Ch 11 part II Practice Test **Name: ______________________________**

Find the derivative for the following. Answers should not have negative exponents and must be simplified completely.

Look for the simplest way to solve a problem. Answers are shown in **green bold**. There will be multiple acceptable solutions for many problems.

Here are the key rules you need to know, from the Calculus Handbook on www.mathguy.us.

Definition of a Derivative

$$\frac{d}{dx}f(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \qquad \frac{d}{dx}f(x) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

Basic Derivative Rules

$$\frac{d}{dx}(c) = 0 \qquad \frac{d}{dx}(c\cdot u) = c\cdot\frac{d}{dx}(u)$$

$$\frac{d}{dx}(u+v) = \frac{d}{dx}(u)+\frac{d}{dx}(v) \qquad \frac{d}{dx}(u-v) = \frac{d}{dx}(u)-\frac{d}{dx}(v)$$

Product Rule (two terms)

$$\frac{d}{dx}[f(x)\cdot g(x)] = f(x)\cdot\frac{d}{dx}[g(x)] + g(x)\cdot\frac{d}{dx}[f(x)]$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot\frac{d}{dx}[f(x)] - f(x)\cdot\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

Power Rule

$$\frac{d}{dx}(x^n) = n\cdot x^{n-1} \qquad \frac{d}{dx}(u^n) = n\cdot u^{n-1}\frac{du}{dx}$$

1) $f(x) = 5x^3 - 4x^2 + 3x + 3$

The power rule is sufficient for this problem.

$f(x) = 5x^3 - 4x^2 + 3x + 3$

$f'(x) = 3\cdot 5x^2 - 2\cdot 4x^1 + 3 + 0$

$= \mathbf{15x^2 - 8x + 3}$

2) $y = \sqrt{x}$

Convert the root to a fractional exponent before using the power rule.

$$f(x) = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \mathbf{\frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}}$$

3) $g(x) = 3x^5 - 2x^3 + x^4 - \sqrt[3]{x}$

Convert the root to a fractional exponent before using the power rule. I have re-ordered the terms in decreasing order of exponent, as is the mathematical standard.

$$f(x) = 3x^5 + x^4 - 2x^3 - x^{\frac{1}{3}}$$

$$f'(x) = 15x^4 + 4x^3 - 6x^2 - \frac{1}{3}x^{-\frac{2}{3}}$$

$$= 15x^4 + 4x^3 - 6x^2 - \frac{1}{3}x^{-\frac{2}{3}}$$

$$= \mathbf{15x^4 + 4x^3 - 6x^2 - \frac{1}{3x^{\frac{2}{3}}}}$$

$$= \mathbf{15x^4 + 4x^3 - 6x^2 - \frac{1}{3\sqrt[3]{x^2}}}$$

4) $g(x) = (4x - 1)(5x^2 - 6)$

Either use the product rule for this one or FOIL it out and take the derivative. This problem is simpler to FOIL and derive.

$$f(x) = (4x - 1)(5x^2 - 6)$$

$$= 20x^3 - 5x^2 - 24x + 6$$

$$\mathbf{f'(x) = 60x^2 - 10x - 24}$$

5) $y = (2x^2 - 5)(x^3 - 3x^2 + 4)$

Either use the product rule for this one or multiply it out and take the derivative. I will show both methods. Use whichever one you feel most confident with.

Product Rule:

$$f(x) = (2x^2 - 5)(x^3 - 3x^2 + 4)$$

$$f'(x) = (2x^2 - 5)(3x^2 - 6x) + 4x(x^3 - 3x^2 + 4)$$

$$= 6x^4 - 12x^3 - 15x^2 + 30x + 4x^4 - 12x^3 + 16x$$

$$= 10x^4 - 24x^3 - 15x^2 + 46x$$

$$= \mathbf{x(10x^3 - 24x^2 - 15x + 46)}$$

Multiply it out

$$f(x) = (2x^2 - 5)(x^3 - 3x^2 + 4)$$

$$= 2x^5 - 6x^4 + 8x^2 - 5x^3 + 15x^2 - 20$$

$$= 2x^5 - 6x^4 - 5x^3 + 23x^2 - 20$$

$$f'(x) = 10x^4 - 24x^3 - 15x^2 + 46x$$

$$= \mathbf{x(10x^3 - 24x^2 - 15x + 46)}$$

6) $f(x) = \frac{7x^2 - 5}{4x^3 - 2x^2}$

We will need to use the Quotient Rule for this one.

