7.1 Notes: Solving Systems of Equations

Substitution:

Elimination:

1) $\begin{cases} 3x + 2y = 4 \\ y = -2x + 1 \end{cases}$

2) $\begin{cases} 2x - 3y = 9 \\ 3x + 4y = 8 \end{cases}$

Special Solutions:

3) $\begin{cases} 5x - 2y = 4 \\ -10x + 4y = 7 \end{cases}$

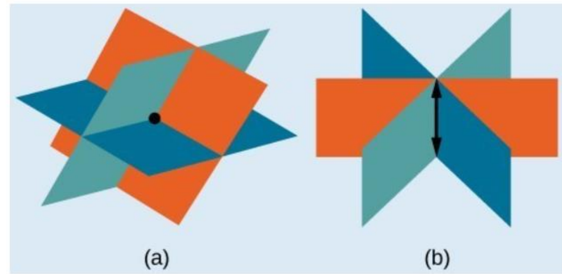
4) $\begin{cases} x = 4y - 8 \\ 5x - 20y = -40 \end{cases}$

7.1 Notes, continued.

Example 5) A chemist needs to mix an 18% acid solution with a 45% acid solution to obtain 12 liters of a 36% acid solution. How many liters of each of the acid solutions must be used?

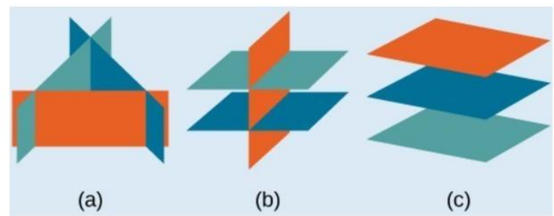
7.2 Notes: Systems with Three Variables

What do equations with three variables represent?



(a)Three planes intersect at a single point, representing a three-by-three system with a single solution. (b) Three planes intersect in a line, representing a three-by-three system with infinite solutions.

What do solutions of equations with 3 variables represent?



Example 1: Is the ordered triple $(-1, -4, 5)$ a solution of the system below?

$$\begin{cases} x - 2y + 3z = 22 \\ 2x - 3y - z = 5 \\ 3x + y - 5z = -32 \end{cases}$$

7.2 Notes, continued.

For examples 2 – 3: Solve each system.

2) $$\begin{cases} x + 4y - z = 20 \\ 3x + 2y + z = 8 \\ 2x - 3y + 2z = -16 \end{cases}$$

3) $$\begin{cases} 2y - z = 7 \\ x + 2y + z = 17 \\ 2x - 3y + 2z = -1 \end{cases}$$

7.3 Notes: Partial Fractions

Exploration: Can you fill in the blanks to make a true statement? $\frac{7}{12} = \frac{\ }{3} + \frac{\ }{4}$

Partial Fraction Decomposition: The process of starting with one simplified fraction and splitting into two fractions whose sum is the original fraction.

- "Decomposing" the final expression into its initial polynomial fractions.
  - Sample: Going from $\frac{x+14}{x^2-2x-8}$ back to $\frac{3}{x-4} - \frac{2}{x+2}$
- Certain processes in calculus can only be done with fractions that have been decomposed.

Forms for Partial Fraction Decomposition: The form depends on the factors of the denominator. Some common forms are below.

Form of denominator	Form of the rational function	Form of the partial function
Product of distinct linear factors	$\frac{px+q}{(x-a)(x-b)}$, $a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
Product of linear factors, one is repeated	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
Product of distinct linear factors	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
Product of linear factors, one is repeated	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
Product includes a prime quadratic factor	$\frac{px^2+qx+r}{(x-a)(x^2-bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2-bx+c}$

* $x^2 - bx + c$ **cannot be factored further**

What would the form look like if the denominator has a repeated prime quadratic factor?

$$\frac{px^4+qx^3+rx^2+sx+t}{(x-a)(x^2-bx+c)^2}$$

Important: The numerator will always be one degree less than the factor of the denominator.

7.3 Notes, continued.

For examples 1 – 4: Write the ***form*** of the partial fraction decomposition of the rational expression. Do not solve for the constants.

1) $\frac{5x+7}{(x-1)(x+3)}$

2) $\frac{3x+16}{(x+1)(x-2)^2}$

3) $\frac{5x^2-6x+7}{(x-1)(x^2+5x+1)}$

4) $\frac{7x^2-9x+3}{(x^2+7)^2}$

Steps for Partial Fraction Decomposition:

1. Write the form of the partial fraction decomposition.
2. Multiply each ratio to have the **same denominator** as the original denominator.
3. Set the new numerators equal.
4. Solve for the constants… use a combination of the strategies below.
    a. Let the variable equal roots of the denominator and solve.
    b. Set like terms equal to create equations (and perhaps systems) to solve.

