Chapter 10 Practice Test Name________________________

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Write the first four terms of the sequence whose general term is given.

1) $a_n = 2(3n - 1)$ 1) ____________

$a_n = 2(3n-1) = 6n - 2$

Start with a_1 and then add 6 for to get each successive term:

$a_1 = 6(1) - 2 = 4$

So, the first 4 terms of the sequence are:

$4,\ \ 4+1(6),\ \ 4+2(6),\ \ 4+3(6) \quad \Rightarrow \quad \mathbf{4, 10, 16, 22}$

Write the first four terms of the sequence defined by the recursion formula.

2) $a_1 = -2$ and $a_n = a_{n-1} - 3$ for $n \geq 2$ 2) ____________

$a_n = a_{n-1} - 3$ indicates we need to subtract 3 for each successive term.

$a_1 = \mathbf{-2}$

$a_2 = a_1 - 3 = -2 - 3 = \mathbf{-5}$

$a_3 = a_2 - 3 = -5 - 3 = -8$

$a_4 = a_3 - 3 = -8 - 3 = -11$

So, the first 4 terms of the sequence are: $\mathbf{-2, -5, -8, -11}$

Evaluate the factorial expression.

3) $\dfrac{(n+5)!}{n+4}$ 3) ____________

$$\frac{(n+5)!}{n+4} = \frac{(n+5)(n+4)(n+3)!}{n+4} = \boldsymbol{(n+5)(n+3)!}$$

Find the indicated sum.

4) $\sum_{i=1}^{3} 4^i$ 4) ____________

Method 1: Add 'em up:

$$\sum_{k=1}^{3} 4^k = 4^1 + 4^2 + 4^3 = 4 + 16 + 64 = \mathbf{84}$$

Method 2: Use the geometric series sum formula: $S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right)$

$a_1 = 4^1 = 4 \qquad r = 4 \qquad n = 3$

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 4 \cdot \left(\frac{4^3 - 1}{4 - 1}\right) = 4 \cdot \left(\frac{63}{3}\right) = 4 \cdot 21 = \mathbf{84}$$

Write the first five terms of the arithmetic sequence.

5) $a_1 = 4$; $d = -1$ 5) ____________

$a_1 = 4$

Start with a_1 and then add -1 (i.e., subtract 1) to get each successive term:

So, the first 5 terms of the sequence are:

$$4,\ 4 + 1(-1),\ 4 + 2(-1),\ 4 + 3(-1),\ 4 + 4(-1) \quad \Rightarrow \quad \mathbf{4, 3, 2, 1, 0}$$

Write a formula for the general term (the nth term) of the arithmetic sequence. Then use the formula for a_n to find a_{20}, the 20th term of the sequence.

6) 1, 5, 9, 13, 17, . . . 6) ____________

Unorthodox, but I like to find $a_0 = a_1 - d$ because a_0 (i.e., the 0th term) is the constant term in the explicit formula for a_n and d is the multiplier of n. So, the explicit formula for an arithmetic sequence is always:

$a_n = a_0 + dn$ (note: you need to calculate a_0; it is not given)

For this sequence, $d = 5 - 1 = 4$, so $a_0 = a_1 - d = 1 - 4 = -3$

Then, the explicit formula is: $\boldsymbol{a_n = -3 + 4n}$

Finally, $\boldsymbol{a_{20}} = -3 + 4(20) = -3 + 80 = \mathbf{77}$

Find the indicated sum.

7) Find 2 + 4 + 6 + 8 + . . ., the sum of the first 40 positive even integers. 7) ______________

Method 1: Think like Gauss

First, we need $a_{40} = a_1 + 39d = 2 + 39(2) = 80$. Then:

$$S = \ \ 2 + \ \ 4 + \ \ 6 + \cdots + 80$$

$$S = 80 + 78 + 76 + \cdots + \ \ 2$$

$$2S = 82 + 82 + 82 + \cdots + 82 = 40(82)$$

Divide both sides by 2, to get

$$S = 20(82) = \mathbf{1{,}640}$$

Method 2: Use the arithmetic series sum formula: $S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n)$

Again, we need $a_{40} = a_1 + 39d = 2 + 39(2) = 80$

$a_1 = 2 \qquad a_{40} = 80 \qquad n = 40$

$$S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n) \ = \ \left(\frac{40}{2}\right) \cdot (2 + 80) \ = \ 20(82) = \ \mathbf{1{,}640}$$

Use the formula for the sum of the first n terms of an arithmetic sequence to find the indicated sum.

