square both sides

$$\cancel{2} - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta = \cancel{2} - 2\cos(\alpha - \beta)$$

$$-\cancel{2}(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = -\cancel{2}(\cos(\alpha - \beta))$$

$$\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos\alpha - \beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$ (cos flips sign)

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$ (tan + ± tan - ∓)

Sum and Difference formulas for Cos, Sin & Tan

$$\cos(\alpha-\beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha \tan\beta}$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha \tan\beta}$$

1. Find the exact value of $\cos 15^\circ$

$\cos 15 = \cos(60-45)$

$= \cos 60 \cos 45 + \sin 60 \sin 45$

$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

2. Find the exact value of $\cos 100^\circ \cos 55^\circ + \sin 100^\circ \sin 55^\circ$

$\cos(100-55)$

$\cos 45$

$\boxed{\frac{\sqrt{2}}{2}}$

3. Verify the identity: $\frac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta} = 1 + \tan\alpha \tan\beta$

$$\frac{\cos\alpha \cos\beta + \sin\alpha \sin\beta}{\cos\alpha \cos\beta}$$

$$\frac{\cos\alpha \cos\beta}{\cos\alpha \cos\beta} + \frac{\sin\alpha \sin\beta}{\cos\alpha \cos\beta}$$

$$1 + \frac{\sin\alpha}{\cos\alpha} \cdot \frac{\sin\beta}{\cos\beta}$$

$$1 + \tan\alpha \tan\beta = 1 + \tan\alpha \tan\beta$$

4. *Find the exact value f* $\sin\frac{5\pi}{12}$ $\Rightarrow \sin\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)$

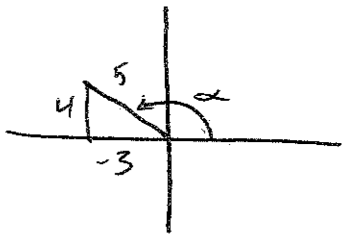
$= \sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

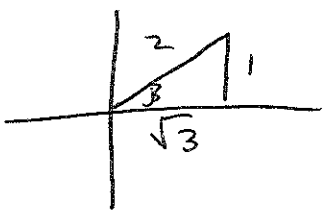
$= \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$

$\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right.$ $\qquad \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

5. *Suppose that* $\sin\alpha = \frac{4}{5}$ *for a quadrant II angle* α *and* $\sin\beta = 1/2$ *for a quadrant I angle* β. *Find the exact value of each of the following:*





($\cos\alpha$) $\quad -3/5$

$\cos\beta \quad \frac{\sqrt{3}}{2}$

$\cos(\alpha+\beta)$ $\cos\alpha\cos\beta - \sin\alpha\sin\beta$

$-\frac{3}{5}\cdot\frac{\sqrt{3}}{2} - \frac{4}{5}\left(\frac{1}{2}\right)$

$\frac{-3\sqrt{3}}{10} - \frac{4}{10} = \boxed{\frac{-3\sqrt{3}-4}{10}}$

$\sin(\alpha+\beta)$ $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\frac{4}{5}\left(\frac{\sqrt{3}}{2}\right) + -\frac{3}{5}\left(\frac{1}{2}\right)$

$\frac{4\sqrt{3}}{10} + \frac{-3}{10} = \boxed{\frac{4\sqrt{3}-3}{10}}$

6. *The graph shows* $y=\cos(x+\frac{3\pi}{2})$ *in a* $[0, 2\pi, \frac{\pi}{2}]$ *by* $[-2, 2, 1]$ *viewing rectangle*