For #5 – 12: Find the partial fraction decomposition for each fraction.

5) $\frac{5x-1}{(x+4)(x-3)}$

6) $\frac{x+2}{x(x-1)^2}$

7.3 Notes, continued.

7) $\frac{8x^2+12x-20}{(x+3)(x^2+x+2)}$

8) $\frac{2x^3+x+3}{(x^2+1)^2}$

9) $\frac{x+14}{(x-4)(x+2)}$

7.3 Notes, continued.

10) $\frac{x-18}{x(x+3)^2}$

11) $\frac{5x^3-3x^2+7x-3}{(x^2+1)^2}$

12) $\frac{3x^2+17x+14}{(x-2)(x^2+2x+4)}$

7.4 Notes: Systems of Nonlinear Equations

Linear versus non-linear equations:

Types of solutions:

Examples 1 – 7: Solve each system for the solution(s).

1) $\begin{cases} x^2 = y - 1 \\ 4x - y = -1 \end{cases}$

By using elimination

By using substitution

7.4 Notes, continued.

2) $\begin{cases} x + 2y = 0 \\ (x-1)^2 + (y-1)^2 = 5 \end{cases}$

3) $\begin{cases} 3x^2 + 2y^2 = 35 \\ 4x^2 + 3y^2 = 48 \end{cases}$

4) $\begin{cases} y = x^2 + 5 \\ x^2 + y^2 = 25 \end{cases}$

7.4 Notes, continued.

5) $\begin{cases} xy = -12 \\ x - 2y + 14 = 0 \end{cases}$

6) $\begin{cases} x + y = 1 \\ x^2 + xy - y^2 = -5 \end{cases}$

7) $\begin{cases} x^2 - y^2 - 4x + 6y - 4 = 0 \\ x^2 + y^2 - 4x - 6y + 12 = 0 \end{cases}$

7.4 Notes, continued.

Examples 8 – 9: Create a system to solve each problem.

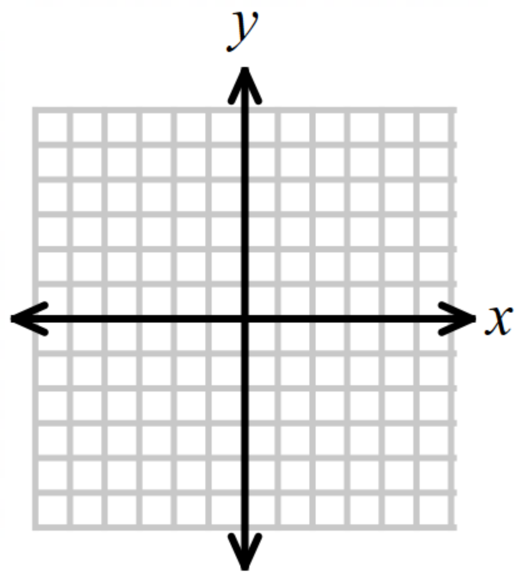
8) Find the length and width of a rectangle whose perimeter is 20 feet and whose area is 21 square feet.

9) A planet's orbit follows a path described by $16x^2 + 4y^2 = 64$. A comet follows the parabolic path y = $x^2 - 4$. Where might the comet intersect the orbiting planet?

7.5 Notes: Systems of Inequalities

Exploration: Try this with your group. Graph the solution set for the following system.

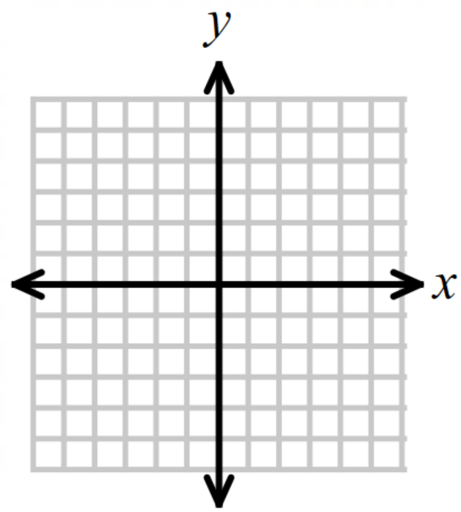
$$\begin{cases} x - y < 1 \\ 2x + 3y \geq 12 \end{cases}$$