$$f(x) = \frac{7x^2 - 5}{4x^3 - 2x^2}$$

$$f'(x) = \frac{(4x^3 - 2x^2)(14x) - (7x^2 - 5)(12x^2 - 4x)}{(4x^3 - 2x^2)^2}$$

$$= \frac{56x^4 - 28x^3 - (84x^4 - 28x^3 - 60x^2 + 20x)}{(2x^2)^2(2x - 1)^2}$$

$$= \frac{-28x^4 + 60x^2 - 20x}{4x^4(2x - 1)^2}$$

$$= \frac{4x(-7x^3 + 15x - 5)}{4x^4(2x - 1)^2}$$

$$= \mathbf{\frac{-7x^3 + 15x - 5}{x^3(2x - 1)^2} = \frac{-(7x^3 - 15x + 5)}{x^3(2x - 1)^2}}$$

7) $y = \frac{2x^3-5}{5x^3-2}$

We will need to use the Quotient Rule for this one.

$$f(x) = \frac{2x^3 - 5}{5x^3 - 2}$$

$$f'(x) = \frac{(5x^3 - 2)(6x^2) - (2x^3 - 5)(15x^2)}{(5x^3 - 2)^2}$$

$$= \frac{30x^5 - 12x^2 - (30x^5 - 75x^2)}{(5x^3 - 2)^2}$$

$$= \mathbf{\frac{63x^2}{(5x^3 - 2)^2}}$$

8) $f(x) = \sqrt[4]{(2x^5 + 7x)}$ *Do not rationalize answer

Notice that we have a function inside another function, so we must use the chain rule.

$$f(x) = (2x^5 + 7x)^{\frac{1}{4}}$$

$$f'(x) = \frac{1}{4}(2x^5 + 7x)^{-\frac{3}{4}} \cdot (10x^4 + 7)$$

$$= \frac{10x^4 + 7}{4 \cdot (2x^5 + 7x)^{\frac{3}{4}}}$$

$$= \mathbf{\frac{10x^4 + 7}{4 \cdot [x(2x^4 + 7)]^{\frac{3}{4}}}}$$

$$= \mathbf{\frac{10x^4 + 7}{4 \cdot \sqrt[4]{x^3(2x^4 + 7)^3}}}$$

9) $y = (3x^4 - 5)^3$

Notice that we have a function inside another function, so we must use the chain rule.

$$y = (3x^4 - 5)^3$$

$$y' = 3(3x^4 - 5)^2 \cdot (12x^3)$$

$$= \mathbf{36x^3 \cdot (3x^4 - 5)^2}$$

10) $y = \frac{x^3 - \sqrt{x}}{4x}$

We will **NOT** need to use the Quotient Rule for this one. Simplify before taking a derivative.

$$y = \frac{x^3 - x^{\frac{1}{2}}}{4x} = \frac{1}{4}\left(\frac{x^3}{x} - \frac{x^{\frac{1}{2}}}{x}\right) = \frac{1}{4}\left(x^2 - x^{-\frac{1}{2}}\right)$$

$$y' = \frac{1}{4}\left(2x + \frac{1}{2}x^{-\frac{3}{2}}\right)$$

$$= \frac{1}{4}\left(2x + \frac{1}{2x^{\frac{3}{2}}}\right)$$

Next, get a common denominator inside the parentheses.