8) $\sum_{i=1}^{30} 4i$ 8) ______________

Method 1: Think like Gauss

First, we need $a_1 = 4(1) = 4$ and $a_{30} = 4(30) = 120$. Then:

$$S = \ \ 4 + \ \ 8 + \ \ 12 + \cdots + 120$$

$$S = 120 + 116 + 112 + \cdots + \ \ 4$$

$$2S = 124 + 124 + 124 + \cdots + 124 = 30(124)$$

Divide both sides by 2, to get

$$S = 15(124) = \mathbf{1{,}860}$$

Method 2: Use the arithmetic series sum formula $S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n)$

We need $a_1 = 4(1) = 4$ and $a_{30} = 4(30) = 120$

$a_1 = 4 \qquad a_{30} = 120 \qquad n = 30$

$$S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n) \ = \ \left(\frac{30}{2}\right) \cdot (4 + 120) \ = \ 15(124) = \ \mathbf{1{,}860}$$

If the given sequence is a geometric sequence, find the common ratio.

9) $\frac{3}{1}, \frac{3}{4}, \frac{3}{16}, \frac{3}{64}, \frac{3}{256}$ 9) ____________

The common ratio can be found by dividing consecutive terms. It is a good idea to do this twice to make sure the ratio between terms is consistent.

$$r = \frac{a_2}{a_1} = \frac{\frac{3}{4}}{\frac{3}{1}} = \frac{3}{4} \cdot \frac{1}{3} = \mathbf{\frac{1}{4}}$$

$$r = \frac{a_3}{a_2} = \frac{\frac{3}{16}}{\frac{3}{4}} = \frac{3}{16} \cdot \frac{4}{3} = \frac{4}{16} = \mathbf{\frac{1}{4}} \quad \checkmark$$

Write the first five terms of the geometric sequence.

10) $a_1 = 12;\ r = \frac{1}{3}$ 10) ____________

$a_1 = 12$

Start with a_1 and then multiply by $\frac{1}{3}$ to get each successive term:

So, the first 5 terms of the sequence are:

$$12,\ 12\left(\frac{1}{3}\right),\ 12\left(\frac{1}{3}\right)^2,\ 12\left(\frac{1}{3}\right)^3,\ 12\left(\frac{1}{3}\right)^4 \quad \Rightarrow \quad \mathbf{12,\ 4,\ \frac{4}{3},\ \frac{4}{9},\ \frac{4}{27}}$$

Write a formula for the general term (the nth term) of the geometric sequence.

11) 5, −15, 45, −135, 405, . . . 11) ____________

The general term of a geometric sequence is: $a_n = a_1 \cdot r^{n-1}$

$$a_1 = 5 \qquad r = \frac{-15}{5} = \frac{45}{-15} = -3$$

Then, $\boldsymbol{a_n = 5 \cdot (-3)^{n-1}}$

Use the formula for the sum of the first n terms of a geometric sequence to solve.

12) Find the sum of the first five terms of the geometric sequence: $\frac{1}{3}, \frac{4}{3}, \frac{16}{3}, \ldots$ 12) ______________

Method 1: Add 'em up (note that $r = 4$):

$$\frac{1}{3}+\frac{4}{3}+\frac{16}{3}+\frac{64}{3}+\frac{256}{3}=\mathbf{\frac{341}{3}}$$

Method 2: Use the geometric series sum formula: $S = a_1 \cdot \left(\frac{r^n-1}{r-1}\right)$

$a_1 = \frac{1}{3}$ $\quad r = 4 \quad$ $n = 5$

$$S = a_1 \cdot \left(\frac{r^n - 1}{r-1}\right) = \frac{1}{3}\left(\frac{4^5-1}{4-1}\right) = \frac{(1024-1)}{3\cdot 3} = \frac{1023}{9} = \mathbf{\frac{341}{3}}$$

Find the sum of the infinite geometric series, if it exists.