a. *Describe the graph using another equation*

b. *Verify that the two equations are equivalent*

a.) $\sin x$

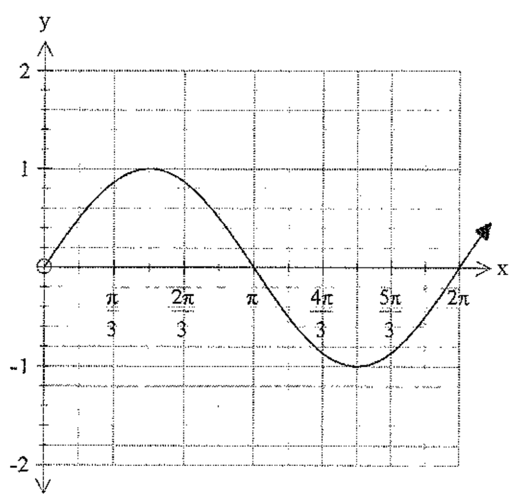
b.) $\cos(x+\frac{3\pi}{2}) = \sin x$

$\cos x\cos\frac{3\pi}{2} - \sin x\sin\frac{3\pi}{2} = \sin x$

$\cos x(0) - \sin x(-1) = \sin x$

$0 + \sin x = \sin x$

$\sin x = \sin x$



7. *Find the exact value of* $\sin(\frac{2\pi}{3} - \frac{\pi}{4})$

$$\sin\frac{2\pi}{3}\cos\frac{\pi}{4} - \cos\frac{2\pi}{3}\sin\frac{\pi}{4}$$

$$\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

8. *Simplify:* $\tan(\frac{5\pi}{4} + \theta)$

$$\frac{\tan\frac{5\pi}{4} + \tan\theta}{1 - \tan\frac{5\pi}{4}\tan\theta}$$

$$\frac{1 + \tan\theta}{1 - (1)\tan\theta} \Rightarrow \frac{\tan\theta + 1}{-\tan\theta + 1}$$

9. *Simplify:* $\tan(\pi - \beta)$

$$\frac{\tan\pi - \tan\beta}{1 + \tan\pi\tan\beta} \qquad \tan\pi = 0$$

$$\frac{0 - \tan\beta}{1 + (0)\tan\beta} = \frac{-\tan\beta}{1} = -\tan\beta$$

5.2 Day 2: Sum and Difference Formulas

Find the exact value of the expression:

1. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12}+\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$\cos\left(\frac{4\pi}{12}\right)$

$\cos \pi/3$

$\boxed{\frac{1}{2}}$

2. $\cos 75^{\circ}$

$\cos(45+30)$

$\cos45\cos30-\sin45\sin30$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

3. $\tan\left(\frac{4\pi}{3}-\frac{\pi}{4}\right)$

$\dfrac{\tan\frac{4\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{4\pi}{3}\tan\frac{\pi}{4}}$

$\dfrac{\sqrt{3}-1}{1+\sqrt{3}}\;\dfrac{(1-\sqrt{3})}{(1-\sqrt{3})}$

$\dfrac{\sqrt{3}-3-1+\sqrt{3}}{1-3}$

$\dfrac{-4+2\sqrt{3}}{-2}=\boxed{2-\sqrt{3}}$

4. $\dfrac{\tan 10^{\circ}+\tan 35^{\circ}}{1-\tan 10^{\circ}\tan 35^{\circ}}$

$\tan(10+35)$

$\tan 45$

$\boxed{1}$

Verify the identity

5. $\cos(x - \frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x + \sin x)$

$\cos x \cos \pi/4 + \sin x \sin \pi/4$

$\cos x\left(\frac{\sqrt{2}}{2}\right) + \sin x\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{2}(\cos x + \sin x) = \frac{\sqrt{2}}{2}(\cos x + \sin x)$

6. $\sin(x+\frac{\pi}{2}) = \cos x$

$\sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2}$

$\sin x(0) + \cos x(1)$

$\cos x = \cos x$

7. $\frac{\sin(\alpha-\beta)}{\cos\alpha \cos\beta} = \tan\alpha - \tan\beta$

$\frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$\frac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} - \frac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta}$

$\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta}$

$\tan\alpha - \tan\beta = \tan\alpha - \tan\beta$

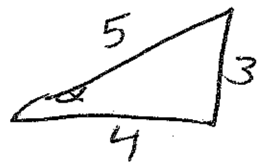
8. $\frac{\cos(x+h)-\cos x}{h} = \cos x \frac{\cos h - 1}{h} - \sin x \frac{\sin h}{h}$

$$\frac{\cos x \cosh - \sin x \sin h - \cos x}{h}$$

$$\frac{(\cos x \cosh - \cos x)}{h} - \frac{\sin x \sin h}{h}$$

$$\cos x \frac{(\cosh - 1)}{h} - \sin x \frac{\sin h}{h}$$

9. *Find the exact value of the following if* $\sin \alpha = \frac{3}{5}$, α *lies in quadrant I, and* $\sin \beta = \frac{5}{13}$, β *lies in quadrant II.*

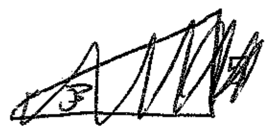


$\cos(\alpha + \beta)$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\frac{4}{5}\left(-\frac{12}{13}\right) - \frac{3}{5}\left(\frac{5}{13}\right)$

$-\frac{48}{65} + \frac{-15}{65} \rightarrow \boxed{-\frac{63}{65}}$

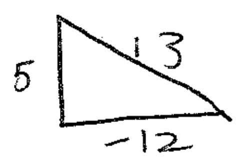


$\sin(\alpha + \beta)$

$\sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\frac{3}{5}\left(-\frac{12}{13}\right) + \frac{4}{5}\left(\frac{5}{13}\right)$

