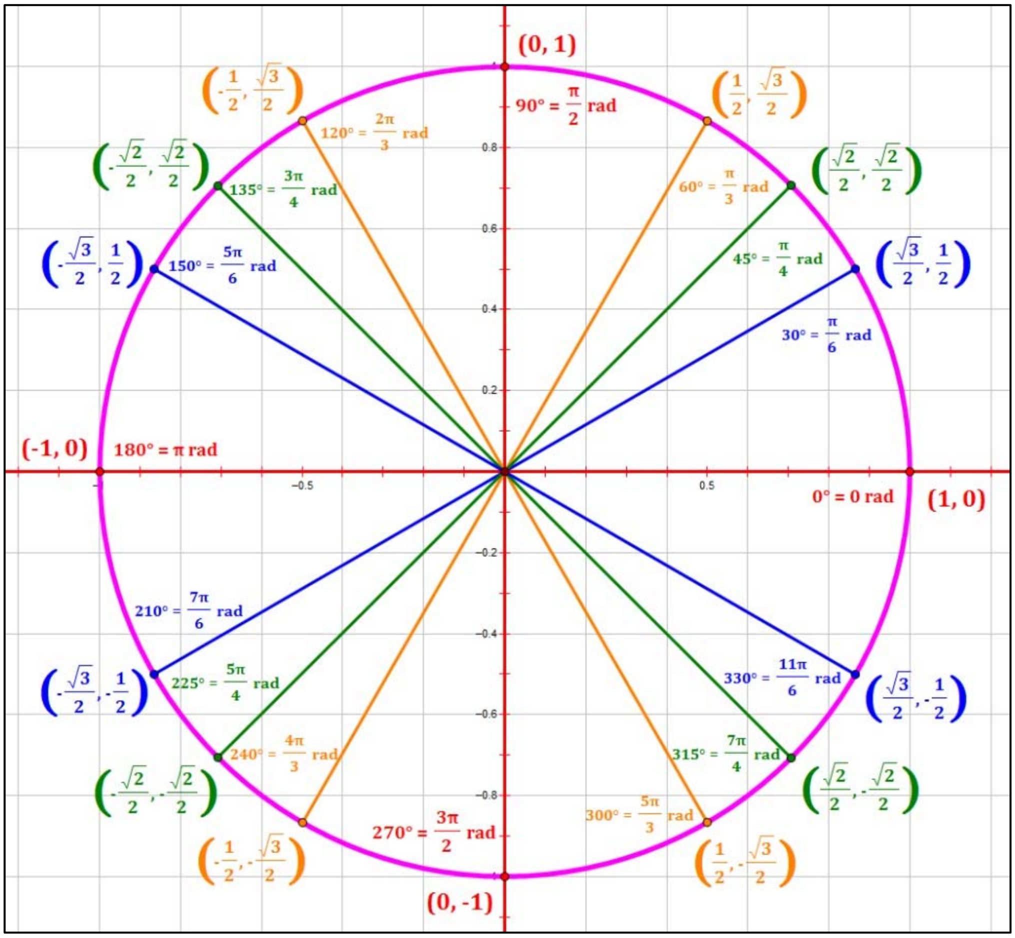
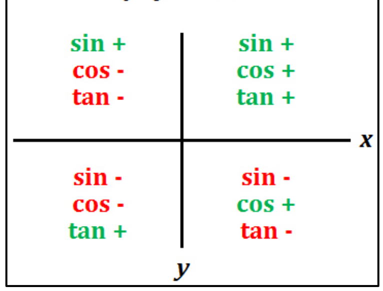
Trigonometry (Chapters 4-5) – Sample Test #1

First, a couple of things to help out:



Trig Functions of Special Angles (θ)				
Radians	Degrees	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0°	$\frac{\sqrt{0}}{2}=0$	$\frac{\sqrt{4}}{2}=1$	$\frac{\sqrt{0}}{\sqrt{4}}=0$
$\pi/6$	30°	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
$\pi/4$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{\sqrt{2}}=1$
$\pi/3$	60°	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{3}}{\sqrt{1}}=\sqrt{3}$
$\pi/2$	90°	$\frac{\sqrt{4}}{2}=1$	$\frac{\sqrt{0}}{2}=0$	*undefined*

Signs of Trig Functions by Quadrant



Use periodic properties of the trigonometric functions to find the exact value of the expression.

Use the unit circle or the chart at the front of this packet.

1. $\cos\frac{10\pi}{3} = \cos\left(\frac{10\pi}{3} - 2\pi\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

2. $\sin\frac{17\pi}{3} = \sin\left(\frac{17\pi}{3} - 4\pi\right) = \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

3. $\cot\left(-\frac{\pi}{3}\right) = \cot\left(2\pi + \left[-\frac{\pi}{3}\right]\right) = \cot\left(\frac{5\pi}{3}\right) = \frac{\cos\left(\frac{5\pi}{3}\right)}{\sin\left(\frac{5\pi}{3}\right)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

Sin t and cos t are given. Use identities to find the indicated value. Where necessary, rationalize denominators.

4. $\sin t = \frac{\sqrt{7}}{4}$, $\cos t = \frac{3}{4}$. Find $\sec t$.

Nothing fancy here. We don't even need a drawing. $\sec t = \frac{1}{\cos t} = \frac{4}{3}$

$\mathbf{0 \le t < \frac{\pi}{2}}$ **and cos t is given. Use the Pythagorean identity $\sin^2 t + \cos^2 t = 1$ to find sin t.**

5. $\cos t = \frac{\sqrt{14}}{4}$

$\sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \sin^2 t + \left(\frac{\sqrt{14}}{4}\right)^2 = 1$

$\sin^2 t + \frac{14}{16} = 1$

$\sin^2 t = \frac{2}{16}$ in Q1 $\quad \Rightarrow \quad \sin t = \frac{\sqrt{2}}{4}$

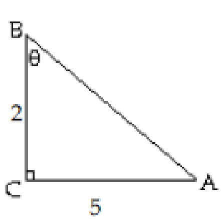
Find a cofunction with the same value as the given expression.

6. $\sin\frac{\pi}{19} = \cos\left(\frac{\pi}{2} - \frac{\pi}{19}\right) = \cos\left(\frac{17\pi}{38}\right)$

7. $\csc 52° = \sec(90° - 52°) = \sec(38°)$

$\sin\theta = \cos(90° - \theta)$	$\cos\theta = \sin(90° - \theta)$
$\tan\theta = \cot(90° - \theta)$	$\cot\theta = \tan(90° - \theta)$
$\sec\theta = \csc(90° - \theta)$	$\csc\theta = \sec(90° - \theta)$

Find all six trig functions for the angle θ.



8.

$\cdot\, 25 = \sqrt{29}$

$\sin\theta = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$

$\cos\theta = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$

$\tan\theta = \frac{5}{2}$

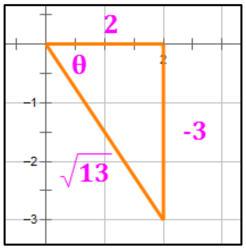
$\cot\theta = \frac{1}{\tan\theta} = \frac{2}{5}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{29}}{2}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{29}}{5}$

A point on the terminal side of angle θ is given. Find the exact value of the six trigonometric functions of θ.

9. $(2, -3)$



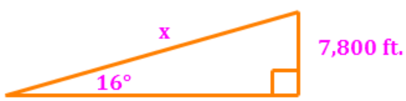
$$\sin\theta = -\frac{3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13} \qquad \cot\theta = \frac{1}{\tan\theta} = -\frac{2}{3}$$

$$\cos\theta = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{13}}{2}$$

$$\tan\theta = -\frac{3}{2} \qquad \csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{13}}{3}$$

Solve the problem.

10. A straight trail with a uniform inclination of 16° leads from a lodge at an elevation of 500 feet to a mountain lake at an elevation of 8,300 feet. What is the length of the trail (to the nearest foot)?

