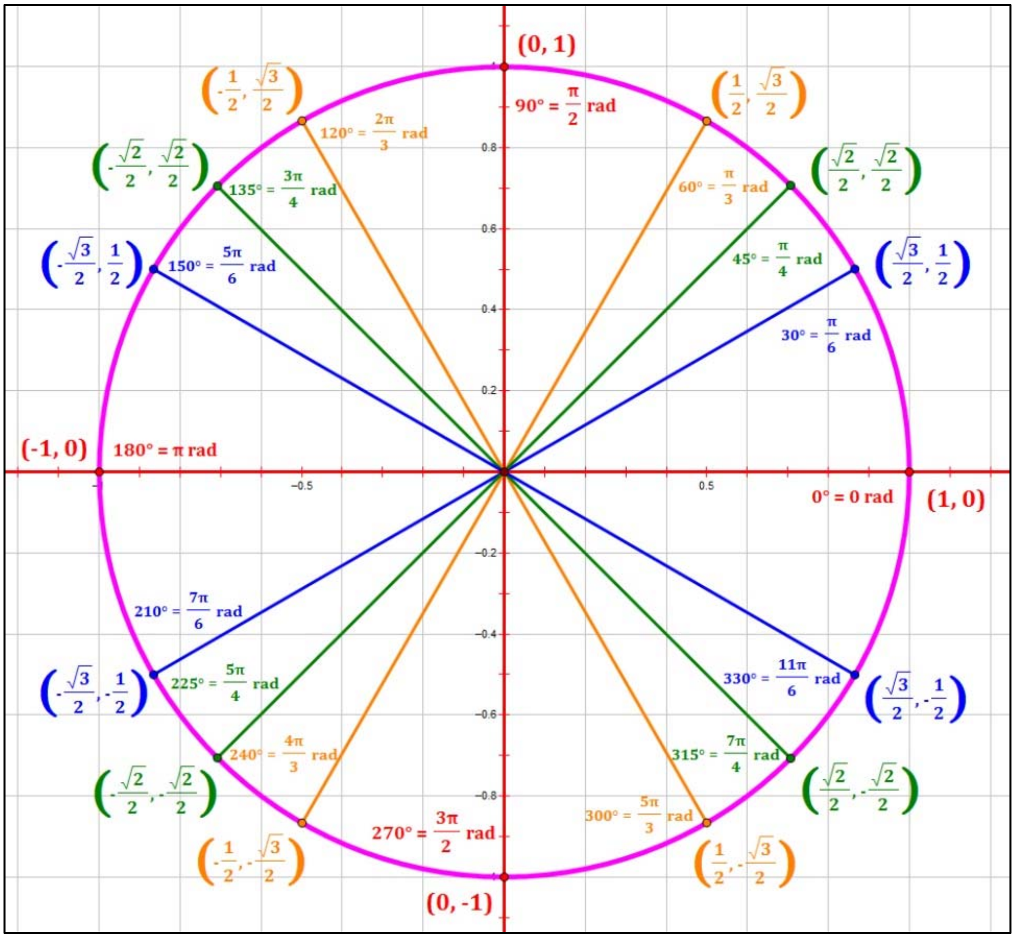
First, a couple of things to help out:



Trig Functions of Special Angles (θ)				
Radians	**Degrees**	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0°	$\frac{\sqrt{0}}{2}=0$	$\frac{\sqrt{4}}{2}=1$	$\frac{\sqrt{0}}{\sqrt{4}}=0$
$^{\pi}/_{6}$	30°	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
$^{\pi}/_{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{\sqrt{2}}=1$
$^{\pi}/_{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{3}}{\sqrt{1}}=\sqrt{3}$
$^{\pi}/_{2}$	90°	$\frac{\sqrt{4}}{2}=1$	$\frac{\sqrt{0}}{2}=0$	*undefined*

Signs of Trig Functions by Quadrant

sin + cos - tan -	sin + cos + tan +	
		x
sin - cos - tan +	sin - cos + tan -	
	y	

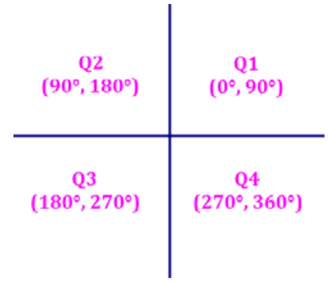
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

The given angle is in standard position. Determine the quadrant in which the angle lies.