$-\frac{36}{65} + \frac{20}{65} \rightarrow \boxed{\frac{-16}{65}}$



$\tan(\alpha + \beta)$

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\frac{\frac{3}{4} + \left(\frac{-5}{12}\right)}{1 - \frac{3}{4}\left(-\frac{5}{12}\right)} \rightarrow \frac{\frac{9-5}{12}}{1 + \frac{15}{48}} = \frac{\frac{1}{3}}{\frac{63}{48}} = \frac{1}{3} \cdot \frac{48}{63} = \boxed{\frac{16}{63}}$

5.3 Day 1: Double Angle Formulas

Double Angles

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Half Angles

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

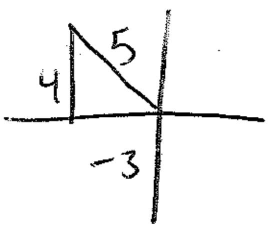
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$$

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$$

Pg. 613

1. If $\sin\theta = \frac{4}{5}$ and θ lies in quadrant II, find the exact value of each of the following:



$\sin 2\theta$

$2\sin\theta\cos\theta$

$2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$

$= \boxed{\frac{-24}{25}}$

$\cos 2\theta$

$\cos^2\theta - \sin^2\theta$

$\left(-\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$

$\frac{9}{25} - \frac{16}{25}$

$\boxed{-7}$

$\tan 2\theta$

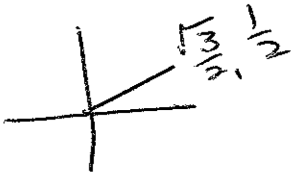
$\frac{2\tan\theta}{1-\tan^2\theta}$

$\frac{2\left(-\frac{4}{3}\right)}{1-\left(-\frac{4}{3}\right)^2} = \frac{-\frac{8}{3}}{\frac{9}{9}-\frac{16}{9}} = \frac{-\frac{8}{3}}{-\frac{7}{9}} = \boxed{\frac{24}{7}}$

2. *Find the exact value of* $\frac{2\tan 15^o}{1-\tan^2 15^o}$

$$= \tan 2\theta = \tan 2(15)$$

$$= \tan 30^\circ$$



$$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \boxed{\frac{\sqrt{3}}{3}}$$

3. *Verify the identity:* $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

* use sum formula

$\cos(2\theta+\theta)$

$\cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$(\cos^2\theta - \sin^2\theta)\cos\theta - [(2\sin\theta\cos\theta)\sin\theta]$

$\cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$

$\cos^3\theta - 3\sin^2\theta\cos\theta$

$\cos^3\theta - 3(1-\cos^2\theta)\cos\theta$

$\cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$4\cos^3\theta - 3\cos\theta =$

4. *Use* $\cos 210^o = \frac{\sqrt{3}}{2}$ *to find the exact value of* $\cos 105^o$

$*$ Quad II so $\cos < 0$

$$\cos 105 = \cos \frac{210}{2} = \pm\sqrt{\frac{1+\cos 210}{2}}$$

$$= \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$= \pm\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} \cdot \frac{1}{2}$$

$$= \pm\sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \pm\frac{\sqrt{2-\sqrt{3}}}{2}$$

$$= -\frac{\sqrt{2-\sqrt{3}}}{2}$$

5. *Verify the identity:* $\tan\theta = \frac{\sin 2\theta}{1+\cos 2\theta}$

$$= \frac{2\sin\theta\cos\theta}{1+\cancel{(1}-2\sin^2\theta)}$$

$$= \frac{2\sin\theta\cos\theta}{2-2\sin^2\theta}$$

$$= \frac{2(\sin\theta\cos\theta)}{2(1-\sin^2\theta)}$$

$$= \frac{\sin\theta\cancel{\cos\theta}}{\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \tan\theta$$

$$\frac{2\sin\theta\cos\theta}{1+(2\cos^2\theta-1)}$$

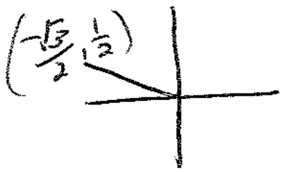
$$= \frac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

6. *Use a ½ angle formula to find the exact value of* $\tan\frac{5\pi}{12}$

* Quad I so $\tan > 0$

$$\tan \frac{\frac{5\pi}{6}}{2} \qquad \alpha = \frac{5\pi}{6}$$



$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos 5\pi/6}{1+\cos 5\pi/6}} = \pm\sqrt{\frac{1-(-\sqrt{3}/2)}{1+(-\sqrt{3}/2)}} = \pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2-\sqrt{3}}{2}}}$$

$$= \pm\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}} = \pm\sqrt{\frac{(2+\sqrt{3})^2}{4-3}} = \pm\frac{2+\sqrt{3}}{1}$$