$$= \frac{1}{4}\left(\frac{2x^{\frac{3}{2}}}{2x^{\frac{3}{2}}} \cdot 2x + \frac{1}{2x^{\frac{3}{2}}}\right)$$

$$= \frac{4x^{\frac{5}{2}} + 1}{4 \cdot 2x^{\frac{3}{2}}} = \frac{\mathbf{4x^{\frac{5}{2}} + 1}}{\mathbf{8x^{\frac{3}{2}}}} = \frac{\mathbf{4\sqrt{x^5} + 1}}{\mathbf{8\sqrt{x^3}}} = \frac{\mathbf{4x^3 + \sqrt{x}}}{\mathbf{8x^2}}$$

Note: the third solution in green in obtained by multiplying either of the other two solutions by $\frac{\sqrt{x}}{\sqrt{x}}$.

11) $y = \frac{x^{-4} + x^{-7}}{x^{-3} + x^{-9}}$

I suggest modifying $f(x)$ to get rid of the negative exponents before taking a derivative.

$$y = \frac{x^{-4} + x^{-7}}{x^{-3} + x^{-9}} = \frac{(x^{-4} + x^{-7})}{(x^{-3} + x^{-9})} \cdot \frac{x^9}{x^9} = \frac{x^5 + x^2}{x^6 + 1}$$

Then,

$$y' = \frac{(x^6 + 1)(5x^4 + 2x) - (x^5 + x^2)(6x^5)}{(x^6 + 1)^2}$$

$$= \frac{5x^{10} + 2x^7 + 5x^4 + 2x - (6x^{10} + 6x^7)}{(x^6 + 1)^2}$$

$$= \frac{-x^{10} - 4x^7 + 5x^4 + 2x}{(x^6 + 1)^2}$$

$$= \frac{\mathbf{-x(x^9 + 4x^6 - 5x^3 - 2)}}{\mathbf{(x^6 + 1)^2}}$$

Old 11) $y = \frac{x^2+3}{2-x^{-3}}$ **Not required**

I suggest modifying $f(x)$ to get rid of the negative exponent before taking a derivative.

$$y = \frac{x^2+3}{2-x^{-3}} = \frac{(x^2+3)}{(2-x^{-3})} \cdot \frac{x^3}{x^3} = \frac{x^5+3x^3}{2x^3-1}$$

Then,

$$y' = \frac{(2x^3-1)(5x^4+9x^2)-(x^5+3x^3)(6x^2)}{(2x^3-1)^2}$$

Next, factor an x^2 out of both orange terms (you could do this later in the problem if you wish).

$$= \frac{x^2[(2x^3-1)(5x^2+9)-(x^3+3x)(6x^2)]}{(2x^3-1)^2}$$

$$= \frac{x^2[(10x^5+18x^3-5x^2-9)-(6x^5+18x^3)]}{(2x^3-1)^2}$$

$$= \frac{x^2(4x^5-5x^2-9)}{(2x^3-1)^2}$$

12) y = $(\sqrt{x} + 2\sqrt[4]{x})(x^4 + 9x^7)$

This one is very ugly. I would FOIL it out before taking the derivative.

$$y = (\sqrt{x} + 2\sqrt[4]{x})(x^4 + 9x^7)$$

$$= \left(x^{\frac{1}{2}} + 2x^{\frac{1}{4}}\right)(9x^7 + x^4)$$

Let's get common denominators for the exponents, which will make adding them easier.

$$= \left(x^{\frac{2}{4}} + 2x^{\frac{1}{4}}\right)\left(9x^{\frac{28}{4}} + x^{\frac{16}{4}}\right) = 9x^{\frac{30}{4}} + 18x^{\frac{29}{4}} + x^{\frac{18}{4}} + 2x^{\frac{17}{4}}$$

$$y' = \frac{270}{4}x^{\frac{26}{4}} + \frac{522}{4}x^{\frac{25}{4}} + \frac{18}{4}x^{\frac{14}{4}} + \frac{34}{4}x^{\frac{13}{4}} = \frac{135}{2}x^{\frac{26}{4}} + \frac{261}{2}x^{\frac{25}{4}} + \frac{9}{2}x^{\frac{14}{4}} + \frac{17}{2}x^{\frac{13}{4}}$$