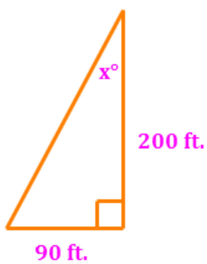


The height of the triangle is found by measuring the distance between the lake and the lodge. $8{,}300 - 500 = 7{,}800$.

$$\sin 16° = \frac{7{,}800}{x}$$

$$x = \frac{7{,}800}{\sin 16°} = \mathbf{28,298} \text{ ft.}$$

11. A building 200 feet tall casts a 90 foot long shadow. If a person looks down from the top of the building, what is the measure of the angle between the end of the shadow and the vertical side of the building (to the nearest degree)? (Assume the person's eyes are level with the top of the building.)

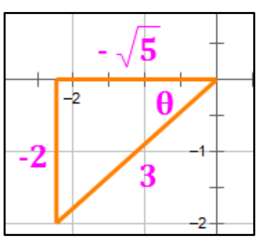


$$\tan x° = \frac{90}{200} = 0.45$$

$$x = \tan^{-1} 0.45 = \mathbf{24°}$$

Find the exact value of the indicated trigonometric function of θ.

$\sin\theta = -\frac{2}{3}, \quad \tan\theta > 0 \qquad$ Find $\sec\theta$. 12.

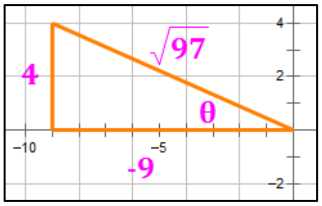


The key on this type of problem is to draw the correct triangle. Notice that $\sin\theta < 0$, $\tan\theta > 0$. Therefore θ is in $Q3$.

Notice that the horizontal leg must have length: $-\sqrt{3^2 - (-2)^2} = -\sqrt{5}$.

Then, $\sec\theta = \frac{1}{\cos\theta} = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$

13. $\cot\theta = -\frac{9}{4}, \quad \cos\theta < 0$ Find $\csc\theta$.



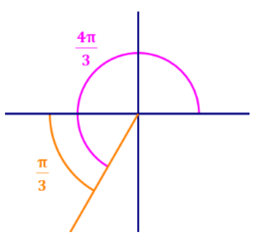
Notice that $\cot\theta < 0$, $\cos\theta < 0$. Therefore θ is in $Q2$.

The hypotenuse has length: $\sqrt{4^2 + (-9)^2} = \sqrt{97}$.

Then, $\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{97}}{4}$

Use reference angles to find the exact value of the expression. Do not use a calculator.

14. $\sin\left(\frac{4\pi}{3}\right)$



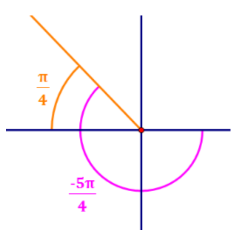
I like to draw the given angle so I can visualize the reference angle and the quadrant it is in.

$\frac{4\pi}{3}$ terminates in Q3. The reference angle is $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$.

The sine function is negative in Q3. So,

$$\sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

15. $\sec\left(\frac{-5\pi}{4}\right)$

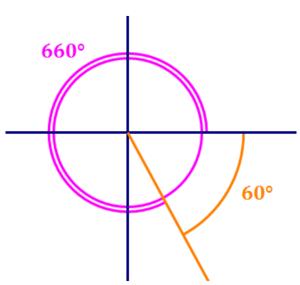


$\frac{-5\pi}{4}$ terminates in Q2. The reference angle is $\frac{\pi}{4}$.

The secant (and cosine) functions are negative in Q3. So,

$$\sec\left(\frac{-5\pi}{4}\right) = -\sec\left(\frac{\pi}{4}\right) = -\frac{1}{\cos\left(\frac{\pi}{4}\right)} = -\sqrt{2}$$

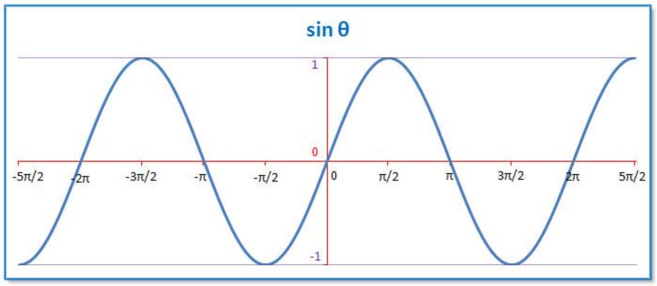
16. $\csc(660°)$

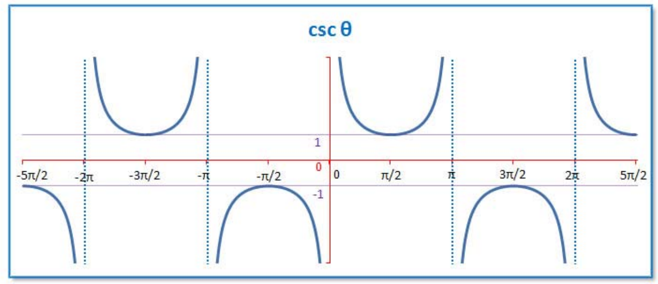


The given angle terminates in Q4. The reference angle is $720° - 660° = 60°$. Also, the cosecant (and sine) functions are negative in Q4. So,

$$\csc(660°) = -\csc(60°) = -\frac{1}{\sin(60°)} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

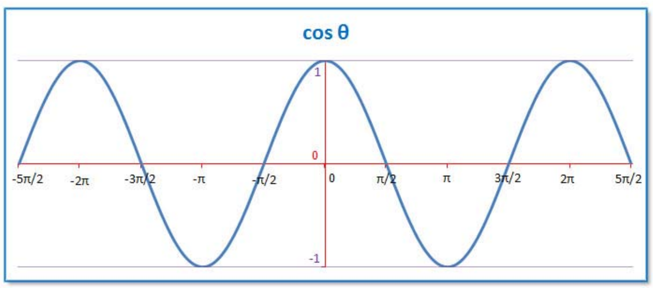
Six Functions – Reference Guide

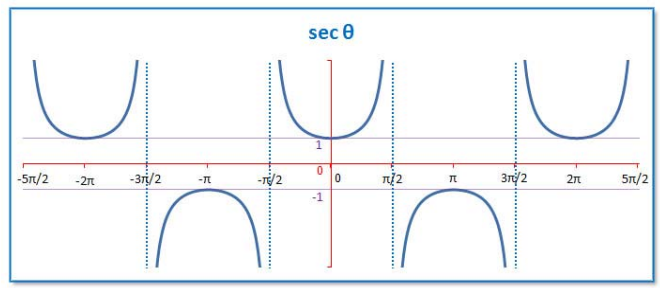




The sine and cosecant functions are inverses. So:

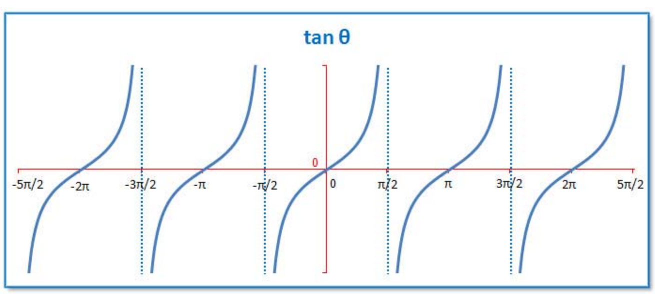
$$\sin\theta = \frac{1}{\csc\theta} \quad \text{and} \quad \csc\theta = \frac{1}{\sin\theta}$$