1) -226° 1) ____________

The easiest way to deal with a negative angle is to identify its smallest positive coterminal angle. To do this, add $360°$ repeatedly until the value of the angle becomes positive.

$-226° + 360° = 134°$



Next, determine which quadrant the new angle is in based on the figure to the right. In the case of this problem, we see that $134°$ is in the upper left quadrant, that is, **Quadrant 2**.

Find the radian measure of the central angle of a circle of radius r that intercepts an arc of length s.

2) $r = \frac{1}{5}$ feet, $s = 4$ feet 2) ____________

The formula for the measure of an arc is: $s = r\theta$, where θ is the measure of the central angle. So,

$$s = r\theta \quad \Rightarrow \quad 4 = \frac{1}{5}\theta \quad \Rightarrow \quad \boldsymbol{\theta = 20 \text{ radians}}$$

Convert the angle in degrees to radians. Express answer as a multiple of π.

3) 144° 3) ____________

$180° = \pi$ radians. Let's use proportions to calculate the desired angle in radians.

$$\frac{144}{180} = \frac{x}{\pi} \quad \Rightarrow \quad \frac{144}{180}\pi = x \quad \Rightarrow \quad \boldsymbol{\frac{4}{5}\pi = x}$$

Convert the angle in radians to degrees.

4) $\frac{5}{4}\pi$ 4) ____________

Use proportions to calculate the desired angle in degrees.

$$\frac{\frac{5}{4}\pi}{\pi} = \frac{x}{180} \quad \Rightarrow \quad \frac{5}{4} = \frac{x}{180} \quad \Rightarrow \quad \frac{5}{4} \cdot 180 = x \quad \Rightarrow \quad \boldsymbol{225° = x}$$

Find a positive angle less than 360° or 2π that is coterminal with the given angle.

5) 558° 5) ____________

To get the desired coterminal angle, add or subtract $360°$ repeatedly until the value of θ is in the interval $(0°, 360°)$.

$558° - 360° = \mathbf{198°}$

6) $\frac{21\pi}{10}$ 6) ____________

To get the desired coterminal angle, add or subtract 2π repeatedly until the value of θ is in the interval $(0, 2\pi)$.

$$\frac{21\pi}{10} - 2\pi = \frac{21\pi}{10} - \frac{20\pi}{10} = \boldsymbol{\frac{\pi}{10}\ radians}$$

The point P(x, y) on the unit circle that corresponds to a real number t is given. Find the value of the indicated trigonometric function at t.

7) $\left(\frac{5}{8}, \frac{\sqrt{39}}{8}\right)$ Find tan t. 7) ____________

In coordinate terms, $\tan t = \frac{y}{x}$. So, we have:

$$\tan t = \frac{\frac{\sqrt{39}}{8}}{\frac{5}{8}} = \boldsymbol{\frac{\sqrt{39}}{5}}$$

Note: Denominators that are the same in the numerator and denominator of a compound fraction cancel each other out.

Solve the problem.

8) What is the domain of the sine function? 8) ____________

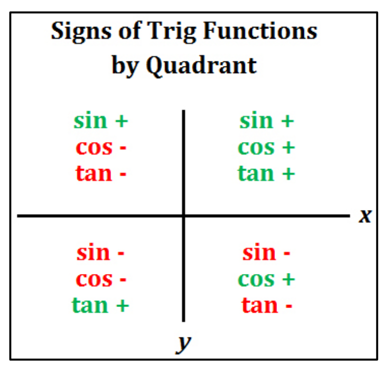
The domain is the set of all x-values that can be used in an equation. The sine function can accept any real value of x, so its domain is the set of all real numbers, i.e., $(-\infty, \infty)$ or $\mathbb{R}$.