* So $\tan\frac{5\pi}{12} = 2+\sqrt{3}$

7. *Verify the identity:* $\tan\frac{\alpha}{2} = \frac{\sec\alpha}{\sec\alpha\csc\alpha + \csc\alpha}$

$$= \frac{\frac{1}{\cos\alpha}}{\left(\frac{1}{\cos\alpha}\right)\left(\frac{1}{\sin\alpha}\right)+\frac{1}{\sin\alpha}}$$

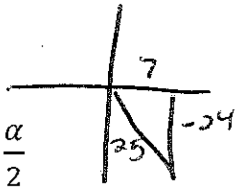
$$= \frac{\frac{1}{\cos\alpha}}{\frac{1}{\cos\sin} + \frac{1}{\sin}\cdot\frac{\cos}{\cos}}$$

$$= \frac{\frac{1}{\cancel{\cos}\alpha}}{\frac{1+\cos\alpha}{\cancel{\cos}\alpha\sin\alpha}}$$

$$= \frac{\sin\alpha}{1+\cos\alpha}$$

$$\tan\frac{\alpha}{2} = \tan\frac{\alpha}{2}$$

8. *If* $\csc\alpha = \frac{-25}{24}$, *and* α *is in the* 4^{th} *quadrant. Find:*



a) $\sin\frac{\alpha}{2}$ b) $\cos\frac{\alpha}{2}$ c) $\tan\frac{\alpha}{2}$

Quad 4 $\frac{-25}{\frac{12}{2}}$

sin +

cos −

tan −

a.) $+\sqrt{\frac{1-\cos\alpha}{2}}$

$+\sqrt{\frac{1-7/25}{2}}$

$=\sqrt{\frac{\frac{18}{25}}{\cancel{2}}\cdot\frac{1}{2}}$

$=\sqrt{\frac{9}{25}}$

$=\boxed{\frac{3}{5}}$

b.) $-\sqrt{\frac{1+\cos\alpha}{2}}$

$=-\sqrt{\frac{1+7/25}{2}}$

$=-\sqrt{\frac{\frac{32}{25}}{\cancel{2}}\cdot\frac{1}{2}}$

$=-\sqrt{\frac{16}{25}}$

$=\boxed{-\frac{4}{5}}$

c.) $-\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

$=-\sqrt{\frac{1-7/25}{1+7/25}}$

$=-\sqrt{\frac{18}{\cancel{25}}\Big/\frac{32}{\cancel{25}}}$

$=-\sqrt{\frac{18}{32}}$

$=-\sqrt{\frac{9}{16}}$

$=\boxed{-\frac{3}{4}}$

9. *Use half-angle to find the exact value of tan* (112.5°)

Quad II so tan −

$\tan\frac{225}{2}$ so $\alpha = 225$

$=-\sqrt{\frac{1-\cos 225}{1+\cos 225}}$

$=-\sqrt{\frac{1-(-\frac{\sqrt{2}}{2})}{1+(-\frac{\sqrt{2}}{2})}}$

$=-\sqrt{\frac{\frac{2+\sqrt{2}}{\cancel{2}}}{\frac{2-\sqrt{2}}{\cancel{2}}}}$

$=-\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}\cdot\frac{2-\sqrt{2}}{2-\sqrt{2}}}$

$=-\frac{\sqrt{4-2}}{2-\sqrt{2}}$

$=-\frac{\sqrt{2}\,(2+\sqrt{2})}{2-\sqrt{2}\,(2+\sqrt{2})}$

$\cancel{\frac{-2\sqrt{2}-2}{4-2}}$

$\cancel{=\boxed{\sqrt{2}-1}}$

or $\frac{1-\cos\alpha}{\sin\alpha}$

$=\left(\frac{1-\cos 225}{\sin 225}\right)$

$=\frac{1-(-\frac{\sqrt{2}}{2})}{-\frac{\sqrt{2}}{2}}$

$=\frac{\frac{2+\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}} = \frac{-2-\sqrt{2}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$=\frac{-2\sqrt{2}-2}{2} = \boxed{-\sqrt{2}-1}$

5.3 Day 2: Double and Half Angles

24, -7, 25

1. If $\cos\theta = \frac{24}{25}$, θ lies in quadrant IV, find the following:

Sin 2θ

$2\sin\theta\cos\theta$

$2\left(\frac{-7}{25}\right)\left(\frac{24}{25}\right)$

$\boxed{\frac{-336}{625}}$

cos 2θ

$\cos^2\theta - \sin^2\theta$

$\left(\frac{24}{25}\right)^2 - \left(-\frac{7}{25}\right)^2$

$\frac{576}{625} - \frac{49}{625}$

$\boxed{\frac{527}{625}}$

tan 2θ

$\frac{2\tan\theta}{1-\tan^2\theta}$

$\frac{2(-7/24)}{1-(-7/24)^2} = \frac{-\frac{7}{12}}{1-\frac{49}{576}} = \frac{-\frac{7}{12}}{\frac{527}{576}}$