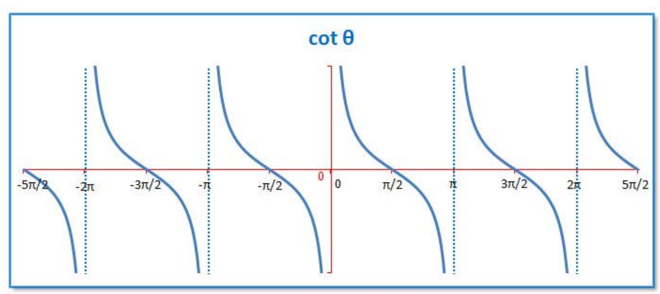




The cosine and secant functions are inverses. So:

$$\cos\theta = \frac{1}{\sec\theta} \quad \text{and} \quad \sec\theta = \frac{1}{\cos\theta}$$





The tangent and cotangent functions are inverses. So:

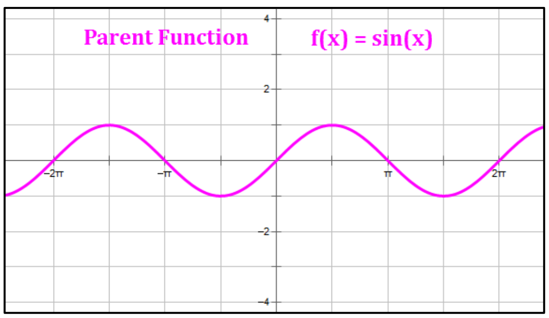
$$\tan\theta = \frac{1}{\cot\theta} \quad \text{and} \quad \cot\theta = \frac{1}{\tan\theta}$$

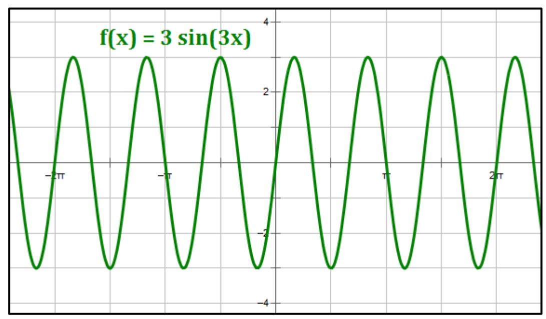
Graph the function.

17. $y = 3\sin 3x$ **Standard Form:** $y = A\sin(Bx - C) + D$

Characteristic	$y = \sin x$	$y = 3\sin 3x$
Amplitude $= \lvert A \rvert$	1	3
Period $= \frac{2\pi}{B}$	2π	$2\pi \div 3 = \frac{2}{3}\pi$
Phase Shift $= \frac{C}{B}$	0	0
Vertical Shift $= D$	0	0

Note: the problem does not require us to show the parent function. I show it for comparison purposes only.

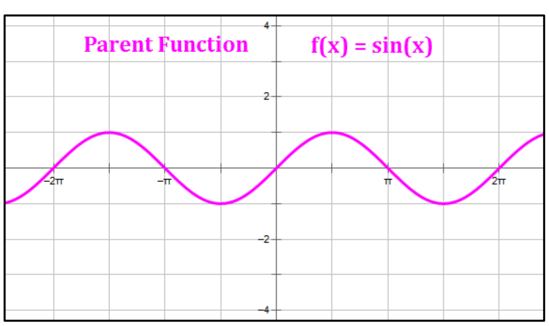


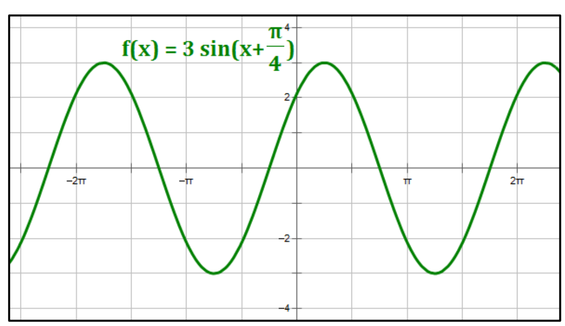


18. $y = 3\sin\left(x + \frac{\pi}{4}\right)$ **Standard Form:** $y = A\sin(Bx - C) + D$

Characteristic	$y = \sin x$	$y = 3\sin\left(x + \frac{\pi}{4}\right)$
Amplitude $= \lvert A \rvert$	1	3
Period $= \frac{2\pi}{B}$	2π	$2\pi \div 1 = 2\pi$
Phase Shift $= \frac{C}{B}$	0	$-\pi/4$
Vertical Shift $= D$	0	0

Note: the problem does not require us to show the parent function. I show it for comparison purposes only.

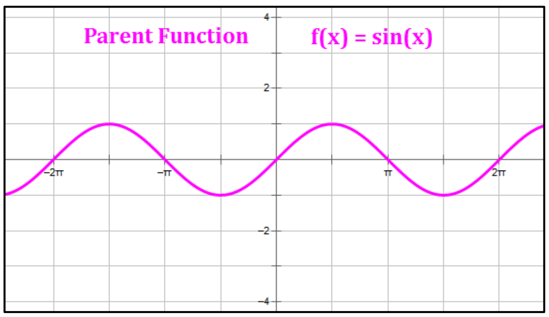


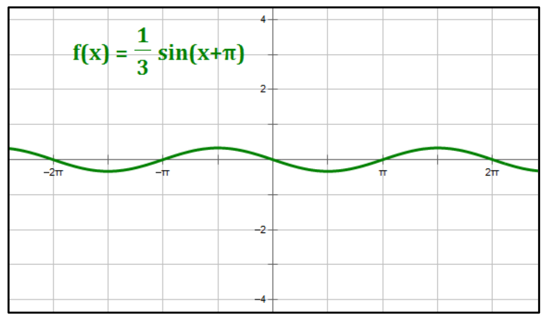


19. $y = \frac{1}{3}\sin(x + \pi)$ **Standard Form: $y = A\sin(Bx - C) + D$**

Characteristic	$y = \sin x$	$y = \frac{1}{3}\sin(x + \pi)$
Amplitude $= \lvert A \rvert$	1	1/3
Period $= \frac{2\pi}{B}$	2π	$2\pi \div 1 = 2\pi$
Phase Shift $= \frac{C}{B}$	0	$-\pi$
Vertical Shift $= D$	0	0

Note: the problem does not require us to show the parent function. I show it for comparison purposes only.

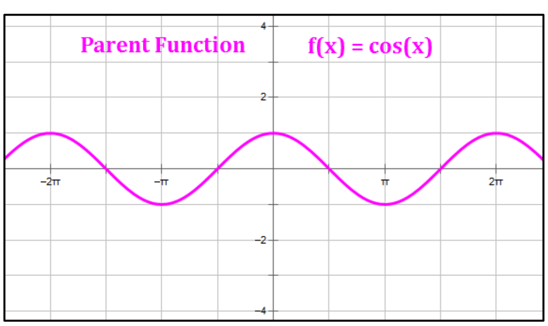


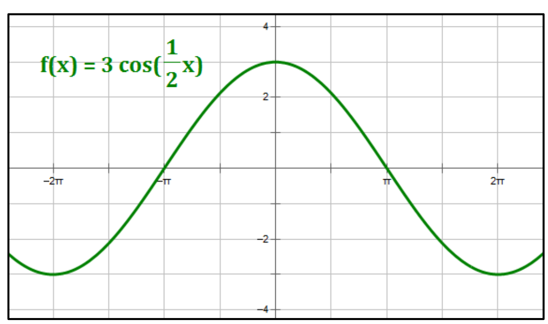


20. $y = 3\cos\left(\frac{1}{2}x\right)$ **Standard Form: $y = A\cos(Bx - C) + D$**

Characteristic	$y = \cos x$	$y = 3\cos\left(\frac{1}{2}x\right)$
Amplitude $= \lvert A \rvert$	1	3
Period $= \frac{2\pi}{B}$	2π	$2\pi \div \frac{1}{2} = 4\pi$
Phase Shift $= \frac{C}{B}$	0	0
Vertical Shift $= D$	0	0

Note: the problem does not require us to show the parent function. I show it for comparison purposes only.