Find the exact value of the trigonometric function. Do not use a calculator.

9) $\sec \frac{\pi}{4}$ 9) ____________

The angle $\frac{\pi}{4}$ is in Quadrant 1, where all Trig function values are positive.

$$\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \boldsymbol{\sqrt{2}}$$



10) $\csc \frac{3\pi}{4}$

The angle $\frac{3\pi}{4}$ is in Quadrant 2, where the sine (and, therefore, cosecant) function is positive. The reference angle (see the reference angle table on page 8) for $\frac{3\pi}{4}$ in Q2 is $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$. Then,

$$\csc\frac{3\pi}{4} = \frac{1}{\sin\frac{3\pi}{4}} = \frac{1}{\sin\frac{\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \boldsymbol{\sqrt{2}}$$

11) $\tan \frac{9\pi}{4}$ 11) ____________

The angle $\frac{9\pi}{4}$ is coterminal with the angle $\frac{9\pi}{4} - 2\pi = \frac{\pi}{4}$ in Quadrant 1, where all of the Trig function values are positive. Then,

$$\tan\frac{9\pi}{4} = \tan\frac{\pi}{4} = \mathbf{1}$$

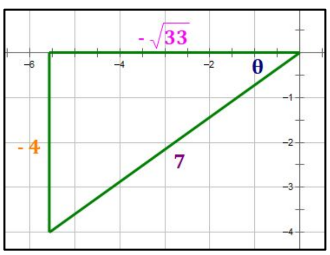
Sin t and cos t are given. Use identities to find the indicated value. Where necessary, rationalize denominators.

12) $\sin t = \frac{\sqrt{7}}{4}$, $\cos t = \frac{3}{4}$. Find sec t. 12) ____________

Nothing fancy here. $\sec t = \frac{1}{\cos t} = \mathbf{\frac{4}{3}}$

Find the exact value of the indicated trigonometric function of θ.

13) $\csc \theta = -\frac{7}{4}$, θ in quadrant III Find cot θ. 13) ____________



The key on this type of problem is to draw the correct triangle. We are given that θ is in $Q3$, where x is negative.

$$\csc\theta = -\frac{7}{4} \quad \Rightarrow \quad \sin\theta = \frac{y}{r} = \frac{-4}{7}$$

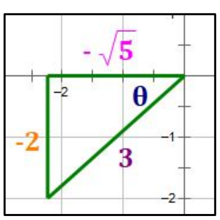
Note: r is always positive.

Then, the horizontal leg must be:

$$x = -\sqrt{7^2 - (-4)^2} = -\sqrt{33}. \quad \text{Then,}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y} = \frac{-\sqrt{33}}{-4} = \mathbf{\frac{\sqrt{33}}{4}}$$

14) $\sin \theta = -\frac{2}{3}$, $\tan \theta > 0$ Find sec θ. 14) ____________



Notice that $\sin\theta < 0$, $\tan\theta > 0$. Therefore θ is in $Q3$, where x is negative.

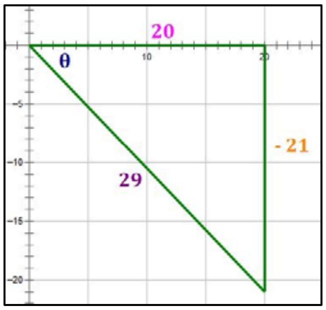
$$\sin\theta = \frac{y}{r} = \frac{-2}{3}$$

Then, the horizontal leg must be:

$$x = -\sqrt{3^2 - (-2)^2} = -\sqrt{5}. \quad \text{Then,}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x} = \frac{3}{-\sqrt{5}} = -\mathbf{\frac{3\sqrt{5}}{5}}$$

15) $\cos\theta = \frac{20}{29}$, $\frac{3\pi}{2} < \theta < 2\pi$ Find $\cot\theta$. 15) ____________



Notice that $\frac{3\pi}{2} < \theta < 2\pi$. Therefore θ is in $Q4$, where y is negative.

$$\cos\theta = \frac{x}{r} = \frac{20}{29}$$

Then, the vertical leg must be:

$$y = -\sqrt{29^2 - (20)^2} = -21. \quad \text{Then,}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y} = \frac{20}{-21} = -\frac{\mathbf{20}}{\mathbf{21}}$$

$0 \le t < \frac{\pi}{2}$ and sin t is given. Use the Pythagorean identity $\sin^2 t + \cos^2 t = 1$ to find cos t.