$= \left(-\frac{7}{12}\right)\left(\frac{576}{527}\right)$

$= \boxed{\frac{-336}{527}}$

2. Find the exact value: $\cos^2 75^\circ - \sin^2 75^\circ$

$\cos(2\cdot 75)$

$= \cos 150$

$= \boxed{-\frac{\sqrt{3}}{2}}$

3. Find the exact value: $\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

$= \tan 2\left(\frac{\pi}{8}\right)$

$= \tan\frac{\pi}{4}$

$= \boxed{1}$

4. *Verify the identity:* $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

$\underbrace{\sin^2\theta + 2\sin\cos + \cos^2}$

$1 + 2\sin\cos$

$1 + \sin 2\theta = 1 + \sin 2\theta$

5. *Verify the identity:* $\cot x = \dfrac{\sin 2x}{1 - \cos 2x}$

$= \dfrac{2\sin x\cos x}{1 - (\cos^2 - \sin^2)}$

$= \dfrac{2\sin x \cos x}{(1 - \cos^2 x) + \sin^2 x}$

$= \dfrac{2\sin x \cos x}{\sin^2 x + \sin^2 x}$

$= \dfrac{\cancel{2}\,\cancel{\sin x}\cos x}{\cancel{2}\sin^{\cancel{2}} x}$

$\dfrac{\cos x}{\sin x} =$

$\cot x = \cot x$

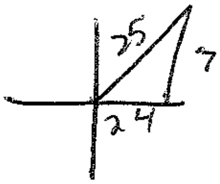
6. *Find the exact value of tan 75°*

Quad I tan +

$\tan\frac{150}{2} = \dfrac{1 - \cos 150}{\sin 150}$

$= \dfrac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} = \boxed{2 + \sqrt{3}}$

7. *If* $\tan\alpha = \frac{7}{24}$, *find* $2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$



$= 2\sqrt{\frac{1-\cos\alpha}{2}} \cdot \sqrt{\frac{1+\cos\alpha}{2}}$

$= 2\sqrt{\frac{1-24/25}{2}} \cdot \sqrt{\frac{1+\frac{24}{25}}{2}}$

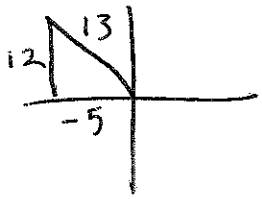
$= 2\sqrt{\frac{1}{50}}\sqrt{\frac{49}{50}}$

$= 2 \cdot \frac{1}{\sqrt{50}} \cdot \frac{7}{\sqrt{50}} = \frac{14}{50} = \boxed{\frac{7}{25}}$

8. *If* $\sec\alpha = -\frac{13}{5}$ *and* $\frac{\pi}{2} < \alpha < \pi$, *find the following:*

since $\frac{\pi}{2} < \alpha < \pi$ then $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$

so QI



$\sin\frac{\alpha}{2}$

$\sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-(-5/13)}{2}} = \sqrt{\frac{18}{26}} = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}} = \boxed{\frac{3\sqrt{13}}{13}}$

$\cos\frac{\alpha}{2}$

$\sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+(-5/13)}{2}} = \sqrt{\frac{8}{26}} = \sqrt{\frac{4}{13}} = \frac{2\sqrt{13}}{13}$

$\tan\frac{\alpha}{2}$

$\frac{1-\cos\alpha}{\sin\alpha} = \frac{1-(-5/13)}{\frac{12}{13}} = \frac{\frac{18}{13}}{\frac{12}{13}} = \frac{18}{12} = \boxed{\frac{3}{2}}$

9. *Verify the identity:* $2\tan\frac{\alpha}{2} = \frac{\sin^2\alpha+1-\cos^2\alpha}{\sin\alpha(1+\cos\alpha)}$

$$= \frac{\sin^2\alpha + \sin^2\alpha}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{2\sin^{\cancel{2}}\alpha}{\cancel{\sin\alpha}(1+\cos\alpha)}$$