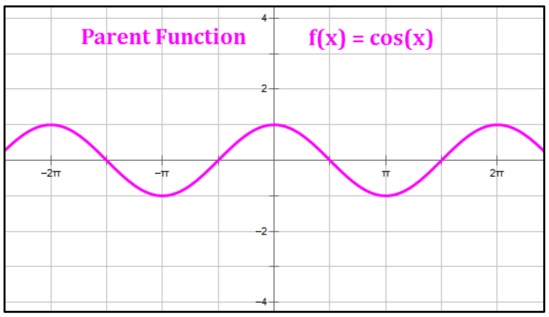


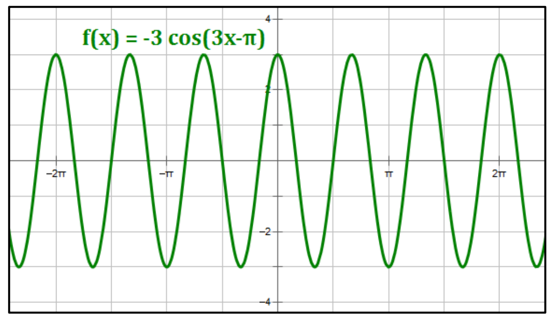


21. $y = -3\ \cos(3x - \pi)$ **Standard Form:** $y = A\cos(Bx - C) + D$

Characteristic	$y = \cos x$	$y = -3\ \cos(3x - \pi)$
Amplitude $= \|A\|$	1	3 (negative)
Period $= \frac{2\pi}{B}$	2π	$2\pi \div 3 = \frac{2}{3}\pi$
Phase Shift $= \frac{C}{B}$	0	$\pi/3$
Vertical Shift $= D$	0	0

Note: the problem does not require us to show the parent function. I show it for comparison purposes only.

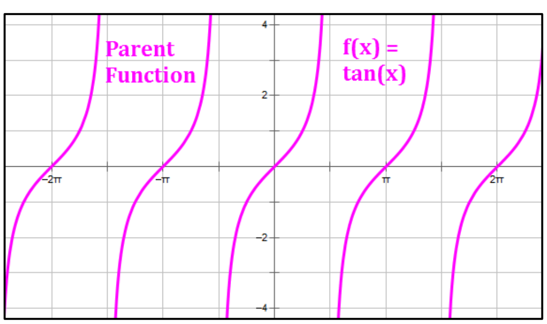


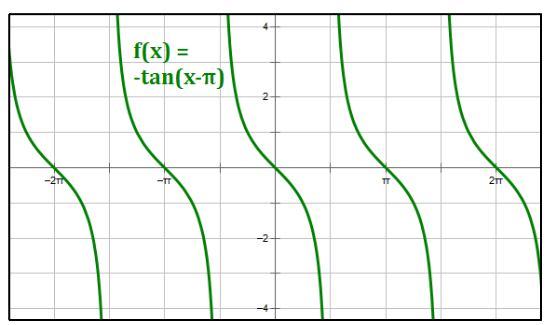


22. $y = -\tan(x - \pi)$ **Standard Form:** $y = A\tan(Bx - C) + D$

Characteristic	$y = \tan x$	$y = -\tan(x - \pi)$
Stretch $= \|A\|$	1	1 (negative)
Period $= \frac{\pi}{B}$	π	$\pi \div 1 = \pi$
Phase Shift $= \frac{C}{B}$	0	π
Vertical Shift $= D$	0	0

Note: the problem does not require us to show the parent function. I show it for comparison purposes only.

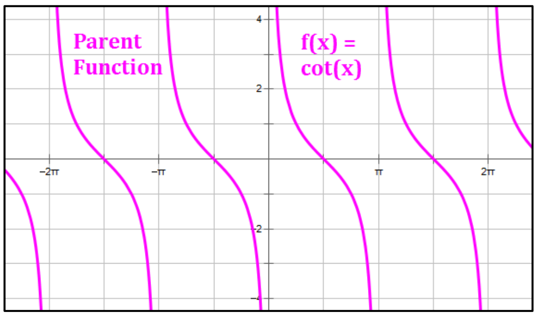


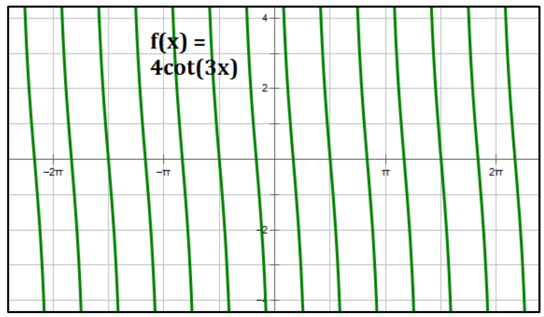


23. $y = 4\cot(3x)$ **Standard Form:** $y = A\cot(Bx - C) + D$

Characteristic	$y = \cot x$	$y = 4\cot(3x)$
Stretch $= \lvert A \rvert$	1	4
Period $= \frac{\pi}{B}$	π	$\pi \div 3 = \frac{1}{3}\pi$
Phase Shift $= \frac{C}{B}$	0	0
Vertical Shift $= D$	0	0

Note: the problem does not require us to show the parent function. I show it for comparison purposes only.

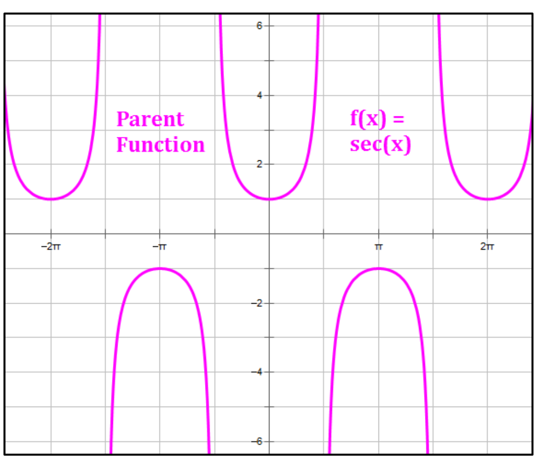


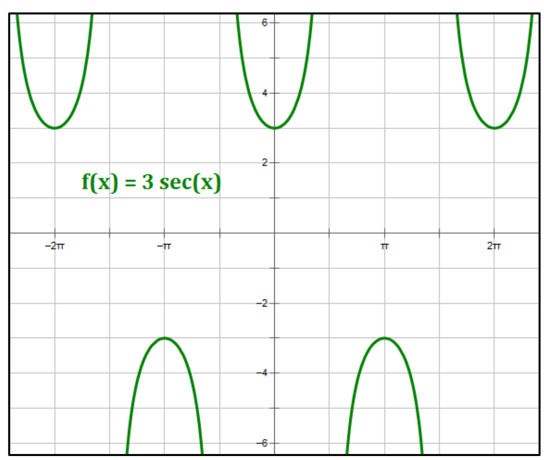


24. $y = 3\sec(x)$ **Standard Form:** $y = A\sec(Bx - C) + D$

Characteristic	$y = \sec x$	$y = 3\sec(x)$
Stretch $= \lvert A \rvert$	1	3
Period $= \frac{2\pi}{B}$	2π	$2\pi \div 1 = 2\pi$
Phase Shift $= \frac{C}{B}$	0	0
Vertical Shift $= D$	0	0

Note: the problem does not require us to show the parent function. I show it for comparison purposes only.