13) $3 - 1 + \frac{1}{3} - \frac{1}{9} + \ldots$ 13) ______________

Method 1: Think like Gauss

$a_1 = 3 \quad r = -\frac{1}{3}$

$$S = 3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \cdots$$

$$+\frac{1}{3}S = \quad 1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} - \cdots$$

$$\frac{4}{3}S = 3$$

Note that this series converges because: $|r| = \left|-\frac{1}{3}\right| = \frac{1}{3} < 1.$

Multiply both sides by $\frac{3}{4}$, to get

$$S = \frac{3}{4}\cdot 3 = \mathbf{\frac{9}{4}}$$

Method 2: Use the infinite geometric series sum formula: $S = a_1 \cdot \left(\frac{1}{1-r}\right)$

$a_1 = 3 \quad r = -\frac{1}{3}$

$$S = a_1 \cdot \left(\frac{1}{1-r}\right) = 3\left(\frac{1}{1-\left(-\frac{1}{3}\right)}\right) = \frac{3}{\frac{4}{3}} = \frac{3}{1}\cdot\frac{3}{4} = \mathbf{\frac{9}{4}}$$

Express the repeating decimal as a fraction in lowest terms.

14) $0.\overline{186}$ 14) ____________

Let $x = 0.\overline{186}$. Then think like our old buddy, Gauss.

$$1000x = 186.\overline{186}$$
$$-x = -0.\overline{186}.$$
$$999x = 186$$
$$x = \frac{186}{999} = \mathbf{\frac{62}{333}}$$

Recall the rule for divisibility by 3: If the sum of its digits is divisible by 3, then the number is divisible by 3.

Same for 9: If the sum of its digits is divisible by 9, then the number is divisible by 9.

Solve the problem. Round to the nearest dollar if needed.

15) Looking ahead to retirement, you sign up for automatic savings in a fixed-income 401K plan that pays 5% per year compounded annually. You plan to invest $3500 at the end of each year for the next 15 years. How much will your account have in it at the end of 15 years? 15) ____________

Note: I don't know what "Round to the nearest dollar if needed" means. I don't think we "need" to.

Let's look at the series that results from this. Note: deposits are made at the **end** of the year.

The first year's deposit will earn 5% per year for 14 years.

The second year's deposit will earn 5% per year for 13 years.

…

The final year's deposit will earn 5% per year for 0 years.

Then, $S = 3{,}500 \cdot [(1.05)^{14} + (1.05)^{13} + \cdots + 1]$

Look at the series inside the brackets in reverse order:

$[1 + (1.05)^1 + (1.05)^2 + \cdots + (1.05)^{14}]$

$a_1 = 3{,}500$ $\quad r = 1.05$ $\quad n = 15$ years

Then,

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 3{,}500 \cdot \left(\frac{1.05^{15} - 1}{1.05 - 1}\right) \sim 3{,}500 \cdot \frac{1.07893}{0.05} = \mathbf{\$75{,}524.97}$$

16) Kurt deposits $150 each month into an account paying annual interest of 6.5% compounded monthly. How much will his account have in it at the end of 5 years? 16) ______________

The answer to this question depends on when during the month Kurt deposits the $150. The formula taught in class **assumes the deposit is made at the end of the month**. If that is true, then,

$$S = \frac{P\left[\left(1+\frac{i}{n}\right)^{nt} - 1\right]}{\frac{i}{n}}$$

Note that I am using "i" for the annual interest rate because using "r" is confusing when relating this formula to other formulas.

$P =$ the amount of the monthly deposit ($P = 150$ in this problem)
$i =$ the annual interest rate ($i = .065$ in this problem)
$n =$ the interest compounding period ($n = 12$ for monthly compounding, 4 for quarterly, etc.)
$t =$ the number of years over which the interest accrues ($t = 5$ in this problem)

Then,

$$S = \frac{P\left[\left(1+\frac{i}{n}\right)^{nt} - 1\right]}{\frac{i}{n}} = \frac{150\left[\left(1+\frac{.065}{12}\right)^{12\cdot(5)} - 1\right]}{\frac{.065}{12}} = \$\mathbf{10,601.10}$$

Alternative to memorizing the above formula: Use the formula from Problem 15.

The first month's deposit will earn $\left(\frac{6.5}{12}\right)\%$ per month for 59 months.

The second month's deposit will earn $\left(\frac{6.5}{12}\right)\%$ per month for 58 months.

...

The final month's deposit will earn $\left(\frac{6.5}{12}\right)\%$ per month for 0 months.