$$= \frac{2\sin\alpha}{1+\cos\alpha}$$

$$= 2\left(\frac{\sin\alpha}{1+\cos\alpha}\right)$$

$$= 2\tan\frac{\alpha}{2}$$

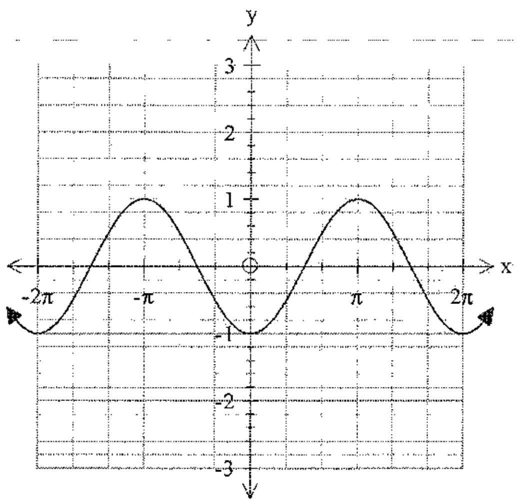
10. *Finish the identity and verify*

$\sin^2\frac{x}{2} - \cos^2\frac{x}{2} = ?$ $-\cos x$

$$= -\left(-\sin^2\frac{x}{2} + \cos^2\frac{x}{2}\right)$$

$$= -\cos\left(2\cdot\frac{x}{2}\right)$$

$$= -\cos x$$



Remember: $\sin\frac{\theta}{2} \neq \frac{1}{2}\sin\theta$

$\cos\frac{\theta}{2} \neq \frac{1}{2}\cos\theta$

$\tan\frac{\theta}{2} \neq \frac{1}{2}\tan\theta$

5.3 *Power Reducing Formulas*

**Double angles are used to derive the power reducing formulas*

In calculus, by reducing the power, we can better explore the relationship between a function and how it changes at every single instant in time. (used by athletes to increase throwing distance)

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

1) Write an equivalent expression for $\cos^4 x$ *that does not contain powers of trigonometric functions greater than 1.*

$(\cos^2 x)^2$

$= \left(\frac{1+\cos 2x}{2}\right)^2$

$= \frac{1+2\cos 2x+\cos^2 2x}{4}$

$= \frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{4}\left(\frac{1+\cos 2(2x)}{2}\right)$

$= \frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{8}+\frac{1}{8}\cos 4x$

$\boxed{= \frac{3}{8}+\frac{1}{2}\cos 2x+\frac{1}{8}\cos 4x}$ ← doesn't contain powers of trig functions greater than 1

2) *Write an equivalent expression for* $8\sin^4 x$ *that does not contain powers of trigonometric functions greater than 1.*

$$8\sin^4 x = 8\left(\frac{1-\cos 2x}{2}\right)^2$$

$$= {}^{2}\cancel{8}\left(\frac{1-2\cos 2x+\cos^2 2x}{{}_{1}\cancel{4}}\right)$$

$$= 2-4\cos 2x+\cancel{2}\left(\frac{1+\cos 4x}{\cancel{2}}\right)$$

$$= 2-4\cos 2x+1+\cos 4x$$

$$= 3-4\cos 2x+\cos 4x$$

3) *Write an equivalent expression for* $2\sin^2 x\cos^2 x$ *that does not contain powers of trigonometric functions greater than 1.*

$$2\sin^2 x\cos^2 x = 2\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$$

$${}^{1}\cancel{2}\left(\frac{1-\cancel{\cos 2x}+\cancel{\cos 2x}-\cos^2 2x}{{}^{2}\cancel{4}}\right)$$

$$= \frac{1}{2}\left[1-\left(\frac{1+\cos 4x}{2}\right)\right]$$

$$= \frac{1}{2}\left(1-\frac{1}{2}-\frac{1}{2}\cos 4x\right)$$

$$= \frac{1}{2}-\frac{1}{4}-\frac{1}{4}\cos 4x$$

$$\boxed{= \frac{1}{4}-\frac{1}{4}\cos 4x}$$

4) Verify: $\sin 4t = 4\sin t\cos^3 t - 4\sin^3 t\cos t$

$\sin(2t+2t) =$

$\sin 2t\cos 2t + \cos 2t\sin 2t =$

$(2\sin t\cos t)(1-2\sin^2 t) + (2\cos^2 t - 1)(2\sin t\cos t) =$

$\cancel{2\sin t\cos t} - 4\sin^3 t\cos t + 4\sin t\cos^3 t - \cancel{2\sin t\cos t} =$

$-4\sin^3 t\cos t + 4\sin t\cos^3 t$

$\boxed{4\sin t\cos^3 t - 4\sin^3 t\cos t =}$

5) Verify: $2\tan\frac{x}{2} = \frac{\sin^2 x + 1 - \cos^2 x}{\sin x(1+\cos x)}$

$= \frac{\sin^2 x + \sin^2 x}{\sin x(1+\cos x)}$

$= \frac{2\sin^{\cancel{2}} x}{\cancel{\sin x}(1+\cos x)}$

$= \frac{2\sin x}{1+\cos x}$

$= 2\left(\frac{\sin x}{1+\cos x}\right)$

$= \boxed{2\tan\frac{x}{2}}$

5.5 Day 1: Trig Equations

1. Solve: $5\sin x = 3\sin x + \sqrt{3}$

$5\sin x - 3\sin x = \sqrt{3}$

$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

No restrictions on x (domain)

$\rightarrow n$ is any integer.