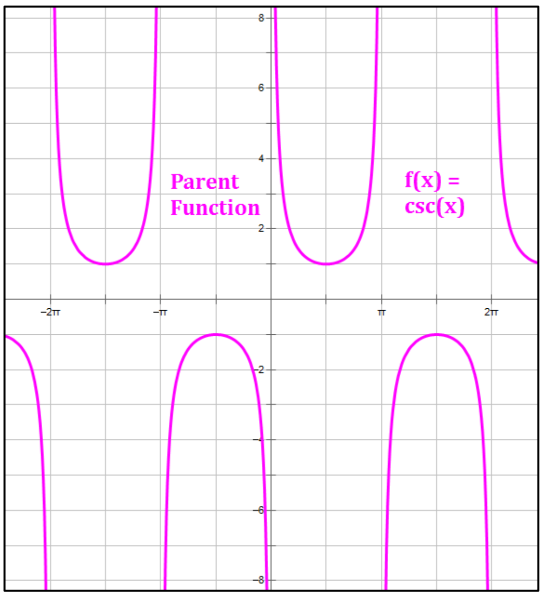


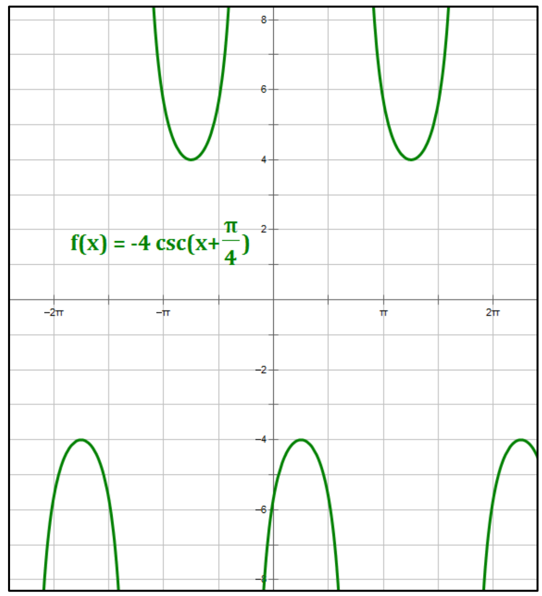
25. $y = -4\csc\left(x + \frac{\pi}{4}\right)$

Standard Form: $y = A\csc(Bx - C) + D$

Characteristic	$y = \csc x$	$y = -4\csc\left(x + \frac{\pi}{4}\right)$
Stretch $= \lvert A \rvert$	1	4 (negative)
Period $= \frac{2\pi}{B}$	2π	$2\pi \div 1 = 2\pi$
Phase Shift $= \frac{C}{B}$	0	$-\pi/4$
Vertical Shift $= D$	0	0

Note: the problem does not require us to show the parent function. I show it for comparison purposes only.





Find the exact value of the expression.

Use the unit circle or the chart at the front of this packet.

26. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

27. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

28. $\cos^{-1}(1) = \mathbf{0}$

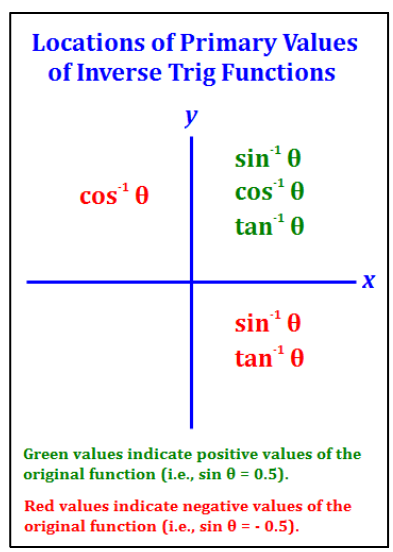
29. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

Know where the primary values for the inverse trig functions are defined.

$\sin^{-1}\theta$ is defined in Q1 and Q4.

$\cos^{-1}\theta$ is defined in Q1 and Q2.

$\tan^{-1}\theta$ is defined in Q1 and Q4.



Find the exact value of the expression, if possible. Do not use a calculator.

30. $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$

The angle $\frac{3\pi}{5}$ is in Q2, but tangent is defined only in Q1 and Q4. Further, $\tan\frac{3\pi}{5} < 0$ in Q2.

So we seek the angle in Q4, where tangent is also < 0, with the same tangent value as $\frac{3\pi}{5}$.

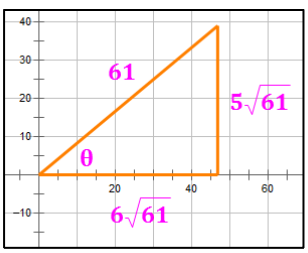
Recall that the tangent function has a period of π radians. Then,

$$\tan^{-1}\left(\tan\frac{3\pi}{5}\right) = \frac{3\pi}{5} - \pi = -\frac{2\pi}{5}$$

Use a sketch to find the exact value of the expression.

31. $\cot\left(\sin^{-1}\left[\frac{5\sqrt{61}}{61}\right]\right)$

First, calculate the horizontal leg of the triangle: $x = \sqrt{61^2 - \left(5\sqrt{61}\right)^2} = 6\sqrt{61}$. Then draw.

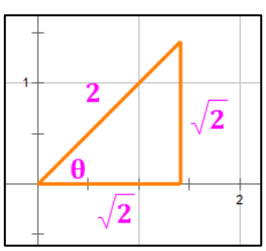


Based on the diagram, then,

$$\cot\left(\sin^{-1}\left[\frac{5\sqrt{61}}{61}\right]\right) = \cot\theta = \frac{6\sqrt{61}}{5\sqrt{61}} = \frac{6}{5}$$

32. $\cot\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$

First, calculate the horizontal leg of the triangle: $x = \sqrt{2^2 - \left(\sqrt{2}\right)^2} = \sqrt{2}$. Then draw.



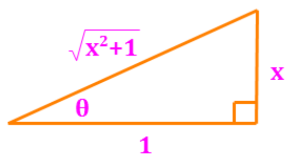
Based on the diagram, then,

$$\cot\left(\sin^{-1}\left[\frac{\sqrt{2}}{2}\right]\right) = \cot\theta = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

Use a right triangle to write the expression as an algebraic expression. Assume that x is positive and in the domain of the given inverse trigonometric function.

33. $\cos(\tan^{-1} x)$

Since the tangent value is x, let's set up a triangle with the side opposite θ equal to x, and the side adjacent to θ equal to 1. The hypotenuse, then is $\sqrt{x^2+1}$.

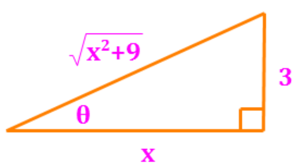


Then,

$$\cos(\tan^{-1} x) = \cos\theta = \frac{1}{\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}}{x^2+1}$$

34. $\sin\left(\sec^{-1}\left[\frac{\sqrt{x^2+9}}{x}\right]\right)$

The cosine of the angle is $\frac{x}{\sqrt{x^2+9}}$, so let's set up a triangle with the side adjacent to θ equal to x, and the hypotenuse equal to $\sqrt{x^2+9}$. The side opposite θ, then, would be 3 in order to have a right triangle.

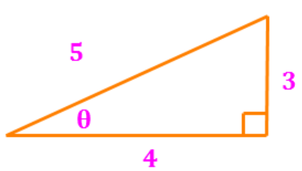


Then,

$$\sin\left(\sec^{-1}\left[\frac{\sqrt{x^2+9}}{x}\right]\right) = \sin\theta = \frac{3}{\sqrt{x^2+9}} = \frac{3\sqrt{x^2+9}}{x^2+9}$$

35. $\cos\left(\sin^{-1}\left[\frac{3}{5}\right]\right)$

The sine of the angle is $\frac{3}{5}$, so let's set up a triangle with the side opposite θ equal to 3, and the hypotenuse equal to 5. The side adjacent to θ, then, would be 4 in order to have a right triangle.



Then,

$$\cos\left(\sin^{-1}\left[\frac{3}{5}\right]\right) = \cos\theta = \frac{4}{5}$$

Find the exact value of the expression, if possible. Do not use a calculator.