Factor out the GCF: $x^{\frac{13}{4}}$ in the numerator and 2 in the denominator.

$$= \frac{x^{\frac{13}{4}}\left(135x^{\frac{13}{4}} + 261x^{\frac{12}{4}} + 9x^{\frac{1}{4}} + 17\right)}{2}$$

$$= \frac{x^{\frac{13}{4}}\left(135x^{\frac{13}{4}} + 261x^3 + 9x^{\frac{1}{4}} + 17\right)}{2} = \frac{\sqrt[4]{x^{13}}\left(135\sqrt[4]{x^{13}} + 261x^3 + 9\sqrt[4]{x} + 17\right)}{2}$$

13) y = $\sqrt{6x^4 - 3x^2 + 2x}$ *do not rationalize answer

This one requires us to re-write the equation before applying the Power and Chain Rules.

$$y = (6x^4 - 3x^2 + 2x)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(6x^4 - 3x^2 + 2x)^{-\frac{1}{2}} \cdot (24x^3 - 6x + 2)$$

$$y' = \frac{24x^3 - 6x + 2}{2(6x^4 - 3x^2 + 2x)^{\frac{1}{2}}} = \frac{2(12x^3 - 3x + 1)}{2(6x^4 - 3x^2 + 2x)^{\frac{1}{2}}}$$

$$= \frac{\mathbf{12x^3 - 3x + 1}}{[\mathbf{x(6x^3 - 3x + 2)}]^{\frac{1}{2}}} = \frac{\mathbf{12x^3 - 3x + 1}}{\sqrt{\mathbf{x(6x^3 - 3x + 2)}}}$$

Depending on the context of the questions and the desire of your teacher, it may not be necessary to factor out the x under the square root sign in the denominator.

14) $y = (x+1)^2(x^2 + 2x)$

Lots of rules to use on this one. Given the factoring required at the end, I would not recommend multiplying this one out to take the derivative.

$$y = (x+1)^2 \cdot (x^2 + 2x)$$

$$y' = (x+1)^2 \cdot (2x + 2) + (x^2 + 2x) \cdot 2(x+1)$$

$$= (x+1)^2 \cdot 2(x+1) + (x^2 + 2x) \cdot 2(x+1)$$

$$= 2(x+1)[(x+1)^2 + (x^2 + 2x)]$$

$$= 2(x+1)(x^2 + 2x + 1 + x^2 + 2x)$$

$$= \mathbf{2(x+1)(2x^2 + 4x + 1)}$$

15) $y = \frac{2}{3x^4} - \frac{4}{7x^{11}} + \sqrt[4]{5x^3} - \sqrt[7]{6x^9}$

$$y = \frac{2}{3}x^{-4} - \frac{4}{7}x^{-11} + \sqrt[4]{5}\, x^{\frac{3}{4}} - \sqrt[7]{6}\, x^{\frac{9}{7}}$$

$$y' = \frac{-8}{3}x^{-5} + \frac{44}{7}x^{-12} + \frac{3}{4}\sqrt[4]{5}\, x^{-\frac{1}{4}} - \frac{9}{7}\sqrt[7]{6}\, x^{\frac{2}{7}}$$

$$= \frac{-8}{3x^5} + \frac{44}{7x^{12}} + \frac{3\sqrt[4]{5}}{4x^{\frac{1}{4}}} - \frac{9\sqrt[7]{6}}{7}\, x^{\frac{2}{7}}$$

$$= \frac{-8}{3x^5} + \frac{44}{7x^{12}} + \frac{3\sqrt[4]{5}}{4\sqrt[4]{x}} - \frac{9\sqrt[7]{6x^2}}{7}$$

For #16-17: Find the slope of the tangent line for the function at the given point.