Then, $S = 150 \cdot \left[\left(1+\frac{.065}{12}\right)^{59} + \left(1+\frac{.065}{12}\right)^{58} + \cdots + 1\right]$

Look at the series inside the brackets in reverse order:

$$\left[1 + \left(1+\frac{.065}{12}\right)^{1} + \left(1+\frac{.065}{12}\right)^{2} \ldots + \left(1+\frac{.065}{12}\right)^{59}\right]$$

$a_1 = 150$ $\quad r = \left(1+\frac{.065}{12}\right)$ $\quad n = 60$ months

Then,

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 150 \cdot \left[\frac{\left(1+\frac{.065}{12}\right)^{60} - 1}{\left(1+\frac{.065}{12}\right) - 1}\right] \sim \$\mathbf{10,601.10}$$

Note that if the deposits were made at the beginning of the month, they would all earn an extra month's interest, so the total at the end of 15 years would be: $\$10{,}601.10 \cdot \left(1+\frac{.065}{12}\right) = \$10{,}658.52$.

17) Laura invests $225 each quarter in a fixed-interest mutual fund paying annual interest of 5% compounded quarterly. How much will her account have in it at the end of 6 years? 17) ____________

The answer to this question depends on when during the quarter Laura deposits the $225. The formulas taught in class **assume the deposit is made at the end of the quarter**. If that is true, then, **using the formula from Problem 15:**

5% annual interest, compounded quarterly is $\frac{5\%}{4} = 1.25\%$ per quarter.

The first quarter's deposit will earn 1.25% per quarter for 23 quarters.

The second quarter's deposit will earn 1.25% per quarter for 22 quarters.

…

The final quarter's deposit will earn 1.25% per quarter for 0 quarters.

Then, $S = 150 \cdot [(1.0125)^{23} + (1.0125)^{22} + \cdots + 1]$

Look at the series inside the brackets in reverse order:

$[1 + (1.0125)^1 + (1.0125)^2 \ldots + (1.0125)^{23}]$

$a_1 = 225 \qquad r = 1.0125 \qquad n = 24$ quarters

Then,

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 225 \cdot \left[\frac{(1.0125)^{24} - 1}{(1.0125) - 1}\right] \sim \mathbf{\$6,252.32}$$

Solve the problem.

18) A small business owner made $50,000 the first year he owned his store and made an additional 9% over the previous year in each subsequent year. Find how much he made during his fourth year of business. Find his total earnings during the first four years. (Round to the nearest cent, if necessary.) 18) ____________

Method 1: Add 'em up

1st year: $\$50{,}000$

2nd year: $\$50{,}000 \cdot 1.09 = \$54{,}500$

3rd year: $\$50{,}000 \cdot 1.09^2 = \$59{,}405$

4th year: $\$50{,}000 \cdot 1.09^3 = \mathbf{\$64,751.45}$

Total: $\$(50{,}000 + 54{,}500 + 59{,}405 + 64{,}751.45) = \mathbf{\$228,656.45}$

Method 2: By formula,

$a_1 = 50{,}000 \qquad r = 1.09 \qquad n = 4$

4th year: $\$50{,}000 \cdot 1.09^{(4-1)} = \$50{,}000 \cdot 1.09^3 = \mathbf{\$64,751.45}$

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 50{,}000\left(\frac{1.09^4 - 1}{1.09 - 1}\right) = \mathbf{\$228,656.45}$$

19) A job pays a salary of 34,000 the first year. During the next 8 years, the salary increases by 4% each year. What is the salary for the 9th year? What is the total salary over the 9–year period? (Round to the nearest cent.) 19) ____________

9th year salary:

$$a_9 = a_1 \cdot r^{(9-1)} = a_1 \cdot r^8$$

$$a_1 = 34{,}000 \qquad r = 1.04 \qquad n = 9$$

$$a_9 = 34{,}000 \cdot 1.04^8 = \$46,531.35$$

Total salary over 9 years:

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 34{,}000\left(\frac{1.04^9 - 1}{1.04 - 1}\right) = \$359,815.04$$

20) A hockey player signs a contract with a starting salary of $810,000 per year and an annual increase of 6.5% beginning in the second year. What will the athlete's salary be, to the nearest dollar, in the eighth year? 20) ____________

8th year salary:

$$a_8 = a_1 \cdot r^{(8-1)} = a_1 \cdot r^7$$

$$a_1 = 810{,}000 \qquad r = 1.065 \qquad n = 8$$

$$a_8 = 810{,}000 \cdot 1.065^7 = \$1,258,729$$

Total salary over 8 years (not part of the problem, but still fun):

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = 810{,}000\left(\frac{1.065^8 - 1}{1.065 - 1}\right) = \$8,162,254$$

Maybe it's worth losing a few teeth!