36. $\sin^{-1}\left[\sin\left(\frac{4\pi}{7}\right)\right]$

The angle $\frac{4\pi}{7}$ is in Q2, but sine is defined only in Q1 and Q4. Further, $\sin\frac{4\pi}{7} > 0$ in Q2.

So we seek the angle in Q1, where sine is also > 0 with the same tangent value as $\frac{4\pi}{7}$.

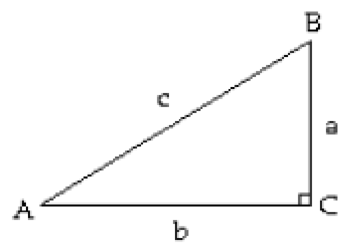
$$\sin^{-1}\left[\sin\left(\frac{4\pi}{7}\right)\right] = \pi - \frac{4\pi}{7} = \mathbf{\frac{3\pi}{7}}$$

Solve the right triangle shown in the figure. Round lengths to one decimal place and express angles to the nearest tenth of a degree.

37. a = 3.8 cm, b = 2.4 cm

$c = \sqrt{3.8^2 + 2.4^2} = \mathbf{4.5\ cm}$ $\quad m\angle A = \tan^{-1}\left(\frac{3.8}{2.4}\right) = \mathbf{57.7°}$

$m\angle B = 90° - 57.7° = \mathbf{32.3°}$



38. a = 3.3 in, A = 55.1°

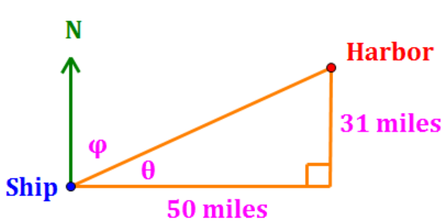
$\sin 55.1° = \frac{3.3}{c} \quad \Rightarrow \quad c = \frac{3.3}{\sin 55.1°} = \mathbf{4.0\ in}$

$\tan 55.1° = \frac{3.3}{b} \quad \Rightarrow \quad b = \frac{3.3}{\tan 55.1°} = \mathbf{2.3\ in}$

$m\angle B = 90° - 55.1° = \mathbf{34.9°}$

Using a calculator, solve the following problems. Round your answers to the nearest tenth.

39. A ship is 50 miles west and 31 miles south of a harbor. What bearing should the Captain set to sail directly to harbor?

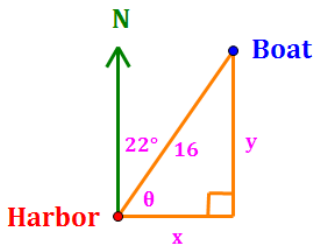


$\theta = \tan^{-1}\left(\frac{31}{50}\right) = 31.8°$

$\varphi = 90° - 31.8° = 58.2°$

Bearing = N 58.2° E

40. A boat leaves the entrance of a harbor and travels 16 miles on a bearing of N 22° E. How many miles north and how many miles east from the harbor has the boat traveled?



$\theta = 90° - 22° = 68°$

$x = 16 \cdot \cos 68° = \mathbf{6.0\ miles\ east}$

$y = 16 \cdot \sin 68° = \mathbf{14.8\ miles\ north}$

CHAPTER 5

Complete the identity.

41. $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = ?$

A) 1 + cot x B) sin x tan x C) sec x csc x D) -2 tan2 x

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin x}{\sin x} \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x\ \cos x}$$

$$= \frac{1}{\sin x\ \cos x} = \csc x\ \sec x = \mathbf{\sec x\ \csc x}$$

Answer C

42. $\tan x\,(\cot x - \cos x)\ = ?$

A) - sec2 x B) 1 - sin x C) 0 D) 1

$$\tan x\,(\cot x - \cos x)$$

$$= \frac{\sin x}{\cos x} \cdot \left(\frac{\cos x}{\sin x} - \cos x\right)$$

$$= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \cdot \cos x$$

$$= \quad 1 \quad - \quad \sin x$$

Answer B

43. $\dfrac{\cos x - \sin x}{\cos x} + \dfrac{\sin x - \cos x}{\sin x} = ?$

A) 1 - sec x csc x B) 2 - sec x csc x C) 2 + sec x csc x D) sec x csc x

$$\frac{\cos x - \sin x}{\cos x} + \frac{\sin x - \cos x}{\sin x}$$

$$= \frac{\sin x}{\sin x} \cdot \frac{\cos x - \sin x}{\cos x} + \frac{\sin x - \cos x}{\sin x} \cdot \frac{\cos x}{\cos x}$$

$$= \frac{\sin x \cos x - \sin^2 x + \sin x \cos x - \cos^2 x}{\sin x \cos x}$$

$$= \frac{2 \sin x \cos x - (\sin^2 x + \cos^2 x)}{\sin x \cos x} = \frac{2 \sin x \cos x - 1}{\sin x \cos x}$$

$$= 2 - \frac{1}{\sin x \cos x} = 2 - \csc x \sec x = \mathbf{2 - \sec x \csc x}$$

Answer B

44. cos (α + β) + cos (α - β) = ?

A) sin β cos α B) 2cos α cos β C) 2sin α cos β D) cos α cos β

$$\cos(\alpha + \beta) + \cos(\alpha - \beta)$$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$= 2 \cos\alpha \cos\beta$$

Answer B

45. $\dfrac{\sin(\alpha+\beta)}{\cos\alpha \cos\beta} = ?$

A) cot α + cot β B) tan β + tan α C) -tan α + cot β D) tan α + tan β

$$\frac{\sin(\alpha + \beta)}{\cos\alpha \cos\beta} = \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha \cos\beta} = \frac{\sin\alpha\cos\beta}{\cos\alpha \cos\beta} + \frac{\sin\beta\cos\alpha}{\cos\alpha \cos\beta}$$

$$= \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \mathbf{\tan\alpha + \tan\beta}$$

Answer D

46. $\frac{1+\cos 2x}{\sin 2x} = ?$

A) cos2 x B) sin2 x C) tan x D) cot x

$$\frac{1+\cos 2x}{\sin 2x} = \frac{1+2\cos^2 x - 1}{2\sin x\cos x} = \frac{2\cos^2 x}{2\sin x\cos x} = \frac{\cos x}{\sin x} = \mathbf{cot}\ \boldsymbol{x}$$ **Answer D**

47. $\sin\left(x+\frac{\pi}{2}\right) = ?$

A) -cos x B) -sin x C) sin x D) cos x

We might be tempted to use the angle addition formula for $\sin(\alpha+\beta)$ to solve this, but the form of the formula indicates that we would likely be barking up the wrong tree. So, the question boils down to which of the answers provided is correct.

A look at the graphs of the sine and cosine functions reveals that the cosine function is, in fact, equal to the sine function with a phase shift of $-\frac{\pi}{2}$ (i.e., $\frac{\pi}{2}$ to the left). Therefore the correct solution is:

$\mathbf{sin}\left(\boldsymbol{x}+\frac{\boldsymbol{\pi}}{\mathbf{2}}\right) = \mathbf{cos}\ \boldsymbol{x}$ **Answer D.**

Find the exact value by using a sum or difference identity.

$\sin\ (\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\sin\ (\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$\cos\ (\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

48. $\sin(215° - 95°) = \sin(210° - 90°)$

$$= \sin 210°\cos 90° - \sin 90°\cos 210° = \left(-\frac{1}{2}\right)(0) - (1)\left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$$

49. $\sin 165° = \sin(120° + 45°)$

$$= \sin 120°\cos 45° + \sin 45°\cos 120° = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) = \frac{\sqrt{2}\,(\sqrt{3}-1)}{4}$$

50. $\cos 285° = \cos(240° + 45°)$

$$= \cos 240°\cos 45° - \sin 240°\sin 45° = \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}\,(\sqrt{3}-1)}{4}$$

Find the exact value of the expression.