$\frac{\pi}{3} + 2n\pi$ or $\frac{2\pi}{3} + 2n\pi$

$\therefore \frac{\pi}{3}, \frac{2\pi}{3}$ + increments of 2π.

2. $\tan 2x = \sqrt{3}$ if $0 \leq x < 2\pi$

Domain Restrictions

(period is π for tan)

$\tan\sqrt{3} = \frac{\pi}{3}$

$2x = \frac{\pi}{3}$

$x = \frac{\pi}{6} + \frac{n\pi}{2}$

$n=0 \quad \frac{\pi}{6} + 0 = \boxed{\pi/6}$

$n=1 \quad \frac{\pi}{6} + \frac{\pi}{2} = \boxed{\frac{2\pi}{3}}$

$n=2 \quad \frac{\pi}{6} + \pi = \boxed{\frac{7\pi}{6}}$

$n=3 \quad \frac{\pi}{6} + \frac{3\pi}{2} = \boxed{\frac{5\pi}{3}}$

3. $\sin\frac{x}{3} = \frac{1}{2}$ if $0 \leq x < 2\pi$

Domain Restriction

$\sin = \frac{1}{2}$ @ $\frac{\pi}{6}$ & $\frac{5\pi}{6}$

$\frac{x}{3} = \frac{\pi}{6} \qquad \frac{x}{3} = \frac{5\pi}{6}$

$x = \frac{\pi}{2} + 2\pi n \qquad x = \frac{5\pi}{2} + 2\pi n$

not in interval $0 \rightarrow 2\pi$

Let $n=1 \quad \frac{\pi}{2}$ & $\cancel{\frac{5\pi}{2}}$

* if $n=2$ then we are adding 4π which is

4. $2 \sin^2 x - 3 \sin x + 1 = 0, \quad 0 \leq x < 2\pi$

$(2\sin x - 1)(\sin x - 1)$

$\sin x = \frac{1}{2} \qquad \sin x = 1$

$x = \pi/6 \qquad x = \pi/2$

$$\boxed{\pi/6,\ \pi/2,\ 5\pi/6}$$

5. $4 \cos^2 x - 3 = 0, \quad 0 \leq x < 2\pi$

$\sqrt{\cos^2 x} = \sqrt{\frac{3}{4}}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2} \qquad \cos x = -\frac{\sqrt{3}}{2}$

$$\boxed{\pi/6 \ ; \ 11\pi/6 \qquad 5\pi/6 \ ; \ 7\pi/6}$$

6. $\sin x \tan x = \sin x, \quad 0 \leq x < 2\pi$

$\sin x \tan x - \sin x = 0$

$\sin x(\tan x - 1) = 0$

$\sin x = 0 \qquad \tan x = 1$

$x = 0 \quad x = \pi \qquad x = \pi/4,\ 5\pi/4$

$$\boxed{0,\ \pi/4,\ \pi,\ 5\pi/4}$$

7. $2\sin^2 x - 3\cos x = 0, \quad 0 \le x < 2\pi$

$2(1-\cos^2 x) - 3\cos x = 0$

$2 - 2\cos^2 x - 3\cos x = 0$

$2\cos^2 x + 3\cos x - 2 = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$\cos x = 1/2 \qquad \cancel{\cos x = -2}$

$\boxed{\pi/3,\ 5\pi/3}$

8. $\cos 2x + \sin x = 0, \quad 0 \le x < 2\pi$

$1 - 2\sin^2 x + \sin x = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -1/2 \qquad \sin x = 1$

$x = \pi + \pi/6 = 7\pi/6 \qquad x = \pi/2$

$x = 2\pi - \pi/6 = 11\pi/6$

$\boxed{\pi/2,\ 7\pi/6,\ 11\pi/6}$

9. $\sin x \cos x = -\frac{1}{2} \quad 0 \le x < 2\pi$

$2\sin x \cos x = -1$

$\sin 2x = -1$

$2x = \frac{3\pi}{2} + 2\pi n$

$x = \frac{3\pi}{4} + \pi n$

$\boxed{\frac{3\pi}{4} \ \& \ \frac{7\pi}{4}}$

*

10. $\cos x - \sin x = -1 \quad 0 \le x < 2\pi$

when you square, you must check for extranious roots

$(\cos x - \sin x)^2 = (-1)^2$

$\cos^2 x - 2\cos\sin + \sin^2 = 1$

$1 - 2\cos\sin = 1$

$\cos x \sin x = 0$ * look into.