Vertex Form of a Parabola	Standard Form of a Circle

Review of graphing with inequalities

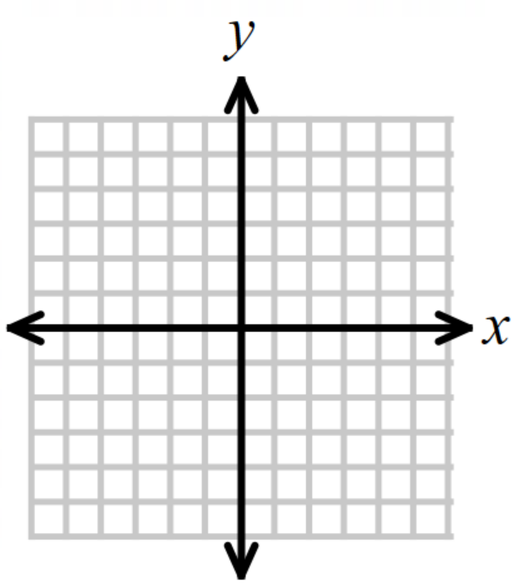
1) Graph: $x^2 + y^2 \geq 16$

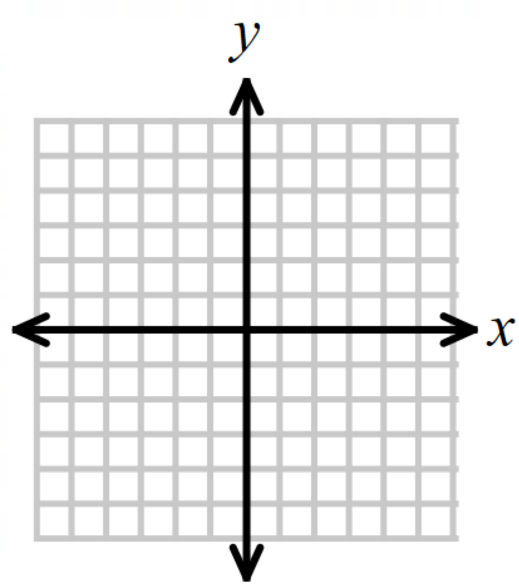


2) Show that the point (66, 130) is a solution to the system of inequalities: $\begin{cases} 4.9x - y \geq 165 \\ 3.7x - y \leq 125 \end{cases}$

3) $\begin{cases} x - 3y < 6 \\ 2x + 3y \leq 0 \end{cases}$

4) $\begin{cases} y \geq x^2 - 4 \\ x + y \leq 2 \end{cases}$

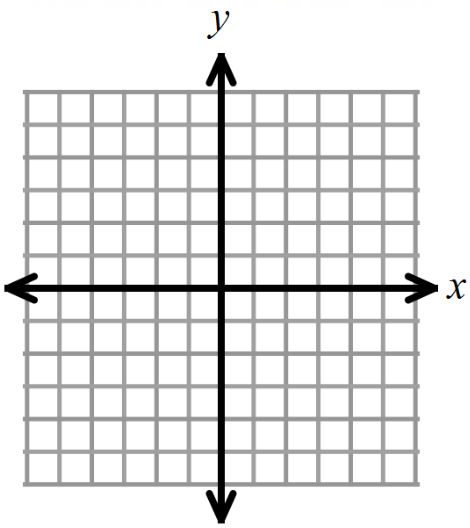


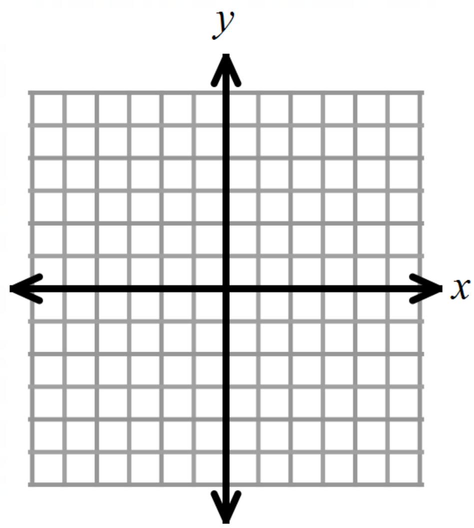


7.5 Notes, continued.

5) $\begin{cases} 3y \geq -2x+6 \\ -2x+4y<0 \end{cases}$

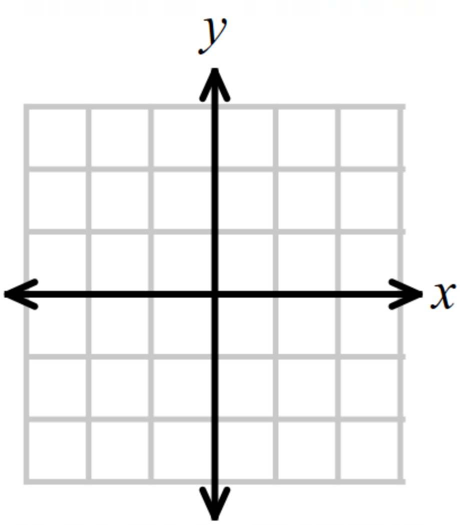
6) $\begin{cases} x+y<2 \\ -2 \leq x < 1 \\ y > -3 \end{cases}$