51. $\sin 265° \cos 25° - \cos 265° \sin 25° = \sin(265° - 25°) = \sin 240° = -\frac{\sqrt{3}}{2}$

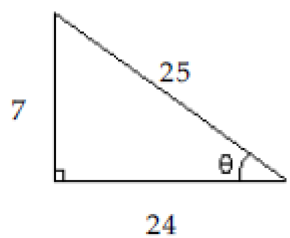
52. $\cos\frac{2\pi}{9}\sin\frac{\pi}{18} - \cos\frac{\pi}{18}\sin\frac{2\pi}{9} = \sin\left(\frac{\pi}{18} - \frac{2\pi}{9}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

53. $\sin 185° \cos 65° - \cos 185° \sin 65° = \sin(185° - 65°) = \sin 120° = \frac{\sqrt{3}}{2}$

Use the figure to find the exact value of sin 2θ, cos 2θ, and tan 2θ.

$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta \qquad \tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

54.

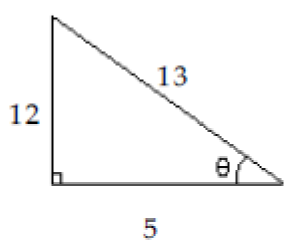


$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{7}{25}\cdot\frac{24}{25} = \frac{336}{625}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2 = \frac{527}{625}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{336}{527}$

55.



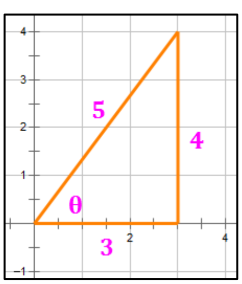
$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{12}{13}\cdot\frac{5}{13} = \frac{120}{169}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2 = -\frac{119}{169}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\frac{120}{119}$

Use the given information to find the sin 2θ, cos 2θ, and tan 2θ.

56. $\sin\theta = \frac{4}{5}$, and θ lies in quadrant I

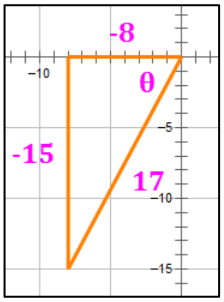


$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{4}{5}\cdot\frac{3}{5} = \frac{24}{25}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = -\frac{7}{25}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\frac{24}{7}$

57. $\tan\theta = \frac{15}{8}$, and θ lies in quadrant III



$$\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\left(-\frac{15}{17}\right)\cdot\left(-\frac{8}{17}\right) = \mathbf{\frac{240}{289}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{8}{17}\right)^2 - \left(-\frac{15}{17}\right)^2 = -\mathbf{\frac{161}{289}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\mathbf{\frac{240}{161}}$$

Write the expression as the sine, cosine, or tangent of a double angle. Then find the exact value of the expression.

58. $2\sin 120°\cos 120° = \sin(2\cdot 120°) = \mathbf{\sin 240°} = -\frac{\sqrt{3}}{2}$

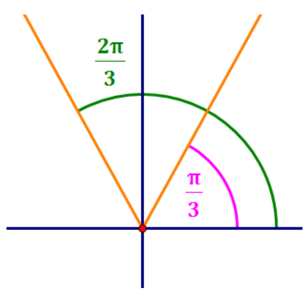
59. $\frac{2\tan\frac{5\pi}{8}}{1-\tan^2\frac{5\pi}{8}} = \tan\left(2\cdot\frac{5\pi}{8}\right) = \mathbf{\tan\left(\frac{5\pi}{4}\right) = 1}$

Find all solutions of the equation.

60. $2\sin x - \sqrt{3} = 0$

$2\sin x = \sqrt{3}$

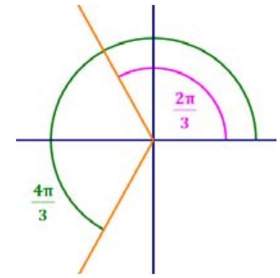
$\sin x = \frac{\sqrt{3}}{2}$



The drawing at left illustrates the two angles in $[0, 2\pi)$ for which $\sin x = \frac{\sqrt{3}}{2}$. To get all solutions, we need to add all integer multiples of 2π to these solutions. So,

$$\boldsymbol{x \in \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\}}$$

61. $\tan x \sec x = -2\tan x$



$\tan x \sec x + 2\tan x = 0$

$\tan x\,(\sec x + 2) = 0$

$\tan x = 0$ or $(\sec x + 2) = 0$

$(\sec x + 2) = 0$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

$x = 0 + n\pi = n\pi$

$x = \frac{2\pi}{3} + 2n\pi$ or $x = \frac{4\pi}{3} + 2n\pi$

Collecting the various solutions, $\boldsymbol{x \in \{n\pi\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}}$

Note: the solution involving the tangent function has two answers in the interval $[0, 2\pi)$. However, they are π radians apart, as most solutions involving the tangent function are. Therefore, we can simplify the answers by showing only one base answer and adding $n\pi$, instead of showing two base answers that are π apart, and adding $2n\pi$ to each.

For example, the following two solutions for $\tan x = 0$ are telescoped into the single solution given above:

$x = 0 + 2n\pi = \{\dots, -4\pi, -2\pi, 0, 2\pi, 4\pi, \dots\}$

$x = \pi + 2n\pi = \{\dots, -3\pi, -\pi, \pi, 3\pi, 5\pi \dots\}$

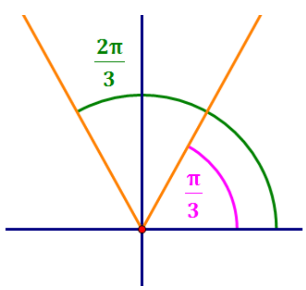
$\to x = 0 + n\pi = \{\dots, -2\pi, -\pi, 0, \pi, 2\pi, \dots\}$

Solve the equation on the interval [0, 2π).

62. $\sin 4x = \frac{\sqrt{3}}{2}$

When working with a problem in the interval $[0, 2\pi)$ that involves a function of kx, it is useful to expand the interval to $[0, 2k\pi)$ for the first steps of the solution.

So, we want all solutions to $\sin u = \frac{\sqrt{3}}{2}$ where $u = 4x$ is an angle in the interval $[0, 8\pi)$. Note that, beyond the two solutions suggested by the diagram, additional solutions are obtained by adding multiples of 2π to those two solutions.



Note that there are 8 solutions because the usual number of solutions (i.e., 2) is increased by a factor of $k = 4$.

Using the diagram at left, we get the following solutions:

$$u = 4x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}, \frac{19\pi}{3}, \frac{20\pi}{3}$$

Then, dividing by 4, we get:

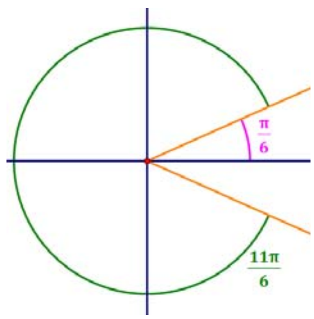
$$x = \frac{\pi}{12}, \frac{2\pi}{12}, \frac{7\pi}{12}, \frac{8\pi}{12}, \frac{13\pi}{12}, \frac{14\pi}{12}, \frac{19\pi}{12}, \frac{20\pi}{12}$$

And simplifying, we get:

$$\boldsymbol{x = \frac{\pi}{12}, \frac{\pi}{6}, \frac{7\pi}{12}, \frac{2\pi}{3}, \frac{13\pi}{12}, \frac{7\pi}{6}, \frac{19\pi}{12}, \frac{5\pi}{3}}$$

63. $\cos 2x = \frac{\sqrt{3}}{2}$

So, we want all solutions to $\cos u = \frac{\sqrt{3}}{2}$ where $u = 2x$ is an angle in the interval $[0, 4\pi)$. Note that, beyond the two solutions suggested by the diagram, additional solutions are obtained by adding 2π to those two solutions.



Note that there are 4 solutions because the usual number of solutions (i.e., 2) is increased by a factor of $k = 2$.