16) $\sin t = \frac{\sqrt{5}}{3}$ 16) ____________

Since $0 < \theta < \frac{\pi}{2}$, we know that θ is in $Q1$, so $\cos\theta$ is positive. Then,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{\sqrt{5}}{3}\right)^2 + \cos^2\theta = 1$$

$$\frac{5}{9} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{4}{9}$$

$$\cos\theta = +\sqrt{\frac{4}{9}} = \frac{\mathbf{2}}{\mathbf{3}}$$ since we established above that $\cos\theta$ is positive.

Use periodic properties of the trigonometric functions to find the exact value of the expression.

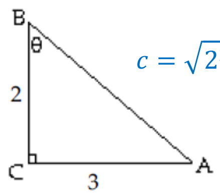
17) $\cos\frac{10\pi}{3}$ 17) ____________

$$\cos\left(\frac{10\pi}{3}\right) = \cos\left(\frac{10\pi}{3} - 2\pi\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{\mathbf{1}}{\mathbf{2}}$$

Further explanation: the reference angle for $\frac{4\pi}{3}$ in Q3 is: $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$ (see the reference angle table on page 8) and $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$. In Q3, the cosine function is negative, so the desired cosine value is $-\frac{1}{2}$

Use the Pythagorean Theorem to find the length of the missing side.Then find the indicated trigonometric function of the given angle. Give an exact answer with a rational denominator.

18) Find sec θ. 18) ______



$c = \sqrt{2^2 + 3^2} = \mathbf{\sqrt{13}}$

Let's find all six Trig functions

$$\sin\theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{2}{3}$$

$$\cos\theta = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13} \qquad \mathbf{\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{13}}{2}}$$

$$\tan\theta = \frac{3}{2} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{13}}{3}$$

Evaluate the expresssions.

19) $\sec\frac{\pi}{3} - \cos\frac{\pi}{6}$ 19) ______

$$\sec\frac{\pi}{3} - \cos\frac{\pi}{6} = \frac{1}{\cos\frac{\pi}{3}} - \cos\frac{\pi}{6}$$

$$= \frac{1}{\frac{1}{2}} - \frac{\sqrt{3}}{2} = 2 - \frac{\sqrt{3}}{2} = \mathbf{\frac{4-\sqrt{3}}{2}}$$

20) $\cos\frac{\pi}{6}$ 20) ______

This can be read right of the unit circle or the table at the bottom of page 1 of this packet (not to mention that we used this value in the previous problem).

$$\cos\frac{\pi}{6} = \mathbf{\frac{\sqrt{3}}{2}}$$

21) $\cos\frac{\pi}{3}\sec\frac{\pi}{3} - \cot\frac{\pi}{6}$ 21) ______

$$\cos\frac{\pi}{3}\cdot\sec\frac{\pi}{3} - \cot\frac{\pi}{6} = \cos\frac{\pi}{3}\cdot\frac{1}{\cos\frac{\pi}{3}} - \frac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}$$

$$= 1 - \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \mathbf{1 - \sqrt{3}}$$

Note: Denominators that are the same in the numerator and denominator of a compound fraction cancel each other out.

Find a cofunction with the same value as the given expression.

22) $\cos 62°$ 22) ____________

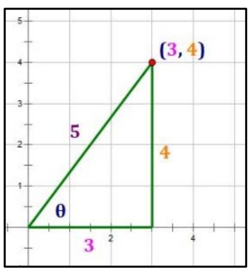
$$\cos 62° = \sin(90° - 62°) = \mathbf{\sin 28°}$$

23) $\tan \frac{\pi}{14}$ 23) ____________

$$\tan\frac{\pi}{14} = \cot\left(\frac{\pi}{2} - \frac{\pi}{14}\right) = \cot\left(\frac{7\pi}{14} - \frac{\pi}{14}\right) = \cot\left(\frac{6\pi}{14}\right) = \mathbf{\cot\left(\frac{3\pi}{7}\right)}$$

A point on the terminal side of angle θ is given. Find the exact value of the indicated trigonometric function of θ.