16) $f(x) = 4x^2 + 5x + 1$ at $x = -2$ and $x = 3$

We get the slope of a tangent line from the derivative of the function.

$f(x) = 4x^2 + 5x + 1$

$f'(x) = 8x + 5$

When $x = -2$

$f'(-2) = 8(-2) + 5 = -11$, **so our slope is -11.**

When $x = 3$

$f'(3) = 8(3) + 5 = 29$, **so our slope is 29.**

17) $f(x) = [(x^4 - x^2)(x^3 - 2x)]^3$ at $x = 1$

That's a lot of x's. I would probably do this one different from many students. See if you like it.

$f(x) = [(x^4 - x^2)(x^3 - 2x)]^3$

$= [x^2(x^2 - 1) \cdot x(x^2 - 2)]^3$

$= [x^3(x^2 - 1)(x^2 - 2)]^3$

$= x^9[x^4 - 3x^2 + 2]^3$

Now, use the Product Rule

$f'(x) = x^9 \cdot 3(x^4 - 3x^2 + 2)^2(4x^3 - 6x) + [x^4 - 3x^2 + 2]^3 \cdot 9x^8$

It is not necessary to simplify this expression. Just substitute 1 for x and calculate the slope.

$f'(1) = 1^9 \cdot 3(1^4 - 3 \cdot 1^2 + 2)^2(4 \cdot 1^3 - 6 \cdot 1) + [1^4 - 3 \cdot 1^2 + 2]^3 \cdot 9 \cdot 1^8$

$= 1 \cdot 3(0)^2(-2) + [0]^3 \cdot 9 \cdot 1$

$= \mathbf{0}$

Write the equation of the tangent line at the given point.

18) $f(x) = 11x^3 + 8x^2 - \sqrt{x}$ at $x = 4$

To obtain the equation of a tangent line, we need a point and a slope. We are given the x-coordinate of a point. We get the slope from the derivative of the function. $\frac{2367}{4}$

$$f(x) = 11x^3 + 8x^2 - \sqrt{x}$$

$$f'(x) = 33x^2 + 16x - \frac{1}{2\sqrt{x}}$$

When $x = 4$

$f(4) = 11(4)^3 + 8(4)^2 - \sqrt{4} = 830$, so our point is $(4, 830)$

$f'(4) = 33(4)^2 + 16(4) - \frac{1}{2\sqrt{4}} = \frac{2367}{4}$, so our slope is $\frac{2367}{4}$

The equation of the line, then is: $\boldsymbol{y = \frac{2367}{4}(x - 4) + 830}$

19) $f(x) = \left(\sqrt[3]{x} - 4x\right)(5x - 2)$ at $x = 8$

To obtain the equation of a tangent line, we need a point and a slope. We are given the x-coordinate of a point. We get the slope from the derivative of the function.

$$f(x) = \left(x^{\frac{1}{3}} - 4x\right)(5x - 2)$$

$$= 5x^{\frac{4}{3}} - 2x^{\frac{1}{3}} - 20x^2 + 8x$$

$$= 5\sqrt[3]{x^4} - 2\sqrt[3]{x} - 20x^2 + 8x$$

$$f'(x) = \frac{20}{3}x^{\frac{1}{3}} - \frac{2}{3}x^{-\frac{2}{3}} - 40x + 8$$

$$= \frac{20}{3}\sqrt[3]{x} - \frac{2}{3\sqrt[3]{x^2}} - 40x + 8$$

When $x = 8$

$f(8) = 5\sqrt[3]{8^4} - 2\sqrt[3]{8} - 20(8)^2 + 8(8) = -1140$, so our point is $(8, -1140)$

$f'(8) = \frac{20}{3}\sqrt[3]{\mathbf{8}} - \frac{2}{3\sqrt[3]{\mathbf{8}^2}} - 40(8) + 8 =$, so our slope is $\frac{-1793}{6}$

The equation of the line, then is: $\boldsymbol{y = \frac{-1793}{6}(x - 8) - 1140}$