

Differentiation - Quotient Rule

Differentiate each function with respect to x .

1) $y = \frac{2}{2x^4 - 5}$

2) $f(x) = \frac{2}{x^5 - 5}$

3) $f(x) = \frac{5}{4x^3 + 4}$

* 4) $y = \frac{4x^3 - 3x^2}{4x^5 - 4}$

5) $y = \frac{3x^4 + 2}{3x^3 - 2}$

6) $y = \frac{4x^5 + 2x^2}{3x^4 + 5}$

7) $y = \frac{4x^5 + x^2 + 4}{5x^2 - 2}$

8) $y = \frac{3x^4 + 5x^3 - 5}{2x^4 - 4}$

$$9) y = \frac{x^3 - x^2 - 3}{x^5 + 3}$$

$$10) y = \frac{x^4 + 6}{3 - 4x^{-4}}$$

$$11) y = \frac{4x^4 - 4x^2 + 5}{\frac{5}{2x^3 + 3}}$$

Critical thinking question:

- 12) A classmate claims that $\left(\frac{f}{g}\right)' = \frac{f'}{g'}$ for any functions f and g . Show an example that proves your classmate wrong.

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Differentiate each function with respect to x .

1) $y = \frac{2}{2x^4 - 5}$

$$\begin{aligned}\frac{dy}{dx} &= -\frac{2 \cdot 8x^3}{(2x^4 - 5)^2} \\ &= -\frac{16x^3}{4x^8 - 20x^4 + 25}\end{aligned}$$

2) $f(x) = \frac{2}{x^5 - 5}$

$$\begin{aligned}f'(x) &= -\frac{2 \cdot 5x^4}{(x^5 - 5)^2} \\ &= -\frac{10x^4}{x^{10} - 10x^5 + 25}\end{aligned}$$

3) $f(x) = \frac{5}{4x^3 + 4}$

$$\begin{aligned}f'(x) &= -\frac{5 \cdot 12x^2}{(4x^3 + 4)^2} \\ &= -\frac{15x^2}{4x^6 + 8x^3 + 4}\end{aligned}$$

4) $y = \frac{4x^3 - 3x^2}{4x^5 - 4}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(4x^5 - 4)(12x^2 - 6x) - (4x^3 - 3x^2) \cdot 20x^4}{(4x^5 - 4)^2} \\ &= \frac{-8x^7 + 9x^6 - 12x^2 + 6x}{4x^{10} - 8x^5 + 4}\end{aligned}$$

5) $y = \frac{3x^4 + 2}{3x^3 - 2}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(3x^3 - 2) \cdot 12x^3 - (3x^4 + 2) \cdot 9x^2}{(3x^3 - 2)^2} \\ &= \frac{9x^6 - 24x^3 - 18x^2}{9x^6 - 12x^3 + 4}\end{aligned}$$

6) $y = \frac{4x^5 + 2x^2}{3x^4 + 5}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(3x^4 + 5)(20x^4 + 4x) - (4x^5 + 2x^2) \cdot 12x^3}{(3x^4 + 5)^2} \\ &= \frac{12x^8 - 12x^5 + 100x^4 + 20x}{9x^8 + 30x^4 + 25}\end{aligned}$$

7) $y = \frac{4x^5 + x^2 + 4}{5x^2 - 2}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(5x^2 - 2)(20x^4 + 2x) - (4x^5 + x^2 + 4) \cdot 10x}{(5x^2 - 2)^2} \\ &= \frac{60x^6 - 40x^4 - 44x}{25x^4 - 20x^2 + 4}\end{aligned}$$

8) $y = \frac{3x^4 + 5x^3 - 5}{2x^4 - 4}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(2x^4 - 4)(12x^3 + 15x^2) - (3x^4 + 5x^3 - 5) \cdot 8x^3}{(2x^4 - 4)^2} \\ &= \frac{-5x^6 - 4x^3 - 30x^2}{2x^8 - 8x^4 + 8}\end{aligned}$$

$$9) y = \frac{x^3 - x^2 - 3}{x^5 + 3}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x^5 + 3)(3x^2 - 2x) - (x^3 - x^2 - 3) \cdot 5x^4}{(x^5 + 3)^2} \\ &= \frac{-2x^7 + 3x^6 + 15x^4 + 9x^2 - 6x}{x^{10} + 6x^5 + 9} \end{aligned}$$

$$10) y = \frac{x^4 + 6}{3 - 4x^{-4}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(3 - 4x^{-4}) \cdot 4x^3 - (x^4 + 6) \cdot 16x^{-5}}{(3 - 4x^{-4})^2} \\ &= \frac{12x^{11} - 32x^7 - 96x^3}{9x^8 - 24x^4 + 16} \end{aligned}$$

$$11) y = \frac{4x^4 - 4x^2 + 5}{2x^{\frac{5}{3}} + 3}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\left(2x^{\frac{5}{3}} + 3\right)(16x^3 - 8x) - (4x^4 - 4x^2 + 5) \cdot \frac{10}{3}x^{\frac{2}{3}}}{\left(2x^{\frac{5}{3}} + 3\right)^2} \\ &= \frac{56x^{\frac{14}{3}} + 144x^3 - 8x^{\frac{8}{3}} - 72x - 50x^{\frac{2}{3}}}{12x^{\frac{10}{3}} + 36x^{\frac{5}{3}} + 27} \end{aligned}$$

Critical thinking question:

12) A classmate claims that $\left(\frac{f}{g}\right)' = \frac{f'}{g'}$ for any functions f and g . Show an example that proves your classmate wrong.

Many answers. Ex: $f = 4$, $g = 2x$, $-\frac{2}{x^2} \neq 0$