Using the diagram at left, we get the following solutions:

$$u = 2x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$$

Then, dividing by 2, we get:

$$\boldsymbol{x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}}$$

We cannot simplify these solutions any further.

64. $\cos^2 x + 2\cos x + 1 = 0$

The trick on this problem is to replace the trigonometric function, in this case, $\cos x$, with a variable, like u, that will make it easier to see how to factor the expression. If you can see how to factor the expression without the trick, by all means proceed without it.

Let $u = \cos x$, and our equation becomes: $u^2 + 2u + 1 = 0$.

This equation factors to get: $(u+1)^2 = 0$

Substituting $\cos x$ back in for u gives: $(\cos x + 1)^2 = 0$

And finally: $\cos x + 1 = 0 \quad \Rightarrow \quad \cos x = -1$

The only solution for this on the interval $[0, 2\pi)$ is: $\boldsymbol{x = \pi}$

65. $\cos x = \sin x$

This problem is most easily solved by inspection. Where are the cosine and sine functions equal? At the angles with a reference angle of $\frac{\pi}{4}$ in Q1 and Q3.

Therefore, $\boldsymbol{x = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}}$

Another method that can be used to solve this kind of problem is shown in the solution to Problem 66, below.

66. $\sin^2 x - \cos^2 x = 0$

$(\sin x + \cos x)\ (\sin x - \cos x) = 0$

$(\sin x + \cos x) = 0$ or $(\sin x - \cos x) = 0$

$\sin x = -\cos x$ | $\sin x = \cos x$

$\tan x = -1$ | $\tan x = 1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ | $x = \frac{\pi}{4}, \frac{5\pi}{4}$

$$\boldsymbol{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

In this problem, we take a different approach to solving $\sin x = \cos x$, which could, as in Problem 65, above, be solved by inspection. Since $\sin x$ and $\cos x$ are never both zero, we can divide both sides by $\cos x$ to get the resulting $\tan x$ equations.

67. $\sin^2 x + \sin x = 0$

$\sin x\ (\sin x + 1) = 0$

$\sin x = 0$ or $(\sin x + 1) = 0$

$x = 0, \pi$ | $\sin x = -1$

$x = \frac{3\pi}{2}$

$$\boldsymbol{x = 0, \pi, \frac{3\pi}{2}}$$

68. $\tan^2 x \sin x = \tan^2 x$

$\tan^2 x \sin x - \tan^2 x = 0$

$\tan^2 x\ (\sin x - 1) = 0$

$\tan x = 0$ or $(\sin x - 1) = 0$

$x = 0, \pi$ | $\sin x = 1$

$x = \frac{\pi}{2}$

$$\boldsymbol{x = 0, \pi}$$

Be extra careful when dealing with functions other than sine and cosine, because there are values at which these functions are undefined.

While $x = \frac{\pi}{2}$ is a solution to the equation $\sin x = 1$, $\tan x$ is undefined at $x = \frac{\pi}{2}$, so $\frac{\pi}{2}$ is not a solution to this equation.

69. $\cos x + 2\cos x \sin x = 0$

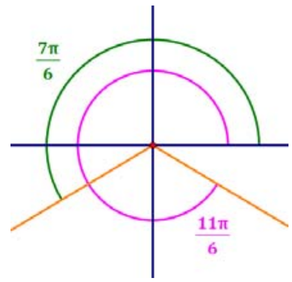
$$\cos x\ (1 + 2\sin x) = 0$$

$\cos x = 0$ or $(1 + 2\sin x) = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$



$$\boldsymbol{x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}}$$

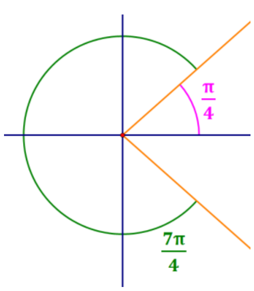
70. $\cos 2x = \sqrt{2} - \cos 2x$

$$2\cos 2x = \sqrt{2}$$

$$\cos 2x = \frac{\sqrt{2}}{2}$$

Recall that working with a problem in the interval $[0, 2\pi)$ that involves a function of kx, it is useful to expand the interval to $[0, 2k\pi)$ for the first steps of the solution.

So, we want all solutions to $\cos u = \frac{\sqrt{2}}{2}$ where $u = 2x$ is an angle in the interval $[0, 4\pi)$. Note that, beyond the two solutions suggested by the diagram, additional solutions are obtained by adding 2π to those two solutions.



Note that there are 4 solutions because the usual number of solutions (i.e., 2) is increased by a factor of $k = 2$.

Using the diagram at left, we get the following solutions:

$$u = 2x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}$$

Then, dividing by 2, we get:

$$\boldsymbol{x = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}}$$

We cannot simplify these answers any further.

71. $2\cos^2 x + \sin x - 2 = 0$

When an equation contains more than one function, try to convert it to one that contains only one function.

$$2\cos^2 x + \sin x - 2 = 0$$

$$2(1 - \sin^2 x) + \sin x - 2 = 0$$

$$2 - 2\sin^2 x + \sin x - 2 = 0$$

$$-2\sin^2 x + \sin x = 0$$

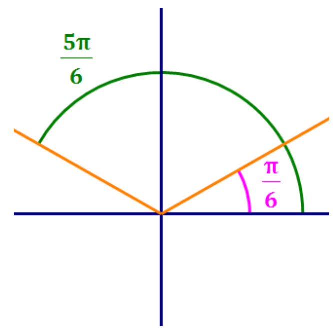
$$\sin x\,(-2\sin x + 1) = 0$$

$\sin x = 0$ or $(-2\sin x + 1) = 0$

$x = 0, \pi$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$



$$\boldsymbol{x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi}$$

72. $\cos\left(x + \frac{\pi}{3}\right) + \cos\left(x - \frac{\pi}{3}\right) = 1$

The following formulas will help us solve this problem.

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad\quad$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos\left(x + \frac{\pi}{3}\right) + \cos\left(x - \frac{\pi}{3}\right) = 1$$

$$\cos x\cos\frac{\pi}{3} - \sin x\sin\frac{\pi}{3} + \cos x\cos\frac{\pi}{3} + \sin x\sin\frac{\pi}{3} = 1$$

$$2\cos x\cos\frac{\pi}{3} = 1$$

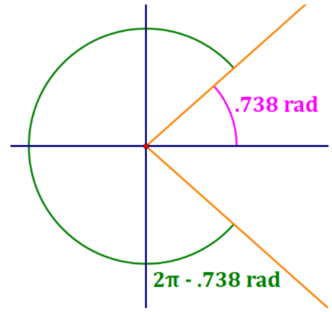
$$2\cos x \cdot \frac{1}{2} = 1$$

$$\cos x = 1$$

$$\boldsymbol{x = 0}$$

Use a calculator to solve the equation on the interval [0, 2π). Round the answer to two decimal places.

73. $\cos x = .74$



$x =$ 0.738 radians (by calculator)

$x = 2\pi - .738 = 6.283 - .738 =$ 5.545 radians

Rounding to 2 decimal places gives: $\boldsymbol{x = \{.74, 5.55\}}$

Use a calculator to solve the equation on the interval [0, 2π). Round to the nearest hundredth of a radian.

74. $\sin 2x - \sin x = 0$

$\sin 2x - \sin x = 0$

$2 \sin x \cos x - \sin x = 0$

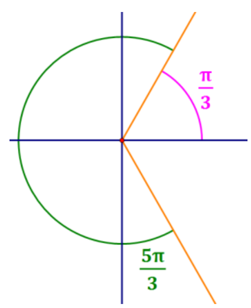
$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0$ or $(2\cos x - 1) = 0$

$x = 0, \pi$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$



$x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

Rounding to the nearest hundredth of a radian gives: $\boldsymbol{x = \{0, 1.05, 3.14, 5.24\}}$