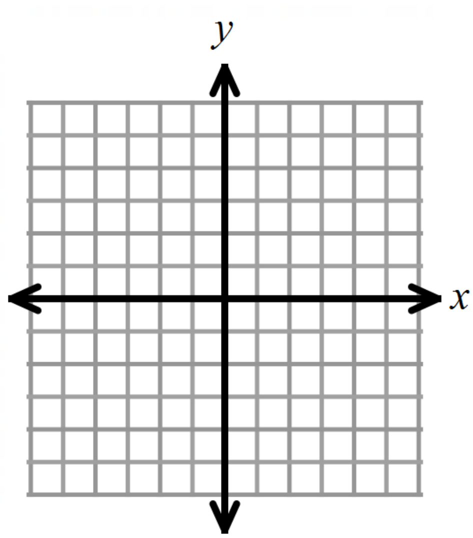




7) $\begin{cases} x^2+y^2 \leq 1 \\ y-x^2>0 \end{cases}$

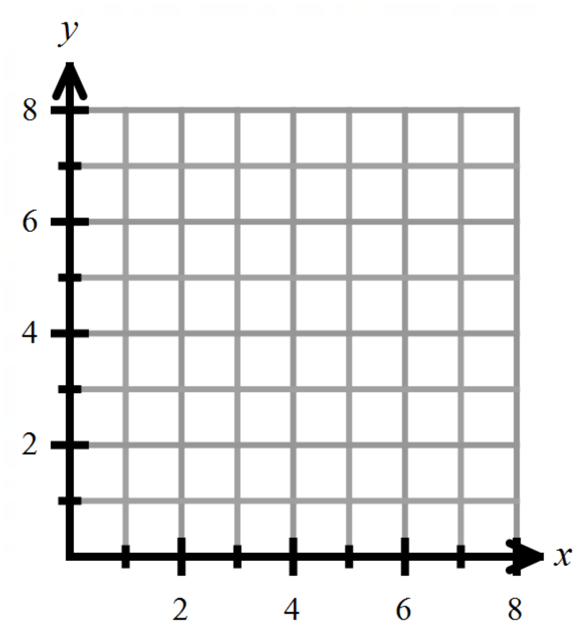
8) $\begin{cases} x^2+y^2<16 \\ y \geq 2^x \end{cases}$





7.6 Notes: Linear Programming

Example 1) A gold processor has two sources of gold ore, source A and Source B. To keep this plant running, at least three tons of ore must be processed each day. Ore from source A costs \$20 per ton to process, and ore from source B costs \$10 per ton to process. Costs must be kept to no more than \$80 per day. Moreover, Federal Regulations require that the amount of ore from source B cannot exceed twice the amount of ore from source A. If ore from source A yields 2 oz. of gold per ton, and ore from source B yields 3 oz. of gold per ton, how many tons of ore from both sources must be processed each day to maximize the amount of gold extracted subject to the above constraints?



Let x =

Let y =

Objective Function:

Constraints:

Solution:

7.6 Notes, continued.

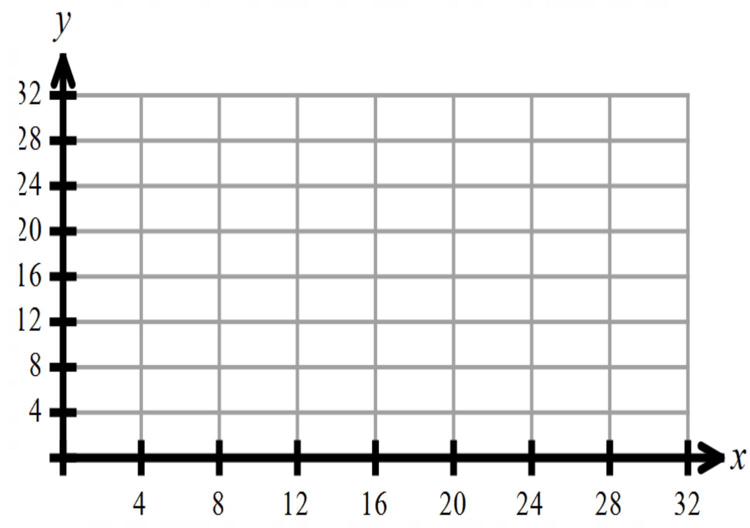
Example 2) A small shoe manufacturer makes two styles of shoes: oxfords and loafers. Two machines are used in the process: a cutting machine and a sewing machine. Each type of shoe requires 15 minutes per pair on the cutting machine. Oxfords require 10 min of sewing per pair, and loafers require 20 min of sewing per pair. Because the manufacturer can only hire one operator for each machine, each process is available for just 8 hours per day. If the profit is $15 on each pair of oxfords and $20 on each pair of loafers, how many pairs of each type should be produced per day for maximum profit?

Let x =

Let y =

Objective Function:

Constraints:



Solution:

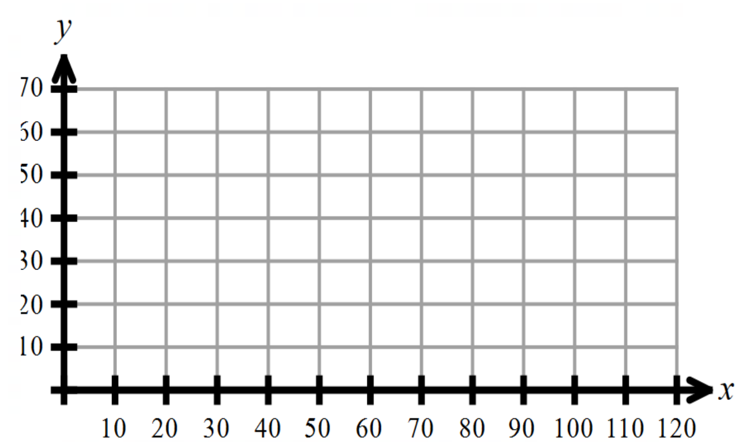
Example 3) A company makes ink-jet and laser printers and can make up to 60 printers per day using 120 labor hours. It takes one labor hour to make an ink-jet printer, which has a profit of $40 per printer. It takes three labor hours to make a laser printer with a profit of $60 per printer. Find the number of each type of printer that should be made to maximize the daily profits.

Let x =

Let y =

Objective Function:

Constraints:



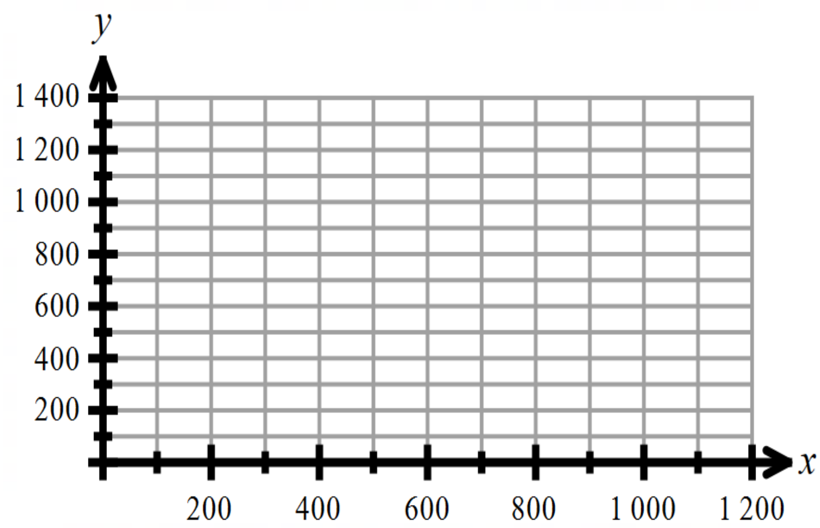
Solution:

7.6 Notes, continued.

Example 4) A company manufactures golf balls and tennis balls. Golf balls are made on Machine A for 10 minutes and Machine B for 6 minutes. Tennis balls need 5 minutes on Machine A, 8 minutes on Machine B, and 4 minutes on Machine C. The company must make at least 100 golf balls and 200 tennis balls per week. Machine A is available up to 6500 minutes per week. Machine B is available for 6400 minutes per week, and Machine C is available for 2300 minutes per week. The profit for one golf ball is $1.25, and each tennis ball earns $1.35 in profit. Find the maximum weekly profit.

Let x =

Let y =



Objective Function:

Constraints:

Solution: