PreCalculus

Exponent Formulas

Word Description of Property	Math Description of Property	Limitations on variables	Examples
Product of Powers	$a^p \cdot a^q = a^{(p+q)}$		$x^4 \cdot x^3 = x^7$ $x^5 \cdot x^{-8} = x^{-3}$
Quotient of Powers	$\frac{a^p}{a^q} = a^{(p-q)}$	$a \neq 0$	$\frac{y^5}{y^2} = y^3$
Power of a Power	$(a^p)^q = a^{(p \cdot q)}$		$(z^4)^3 = z^{12}$ $(x^{-3})^{-5} = x^{15}$
Anything to the zero power is 1	$a^0 = 1$	$a \neq 0$	$91^0 = 1$ $(xyz^3)^0 = 1,$ if $x, y, z \neq 0$
Negative powers generate the reciprocal of what a positive power generates	$a^{(-p)} = \frac{1}{a^p}$	$a \neq 0$	$x^{(-3)} = \frac{1}{x^3}$ $\left(\frac{1}{x}\right)^{-5} = x^5$
Power of a product	$(a \cdot b)^p = a^p \cdot b^p$		$(3y)^3 = 27y^3$ $[(x+1)z]^4 = (x+1)^4 z^4$
Power of a quotient	$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$	$b \neq 0$	$\left(\frac{x}{4}\right)^3 = \frac{x^3}{64}$
Converting a root to a power	$\sqrt[n]{a} = a^{(1/n)}$	$n \neq 0$	$\sqrt{x} = x^{1/2}$

PreCalculus

Logarithm Formulas

Word Description of Property	Math Description of Property	Limitations on variables	Examples
Definition of logarithm	$(\log_b x = a)$ implies $(b^a = x)$	$b, x > 0$ $b \neq 1$	$\log_3 x = 4$ implies $3^4 = x$ $\log_7(-49)$ is undefined
Log (base anything) of 1 is zero	$\log_b 1 = 0$	$b > 0$ $b \neq 1$	$\log_{32} 1 = 0$ $\ln 1 = 0$
Exponents and logs are inverse operators, leaving what you started with	$b^{(\log_b x)} = x$	$b, x > 0$ $b \neq 1$	$3^{(\log_3 92)} = 92$ $e^{(\ln x)} = x$
Logs and exponents are inverse operators, leaving what you started with	$\log_b(b^x) = x$	$b, x > 0$ $b \neq 1$	$\log_6(6^{xyz}) = xyz$ $\ln\ (e^{4y}) = 4y$
The log of a product is the sum of the logs	$\log_b(m \cdot n) = \log_b m + \log_b n$	$m, n, b > 0$ $b \neq 1$	$\log_2(32x) = 5 + \log_2 x$ $\ln\ (8e) = \ln(8) + 1$
The log of a quotient is the difference of the logs	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$	$m, n, b > 0$ $b \neq 1$	$\log_3\left(\frac{3}{x}\right) = 1 - \log_3 x$ $\ln\left(\frac{12}{e}\right) = \ln(12) - 1$
The log of something to a power is the power times the log	$\log_b(m^p) = p \cdot \log_b m$	$m, b > 0$ $b \neq 1$	$\log_4(x^{23}) = 23 \cdot \log_4 x$ $\ln\ (3^z) = z \cdot \ln(3)$
nge the base to whatever you want by dividing by the log of the old base	$\log_b m = \frac{\log_a m}{\log_a b}$	$m, a, b > 0$ $a, b \neq 1$	$\log_{100} x = \frac{\log_{10} x}{2}$

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Table of Exponents and Logarithms

Definition: $b^a = c$ if and only if $\log_b c = a$

$2^0 = 1$	$\log_2 1 = 0$	$6^0 = 1$	$\log_6 1 = 0$
$2^1 = 2$	$\log_2 2 = 1$	$6^1 = 6$	$\log_6 6 = 1$
$2^2 = 4$	$\log_2 4 = 2$	$6^2 = 36$	$\log_6 36 = 2$
$2^3 = 8$	$\log_2 8 = 3$	$6^3 = 216$	$\log_6 216 = 3$
$2^4 = 16$	$\log_2 16 = 4$		
$2^5 = 32$	$\log_2 32 = 5$	$7^0 = 1$	$\log_7 1 = 0$
$2^6 = 64$	$\log_2 64 = 6$	$7^1 = 7$	$\log_7 7 = 1$
$2^7 = 128$	$\log_2 128 = 7$	$7^2 = 49$	$\log_7 49 = 2$
$2^8 = 256$	$\log_2 256 = 8$	$7^3 = 343$	$\log_7 343 = 3$
$2^9 = 512$	$\log_2 512 = 9$		
$2^{10} = 1024$	$\log_2 1024 = 10$	$8^0 = 1$	$\log_8 1 = 0$
		$8^1 = 8$	$\log_8 8 = 1$
$3^0 = 1$	$\log_3 1 = 0$	$8^2 = 64$	$\log_8 64 = 2$
$3^1 = 3$	$\log_3 3 = 1$	$8^3 = 512$	$\log_8 512 = 3$
$3^2 = 9$	$\log_3 9 = 2$		
$3^3 = 27$	$\log_3 27 = 3$	$9^0 = 1$	$\log_9 1 = 0$
$3^4 = 81$	$\log_3 81 = 4$	$9^1 = 9$	$\log_9 9 = 1$
$3^5 = 243$	$\log_3 243 = 5$	$9^2 = 81$	$\log_9 81 = 2$
		$9^3 = 729$	$\log_9 729 = 3$
$4^0 = 1$	$\log_4 1 = 0$		
$4^1 = 4$	$\log_4 4 = 1$	$10^0 = 1$	$\log_{10} 1 = 0$
$4^2 = 16$	$\log_4 16 = 2$	$10^1 = 10$	$\log_{10} 10 = 1$
$4^3 = 64$	$\log_4 64 = 3$	$10^2 = 100$	$\log_{10} 100 = 2$
$4^4 = 256$	$\log_4 256 = 4$	$10^3 = 1000$	$\log_{10} 1000 = 3$
$5^0 = 1$	$\log_5 1 = 0$	$11^0 = 1$	$\log_{11} 1 = 0$
$5^1 = 5$	$\log_5 5 = 1$	$11^1 = 11$	$\log_{11} 11 = 1$
$5^2 = 25$	$\log_5 25 = 2$	$11^2 = 121$	$\log_{11} 121 = 2$
$5^3 = 125$	$\log_5 125 = 3$	$11^3 = 1331$	$\log_{11} 1331 = 3$
$5^4 = 625$	$\log_5 625 = 4$		

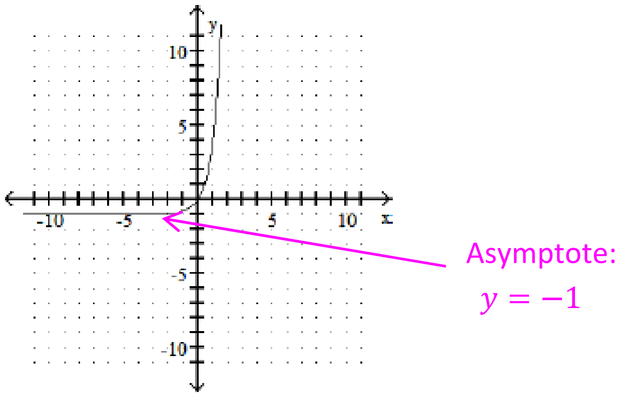
Memorize as much of this as you can!

A note on terminology: Zeros and roots are the same thing. If they are real, as opposed to complex, they are also x-intercepts of your graph.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The graph of an exponential function is given. Select the function for the graph from the functions listed.

1)



A) $f(x) = 5^x$ B) $f(x) = 5^x - 1$ C) $f(x) = 5^x + 1$ D) $f(x) = 5^{x-1}$

For an exponential function of the form: $f(x) = b^{x+c} + d$, the asymptote is at $y = d$.

Since this function's asymptote is at $y = -1$, we can conclude that the constant term of the function is -1. The only answer with a constant of -1 is **B**.

Answer B

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Problems 2 to 8. In evaluating logarithmic expressions, it is often helpful to set the expression equal to x, and then use the "first-last-middle" rule to convert the logarithmic expression to an exponential expression, which is easier to think about. *Note: the "first-last-middle" rule requires that the logarithmic or exponential portion be on the left-hand side of the equation.*

Write the equation in its equivalent exponential form.

2) $\log_b 64 = 2$

In the log expression, $\log_b 64 = 2$, **first** is "b", **last** is "2" and **middle** is "64." We put these in an exponential expression, from left to right, to get: $\boldsymbol{b^2 = 64}$.

3) $\log_6 216 = x$

In the log expression, $\log_6 216 = x$, **first** is "6", **last** is "x" and **middle** is "216." We put these in an exponential expression, from left to right, to get: $\boldsymbol{6^x = 216}$.

Write the equation in its equivalent logarithmic form.

4) $2^3 = x$

First-last-middle works this way too.

In the exponential expression, $2^3 = x$, **first** is "2", **last** is "x" and **middle** is "3." We put these in a logarithmic expression, from left to right, to get: $\mathbf{\log_2 x = 3.}$

5) $2^{-2} = \frac{1}{4}$

In the exponential expression, $2^{-2} = \frac{1}{4}$, **first** is "2", **last** is "$\frac{1}{4}$" and **middle** is "-2." We put these in a logarithmic expression, from left to right, to get: $\mathbf{\log_2 \left(\frac{1}{4}\right) = -2.}$

Evaluate the expression without using a calculator.

6) $\log_{10} 10$

In the log expression, $\log_{10} 10 = x$, **first** is "10", **last** is "x" and **middle** is "10." We put these in an exponential expression, from left to right, to get: $10^x = 10$, then solve:

$\log_{10} 10 = x$ converts to: $10^x = 10$ ⟶ $\boldsymbol{x = 1}$

7) $\log_3 \sqrt{3}$

In the log expression, $\log_3 \sqrt{3} = x$, **first** is "3", **last** is "x" and **middle** is "$\sqrt{3}$." We put these in an exponential expression, from left to right, to get: $3^x = \sqrt{3}$, then solve:

$\log_3 \sqrt{3} = x$ converts to: $3^x = \sqrt{3}$ ⟶ $\boldsymbol{x = \frac{1}{2}}$

8) $\log_6 1$

In the log expression, $\log_6 1 = x$, **first** is "6", **last** is "x" and **middle** is "1." We put these in an exponential expression, from left to right, to get: $6^x = 1$, then solve:

$\log_6 1 = x$ converts to: $6^x = 1$ ⟶ $\boldsymbol{x = 0}$

Problems 9 to 10: Exponentiation and taking logarithms are inverse operations, so when they both exist, *with the same base*, they cancel each other out.

9) $6^{\log_6 15}$

$$6^{(\log_6 15)} = 15$$

10) $\log_7 7^{18}$

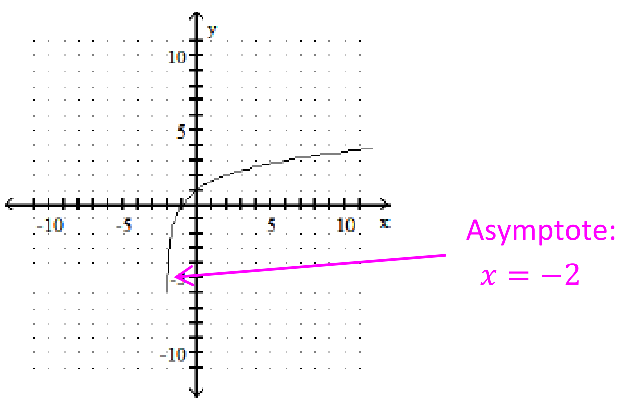
$$\log_7(7^{18}) = 18$$

Note: I like to use parentheses to make it easier to read a problem. For example, Problem 10 has values at four different heights. By using parentheses, I can make it easier to see what is going on in the problem.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The graph of a logarithmic function is given. Select the function for the graph from the options.

11)



A) $f(x) = \log_2 x$

B) $f(x) = \log_2 x + 2$

C) $f(x) = \log_2 (x - 2)$

D) $f(x) = \log_2 (x + 2)$

For an logarithmic function of the form: $f(x) = \log_b(x + c) + d$, the asymptote occurs where $x + c = 0$.

For this problem, the asymptote occurs at: $x = -2$

Add 2 to both sides of the equation: $x + 2 = 0$

So, $x + 2$ must be the object of the logarithm, i.e., the item in the parentheses, $(x + c)$.

The only answer in which this occurs is **D**. **Answer D**

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Evaluate or simplify the expression without using a calculator.

12) log 1000

Back to "**first-last-middle**."

In the log expression, $\log_{10}1000 = x$, **first** is "10", **last** is "x" and **middle** is "1000." We put these in an exponential expression, from left to right, to get: $10^x = 1000$, then solve:

$\log_{10}1000 = x$ converts to: $10^x = 1000$ $\longrightarrow$ $\boldsymbol{x = 3}$

13) $\log 10^7$

In the log expression, $\log_{10}(10^7) = x$, **first** is "10", **last** is "x" and **middle** is "10^7." We put these in an exponential expression, from left to right, to get: $10^x = 10^7$, then solve:

$\log_{10}10^7 = x$ converts to: $10^x = 10^7$ $\longrightarrow$ $\boldsymbol{x = 7}$

14) ln e

Note that $\ln x$ is equivalent to $\log_e x$. Then,

In the log expression, $\log_e e = x$, **first** is "e", **last** is "x" and **middle** is "e." We put these in an exponential expression, from left to right, to get: $e^x = e$, then solve:

$\log_e e = x$ converts to: $e^x = e$ $\longrightarrow$ $\boldsymbol{x = 1}$

Problems 15 to 29: use the properties of logarithms that can be found on page 2 of this packet.

Use properties of logarithms to expand the logarithmic expression as much as possible. Evaluate any logarithms that do not require a calculator.

15) $\log_2 (8x)$

$\log_2 8x = \log_2 8 + \log_2 x = \boldsymbol{3 + \log_2 x}$

16) $\log_5 \left(\frac{125}{x}\right)$

$\log_5 \left(\frac{125}{x}\right) = \log_5 125 - \log_5 x = \boldsymbol{3 - \log_5 x}$

17) $\log_b (yz^4)$

$$\log_b(yz^4) = \log_b(y) + \log_b(z^4) = \mathbf{\log_b y + 4\log_b z}$$

18) $\log_4 \left(\frac{x-6}{x^5}\right)$

$$\log_4\left(\frac{x-6}{x^5}\right) = \log_4(x-6) - \log_4(x^5) = \mathbf{\log_4(x-6) - 5\log_4 x}$$

Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

19) $\log_4 (x - 8) - \log_4 (x - 4)$

$$\log_4(x-8) - \log_4(x-4) = \mathbf{\log_4\left(\frac{x-8}{x-4}\right)}$$

20) $3\log_6 x + 5\log_6 (x - 6)$

$$3\log_6 x + 5\log_6(x-6) = \log_6 x^3 + \log_6(x-6)^5 = \mathbf{\log_6[x^3(x-6)^5]}$$

21) $\log_{20} 400$

$$\log_{20} 400 = \log_{20}(20^2) = \mathbf{2}$$

Solve .

22) $4^{(1 + 2x)} = 64$

Starting equation:	$4^{(1+2x)} = 64$
Take the "$\log_4$" of both sides:	$1 + 2x = 3$
Subtract 1:	$2x = 2$
Divide by 2:	$\mathbf{x = 1}$

23) $e^{x+8} = \frac{1}{e^4}$

Starting equation: $e^{(x+8)} = \frac{1}{e^4}$

Convert to the same form: $e^{(x+8)} = e^{-4}$

Take the "ln" of both sides: $x + 8 = -4$

Subtract 8: $\boldsymbol{x = -12}$

Solve the exponential equation. Express the solution set in terms of natural logarithms.

24) $5^{x+7} = 3$

Starting equation: $5^{(x+7)} = 3$

Take the "ln" of both sides: $\ln 5^{(x+7)} = \ln 3$

Simplify: $(x + 7) \ln 5 = \ln 3$

Divide by $\ln 5$: $x + 7 = \frac{\ln 3}{\ln 5}$

Subtract 7: $\boldsymbol{x = \frac{\ln 3}{\ln 5} - 7}$

25) $e^{x+4} = 2$

Starting equation: $e^{(x+4)} = 2$

Take the $\ln$ of both sides: $\ln e^{(x+4)} = \ln 2$

Simplify: $x + 4 = \ln 2$

Subtract 4: $\boldsymbol{x = \ln 2 - 4}$

Solve the logarithmic equation. Be sure to reject any value that is not in the domain of the original logarithmic expressions. Give the exact answer.

26) $\log_3 (x - 1) = -1$

Starting equation:	$\log_3(x-1) = -1$
Take 3 to the power of both sides:	$3^{\log_3(x-1)} = 3^{-1}$
Simplify:	$x - 1 = \frac{1}{3}$
Add 1:	$\boldsymbol{x = \frac{4}{3}}$

27) $4 + 8 \ln x = 8$

Starting equation:	$4 + 8\ln x = 8$
Subtract 4:	$8 \ln x = 4$
Divide by 8:	$\ln x = \frac{1}{2}$
Take e to the power of both sides:	$e^{\ln x} = e^{1/2}$
Simplify:	$\mathbf{x} = \boldsymbol{e^{1/2} = \sqrt{e}}$

28) $\log_6 x + \log_6 (x - 35) = 2$

Starting equation:	$\log_6 x + \log_6(x-35) = 2$
Combine log terms:	$\log_6[x \cdot (x-35)] = 2$
Take 6 to the power of both sides:	$6^{\log_6[x\cdot(x-35)]} = 6^2$
Simplify:	$x \cdot (x-35) = 36$
Multiply terms:	$x^2 - 35x = 36$
Subtract 36:	$x^2 - 35x - 36 = 0$
Factor:	$(x-36)(x+1) = 0$
Determine solutions for x:	$x = \{36, -1\}$
Test the solutions of x:	$x = 36$: $\log_6 36 + \log_6(36-35) = 2$ ✓
	$x = -1$: $\log_6(-1) + \log_6(-1-35) = 2$ ✗

Final solution: $\mathbf{x = 36}$

These terms are both Invalid because negative numbers are not in the domain of the log function.

29) $\log_6 (5x - 5) = \log_6 (3x + 7)$

Starting equation: $\log_6(5x - 5) = \log_6(3x + 7)$

Take 6 to the power of both sides: $6^{\log_6(5x-5)} = 6^{\log_6(3x+7)}$

Simplify: $5x - 5 = 3x + 7$

Add 5: $5x = 3x + 12$

Subtract $3x$: $2x = 12$

Divide by 2: $x = 6$

Test the solution of x: $\log_6(5 \cdot 6 - 5) = \log_6(3 \cdot 6 + 7)$ ✓

Final solution: $\mathbf{x = 6}$

Pre-Calculus: Chapter 3Practice Test (Calculator's OK)

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Use the compound interest formulas $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$ to solve.

1) Find the accumulated value of an investment of $900 at 12% compounded quarterly for 6 years.

We are given: $P = \$900$ $r = 0.12$ $n = 4$ (quarterly)

$t = 6$ years $\cdot$ 4 quarters $= 24$ periods

$$A = 900 \cdot \left(1 + \frac{0.12}{4}\right)^{24} = 900 \cdot (1.03)^{24} = \$\mathbf{1,829.51}$$

2) Find the accumulated value of an investment of $4000 at 7% compounded continuously for 5 years.

We are given: $P = \$4,000$ $r = 0.07$ continuous compounting (use Pert formula)

$t = 5$ years

$$A = 4000 \cdot (e^{5 \cdot 0.07}) = 4000 \cdot (e^{0.35}) = \$\mathbf{5,676.27}$$

Use common logarithms or natural logarithms and a calculator to evaluate to four decimal places

3) $\log_9 17$

$$\log_9 17 = \frac{\ln 17}{\ln 9} = \mathbf{1.2895} \qquad \text{or} \qquad \log_9 17 = \frac{\log_{10} 17}{\log_{10} 9} = \mathbf{1.2895}$$

Solve.

4) The half-life of silicon-32 is 710 years. If 50 grams is present now, how much will be present in 200 years? (Round your answer to three decimal places.)

The formula for exponential decay is: $A = A_0 e^{kt}$, where:

- A is the amount of substance left at time t.
- A_0 is the starting amount of the substance.
- k is the annual rate of decay.
- t is the number of years.

This is a 2-step problem. First we need to find k based on the half-life of 710 years. Then we need to see how much would be left after 200 years.

Step 1: Determine the value of k

We are given: $t = 710$, $\frac{A}{A_0} = \frac{1}{2}$ (because we are given a "half-life")

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\frac{A}{A_0} = e^{kt}$

Substitute in values: $\frac{1}{2} = e^{710\,k}$

Take natural logs: $\ln \frac{1}{2} = 710k$

Divide by 710: $k = \frac{\ln \frac{1}{2}}{710} = -0.00097626$

Step 2: Find how much is left after 200 years

We are given: are given: $t = 200$, $A_0 = 50$ grams $k = -.00097626$

Starting equation: $A = A_0 e^{kt}$

Substitute in values: $A = 50 \cdot e^{-.00097626 \cdot 200} = \mathbf{41.131}$ **grams**

Solve the problem.

5) The logistic growth function $f(t) = \dfrac{87{,}000}{1 + 1449e^{-1.2t}}$ models the number of people who have become ill with a particular infection t weeks after its initial outbreak in a particular community. How many people were ill after 9 weeks?

This is a simple substitution problem. Substitute 9 for t and calculate the solution.

$$f(9) = \frac{87{,}000}{1 + 1{,}449 \cdot e^{-1.2 \cdot 9}} = \frac{87{,}000}{1 + 1{,}449 \cdot e^{-10.8}} = \mathbf{84{,}502}$$

Rewrite the equation in terms of base e. Express the answer in terms of a natural logarithm, and then round to three decimal places.

6) $y = 900(6)^x$

Original Equation: $y = 900 \cdot 6^x$

Rewrite 6 as $e^{\ln 6}$: $y = 900 \cdot \left(e^{\ln 6}\right)^x$

Simplify: $y = 900\, e^{x \ln 6} = 900\, e^{1.792\, x}$

Solve the exponential equation. Use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

7) $e^{2x} + e^x - 6 = 0$

Let's us a trick called u-substitution for this. Here's how it works.

Let $u = e^x$. Then the equation becomes:

$u^2 + u - 6 = 0$

$(u + 3)(u - 2) = 0$

$u = \{-3, 2\}$

Next we substitute e^x back in for each value of u and solve the resulting equations.

Start with: $u = -3$

Substitute: $e^x = -3$

There is no solution here because e^x can never be negative.

Start with: $u = 2$

Substitute: $e^x = 2$

Take ln's: $x = \ln 2 = \mathbf{0.69}$

Final solution: $x = \mathbf{0.69}$