$\cos x = 0$ $\quad$ $\sin x = 0$

$\pi/2$ $\quad$ $\cancel{0}$

$\cancel{3\pi/2}$ $\quad$ π

$\boxed{\pi/2, \pi}$

Check for extraneous

$\cancel{0}$: $\cos 0 - \sin 0 = -1$

$1 = -1$

$\pi/2$: $\cos \pi/2 - \sin \pi/2 = -1$

$-1 = -1$

π: $\cos \pi - \sin \pi = -1$

$-1 = -1$

$\cancel{3\pi/2}$: $\cos\frac{3\pi}{2} - \sin\frac{3\pi}{2} = -1$

$1 = -1$

11. $\cos^2 x + 5\cos x + 3 = 0, \quad 0 \le x < 2\pi$ to four decimals

$$\frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)}$$

$$\frac{-5 \pm \sqrt{13}}{2}$$

-0.6972 or $\cancel{-4.3028}$ since $\cos x$ is not < 1

$\cos x = -0.6972$

$\cos^{-1}(-0.6972) = 0.7993$

* calc in radians

calc gives QI numbers

cos is neg in II & III

$\pi - 0.7993 \quad \pi + 0.7993$

$\boxed{x = 2.3423 \; \& \; 3.9409}$

5.5 Day 2: Trig Equations

1. Use substitution to determine whether the given x-value is a solution: $\cos x = -\frac{1}{2}$, $x = \frac{2\pi}{3}$

$\cos\frac{2\pi}{3} = -\frac{1}{2}$

$-\frac{1}{2} = -\frac{1}{2}$ ✓ $\qquad \frac{2\pi}{3}$ is a solution

2. Solve $2\cos x + \sqrt{3} = 0$

$\cos x = \frac{-\sqrt{3}}{2}$

$x = \frac{5\pi}{6}$ & $\frac{7\pi}{6} + 2n\pi$

3. Solve on the interval $[0, 2\pi)$ $\quad \sin 4x = -\frac{\sqrt{2}}{2}$

$\sin -\frac{\sqrt{2}}{2}$ @ $\frac{5\pi}{4}$ & $\frac{7\pi}{4}$ $\qquad + \frac{2\pi n}{4} = \frac{+\pi n}{2}$

$4x = \frac{5\pi}{4}$ $\qquad 4x = \frac{7\pi}{4}$

$x = \frac{5\pi}{16} + \frac{8\pi n}{16}$ $\quad x = \frac{7\pi}{16} + \frac{8\pi n}{16}$ for $n = 1, 2, 3, 0$

$\frac{5\pi}{16}, \frac{7\pi}{16}, \frac{13\pi}{16}, \frac{15\pi}{16}, \frac{21\pi}{16}, \frac{23\pi}{16}, \frac{29\pi}{16}$ & $\frac{31\pi}{16}$

4. $\cos^2 x + 2\cos x - 3 = 0, \qquad [0, 2\pi)$

$(\cos x - 1)(\cos x + 3) = 0$

$\cos x = 1 \qquad \cancel{\cos x = -3}$

$\boxed{x = 0}$

5. $9\tan^2 x - 3 = 0 \qquad [0, 2\pi)$

$\sqrt{\tan^2 x} = \sqrt{\frac{3}{9}}$

$\tan = \pm\frac{\sqrt{3}}{3} + \pi n > \frac{6\pi}{6}$

$\boxed{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}}$

6. $\cot x\,(\tan x + 1) = 0 \qquad [0, 2\pi)$

$\cot x = 0 \qquad \tan x = -1$

$\cancel{\pi/2, 3\pi/2} \qquad \boxed{3\pi/4, 7\pi/4}$

$\uparrow$

tan is undefined

7. $4\sin^2 x + 4\cos x - 5 = 0 \qquad [0, 2\pi)$

$$4(1-\cos^2 x) + 4\cos x - 5 = 0$$

$$4 - 4\cos^2 x + 4\cos x - 5 = 0$$

$$4\cos^2 x - 4\cos x + 1 = 0$$

$$(2\cos x - 1)(2\cos x - 1) = 0$$

$$\cos x = \frac{1}{2}$$

$$\boxed{\pi/3,\ 5\pi/3}$$

8. $\cos 2x = \cos x \qquad [0, 2\pi)$

$$2\cos^2 x - 1 - \cos x = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$2\cos x + 1 = 0 \qquad \cos x = 1$$

$$\cos x = -1/2$$

$$\boxed{2\pi/3,\ 4\pi/3,\ \text{?}\ 0}$$

9. *Use a calculator to solve:* $4\tan^2 x - 8\tan x + 3 = 0$ $[0, 2\pi)$ *four decimals*

$(2\tan x - 1)(2\tan x - 3) = 0$

$\tan x = \frac{1}{2}$ $\tan x = \frac{3}{2}$

$$\boxed{\begin{array}{ll} 0.4636, & 0.9828, \\ 3.6052, & 4.1244 \end{array}}$$

10. $\cos x - 5 = 3\cos x + 6$

$-2\cos x = 11$

$\cos x = -11/2$ (cos can't be < -1)

No solution