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Evaluate the given binomial coefficient.

21) $\binom{10}{5}$ 21) ______

A) 126 B) 252 C) 504 D) 30,240

$$\binom{10}{5} = \frac{10!}{5! \cdot (10-5)!} = \frac{10!}{5! \cdot 5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 252$$

Answer B

Use the Binomial Theorem to expand the binomial and express the result in simplified form.

22) $(2x - 1)^5$ 22) ______

A) $(4x^2 - 4x + 1)^5$
B) $32x^5 + 10x^4 - 40x^3 - 40x^2 + 10x - 1$
C) $32x^5 - 16x^4 + 8x^3 - 4x^2 + 2x - 1$
D) $32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1$

General Formula:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Step 1: Start with the binomial coefficients

$$\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$$

Step 2: Add in the powers of the first term of the binomial ($2x$)

$$\binom{5}{0}(2x)^5 + \binom{5}{1}(2x)^4 + \binom{5}{2}(2x)^3 + \binom{5}{3}(2x)^2 + \binom{5}{4}(2x)^1 + \binom{5}{5}(2x)^0$$

Step 3: Add in the powers of the second term of the binomial (-1)

$$\binom{5}{0}(2x)^5(-1)^0 + \binom{5}{1}(2x)^4(-1)^1 + \binom{5}{2}(2x)^3(-1)^2 + \binom{5}{3}(2x)^2(-1)^3 + \binom{5}{4}(2x)^1(-1)^4 + \binom{5}{5}(2x)^0(-1)^5$$

Step 4: Simplify:

$$= (1)(32x^5)(1) + (5)(16x^4)(-1) + (10)(8x^3)(1) + (10)(4x^2)(-1) + (5)(2x)(1) + (1)(1)(-1)$$

$$= \mathbf{32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1}$$

Answer D

Note on this problem: It is not necessary to compute the entire binomial expansion to answer this problem. If you notice that the x^4 term is different in all four answers, **you need only determine the x^4 term to figure out which answer is correct.**

Notice the following about a binomial expansion:

1. There are $(n+1)$ terms, where n is the exponent of the binomial being expanded.
2. n is the top number in every binomial coefficient.
3. The bottom numbers in the binomial coefficients count up from **0** to n.
4. When a term of the original binomial is negative, the signs in the solution alternate.
5. The exponent of the first term in the original binomial counts down from n to **0**.
6. The exponent of the second term in the original binomial counts up from **0** to n.
7. The exponents of the two terms in the original binomial add to n in every term of the expansion.

Find the term indicated in the expansion.

23) $(x^2 + y^4)^9$; 6th term — 23) ______

A) $756x^6y^9$ B) $126x^6y^9$ C) $756x^8y^{20}$ D) $126x^8y^{20}$

General Formula:

$$(\boldsymbol{a} + \boldsymbol{b})^{\boldsymbol{n}} = \sum_{\boldsymbol{k}=0}^{\boldsymbol{n}} \binom{\boldsymbol{n}}{\boldsymbol{k}} \boldsymbol{a}^{\boldsymbol{n}-\boldsymbol{k}} \boldsymbol{b}^{\boldsymbol{k}}$$

KEY POINT: Unfortunately, there are several ways to answer this question, based on how the "6th term" is defined. In order to be consistent with the Pearson textbook and homework problems, we must set the value of k to be one less than the number of the term. Using this approach, the first term has $k = 0$, so the 6th term has $k = 5$. Other sources name the terms differently, but we are concerned with how the solutions to tests and homework are handled in Washoe County.

The terms of the binomial expansion of $(a + b)^n$ are typically given by the formula:

$$\binom{n}{k} a^{n-k} b^k$$

Then, using the approach in the Pearson textbook for this problem:

$a = x^2$ $\quad b = y^4$ $\quad n = 9$ $\quad term = 6$ $\quad$ Pearson textbook: $k = 5$

And, so,

$$\binom{n}{k} a^{n-k} b^k = \binom{9}{5} (x^2)^{9-5} (y^4)^5 = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} (x^2)^4 (y^4)^5 = \mathbf{126}\boldsymbol{x}^{\mathbf{8}}\boldsymbol{y}^{\mathbf{20}}$$

Answer D