24) (3, 4) Find cos θ. 24) ____________



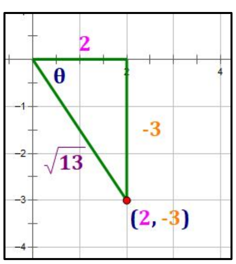
Let's find all six Trig functions

$$\sin\theta = \frac{4}{5} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$$

$$\mathbf{\cos\theta = \frac{3}{5}} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$$

$$\tan\theta = \frac{4}{3} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{5}{4}$$

25) (2, -3) Find sin θ. 25) ____________



Let's find all six Trig functions

$$\mathbf{\sin\theta = -\frac{3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}} \qquad \cot\theta = \frac{1}{\tan\theta} = -\frac{2}{3}$$

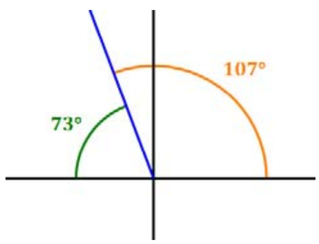
$$\cos\theta = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{13}}{2}$$

$$\tan\theta = -\frac{3}{2} \qquad \csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{13}}{3}$$

Reference Angle Formulas by Quadrant (reference angles are always less than 90° or $\frac{\pi}{2}$ radians)		
Quadrant	**In Degrees**	**In Radians**
Q1	$\rho = \theta$	$\rho = \theta$
Q2	$\rho = 180° - \theta$	$\rho = \pi - \theta$
Q3	$\rho = \theta - 180°$	$\rho = \theta - \pi$
Q4	$\rho = 360° - \theta$	$\rho = 2\pi - \theta$
Notation: ρ is the reference angle of angle θ. To use these formulas, θ must be positive and less than 360° or 2π radians.		

Find the reference angle for the given angle.

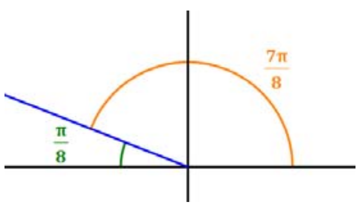
26) 107° 26) ____________



The reference angle is the angle between the original angle's terminal side and the x-axis. It must be between 0° and 90° (0 and $\frac{\pi}{2}$ radians).

In Quadrant 2, $\rho = 180° - 107° = \mathbf{73°}$

27) $\frac{7\pi}{8}$ 27) ____________



The reference angle is the angle between the original angle's terminal side and the x-axis. It must be between 0 and $\frac{\pi}{2}$ radians (0° and 90°).

In Quadrant 2, $\rho = \pi - \frac{7\pi}{8} = \boldsymbol{\frac{\pi}{8}}$

Find a cofunction with the same value as the given expression.

28) csc 52° 28) ____________

$\csc 52° = \sec(90° - 52°) = \mathbf{sec\,38°}$

Find the reference angle for the given angle.

29) $-\frac{2\pi}{3}$ 29) ____________

First, find the positive coterminal angle less than 2π radians.

$-\frac{2\pi}{3} + 2\pi = \frac{4}{3}\pi$. This angle is in Quadrant 3. Then, the reference angle is:

$\rho = \frac{4}{3}\pi - \pi = \boldsymbol{\frac{\pi}{3}}$

Use reference angles to find the exact value of the expression. Do not use a calculator.

30) $\cot \frac{-5\pi}{6}$ 30) ____________

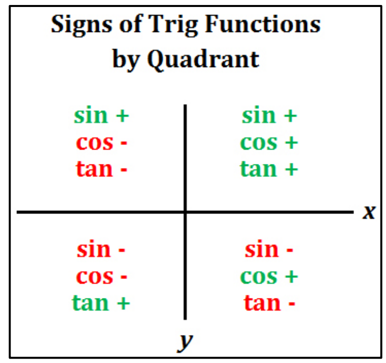
First, find the positive coterminal angle less than 2π radians.

$-\frac{5\pi}{6} + 2\pi = \frac{7}{6}\pi.$ This angle is in Quadrant 3. Then, the reference angle is:

$$\rho = \frac{7}{6}\pi - \pi = \frac{\pi}{6}$$

Next, using the reference angle:

$$\cot\left(\frac{\pi}{6}\right) = \frac{\cos\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$



Finally, consider the sign pattern of the cotangent function. It follows the sign pattern of the tangent function, which is positive in Quadrant 3. Therefore,

$$\cot\left(\frac{-5\pi}{6}\right) = \boldsymbol{\sqrt{3}}$$

31) $\tan \frac{-7\pi}{4}$ 31) ____________

First, find the positive coterminal angle less than 2π radians.

$-\frac{7\pi}{4} + 2\pi = \frac{\pi}{4}.$ This angle is in Quadrant 1. So, the reference angle is $\frac{\pi}{4}$:

Next, using the reference angle:

$$\tan\left(\frac{\pi}{4}\right) = 1$$

Finally, consider the sign pattern of the Tangent function. $-\frac{7\pi}{4}$ is in Quadrant 3, where the tangent function is positive. Therefore,

$$\tan\left(-\frac{7\pi}{4}\right) = \mathbf{1}$$

32) $\csc \frac{4\pi}{3}$ 32) ___________

The angle given, $\frac{4\pi}{3}$, is positive and less than 2π radians, so we can work with it directly. It is in Quadrant 3. Then, the reference angle is:

$$\rho = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$$

Next, using the reference angle:

$$\csc\left(\frac{\pi}{3}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Finally, consider the sign pattern of the cosecant function. It follows the sign pattern of the sine function, which is negative in Quadrant 3. Therefore,

$$\csc\left(\frac{4\pi}{3}\right) = -\frac{\mathbf{2\sqrt{3}}}{\mathbf{3}}$$

Solve the problem.

33) A car wheel has a 14-inch radius. Through what angle (to the nearest tenth of a degree) does the wheel turn when the car rolls forward 2 ft? 33) ___________

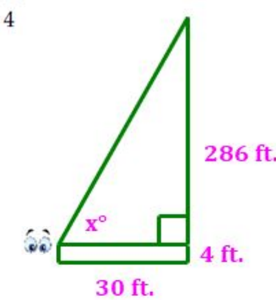
The formula for the measure of an arc is: $s = r\theta$, where θ is the measure of the central angle. So,

$$s = r\theta \quad \Rightarrow \quad 24 = 14\theta \quad \Rightarrow \quad \theta = \frac{12}{7} \text{ radians}$$

Use proportions to calculate the desired angle in degrees.

$$\frac{\frac{12}{7}}{\pi} = \frac{x}{180} \quad \Rightarrow \quad \frac{12 \cdot 180}{7\pi} = x \quad \Rightarrow \quad \mathbf{98.2^\circ = x}$$

34) A building 290 feet tall casts a 30 foot long shadow. If a person stands at the end of the shadow and looks up to the top of the building, what is the angle of the person's eyes to the top of the building (to the nearest hundredth of a degree)? (Assume the person's eyes are 4 feet above ground level.) 34) ___________



Assume that the person's eyes are 30 feet from the building. Then, the height that they are looking up is: $290 - 4 = 286$ ft. So,

$$\tan x = \frac{286}{30} \quad \Rightarrow \quad x = \tan^{-1}\left(\frac{286}{30}\right) = 1.466283 \text{ radians}$$

Use proportions to calculate the desired angle in degrees.

$$\frac{1.466283}{\pi} = \frac{x}{180} \quad \Rightarrow \quad \frac{1.466283 \cdot 180}{\pi} = x \quad \Rightarrow \quad \mathbf{84.01^\circ = x}$$