PreCalculus **Chapter 3 Practice Test** **Name:_______________________**

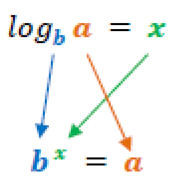
A note on terminology: Zeros and roots are the same thing. If they are real, as opposed to complex, they are also x-intercepts of your graph.

Write the equation in its equivalent exponential or logarithmic form:

To convert between an exponential expression and a logarithmic expression, it is often helpful to use the "first-last-middle" rule to perform the conversion. If necessasy, set the expression equal to x before applying the rule.

Note: the "first-last-middle" rule requires that the logarithmic or exponential portion of the expression be on the left-hand side of the equation.

Converting from Logarithmic Form to Exponential Form



using "first-last-middle"

Examples:

1) Solve for x: $log_4 64 = x$.

First is "4", last is "x" and middle is "64." So, $4^x = 64$.

Then, $4^1 = 4$; $4^2 = 16$; $4^3 = 64$ ✓

So, we have: $x = 3$

2) Solve for x: $\ln e = x$

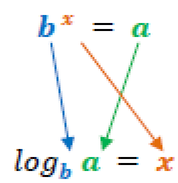
(remember ln is shorthand for log_e)

Using first-last-middle,

$log_e\, e = x$ converts to: $e^x = e$

So, we have: $x = 1$

Converting from Exponential Form to Logarithmic Form



using "first-last-middle"

Examples:

1) Convert the expression, $2^5 = 32$ to logarithmic form.

First is "2", last is "32" and middle is "5".

So, we have: $log_2 32 = 5$

2) Convert the expression, $7^3 = 343$ to logarithmic form.

Using first-last-middle,

$7^3 = 343$ converts to: $log_7 343 = 3$

So, we have: $log_7 343 = 3$

1. $\log_b 64 = 2$

In the log expression, $\log_b 64 = 2$, **first** is "b", **last** is "2" and **middle** is "64." We put these in an exponential expression, from left to right, to get: $b^2 = 64$.

2. $\log_6 216 = x$

In the log expression, $\log_6 216 = x$, **first** is "6", **last** is "x" and **middle** is "216." We put these in an exponential expression, from left to right, to get: $\mathbf{6^x = 216}$.

3. $2^3 = x$

First-last-middle works this way too.

In the exponential expression, $2^3 = x$, **first** is "2", **last** is "x" and **middle** is "3." We put these in a logarithmic expression, from left to right, to get: $\mathbf{\log_2 x = 3.}$

4. $2^{(-2)} = \frac{1}{4}$

In the exponential expression, $2^{-2} = \frac{1}{4}$, **first** is "2", **last** is "$\frac{1}{4}$" and **middle** is "-2." We put these in a logarithmic expression, from left to right, to get: $\mathbf{\log_2 \left(\frac{1}{4}\right) = -2.}$

Evaluate the following without the use of a calculator:

5. $\log_{10} 10$

In the log expression, $\log_{10} 10 = x$, **first** is "10", **last** is "x" and **middle** is "10." We put these in an exponential expression, from left to right, to get: $10^x = 10$, then solve:

$$\log_{10} 10 = x \quad \text{converts to:} \quad 10^x = 10 \longrightarrow \mathbf{x = 1}$$

6. $\log_3 \sqrt{3}$

In the log expression, $\log_3 \sqrt{3} = x$, **first** is "3", **last** is "x" and **middle** is "$\sqrt{3}$." We put these in an exponential expression, from left to right, to get: $3^x = \sqrt{3}$, then solve:

$$\log_3 \sqrt{3} = x \quad \text{converts to:} \quad 3^x = \sqrt{3} \longrightarrow \mathbf{x = \frac{1}{2}}$$

Note: $\sqrt{3} = 3^{1/2}$

7. $\log_6 1$

In the log expression, $\log_6 1 = x$, **first** is "6", **last** is "x" and **middle** is "1." We put these in an exponential expression, from left to right, to get: $6^x = 1$, then solve:

$\log_6 1 = x$ converts to: $6^x = 1$ $\longrightarrow$ $\boldsymbol{x = 0}$

Problems 8 to 9: Exponentiation and taking logarithms are inverse operations, so when they both exist, *with the same base*, they cancel each other out.

8. $6^{(\log_6 15)}$

$$\boldsymbol{6^{(\log_6 15)} = 15}$$

9. $\log_7 7^{(18)}$

$$\boldsymbol{\log_7(7^{18}) = 18}$$

Note: I like to use parentheses to make it easier to read a problem. For example, Problems 8 and 9 have values at multiple levels. By using parentheses, I can make it easier to see what is going on in the problem.

10. $\log 1000$

Back to "**first-last-middle**."

In the log expression, $\log_{10} 1000 = x$, **first** is "10", **last** is "x" and **middle** is "1000." We put these in an exponential expression, from left to right, to get: $10^x = 1000$, then solve:

$\log_{10} 1000 = x$ converts to: $10^x = 1000$ $\longrightarrow$ $\boldsymbol{x = 3}$

11. $\log 10^7$

In the log expression, $\log_{10}(10^7) = x$, **first** is "10", **last** is "x" and **middle** is "10^7." We put these in an exponential expression, from left to right, to get: $10^x = 10^7$, then solve:

$\log_{10} 10^7 = x$ converts to: $10^x = 10^7$ $\longrightarrow$ $\boldsymbol{x = 7}$

12. $\ln e$

Note that $\ln x$ is equivalent to $\log_e x$. Then,

In the log expression, $\log_e e = x$, **first** is "e", **last** is "x" and **middle** is "e." We put these in an exponential expression, from left to right, to get: $e^x = e$, then solve:

$\log_e e = x$ converts to: $e^x = e$ $\longrightarrow$ $\boldsymbol{x = 1}$

Expand or condense the following expressions:

13. $\log_2(8x)$

$$\log_2 8x = \log_2 8 + \log_2 x = \mathbf{3 + \log_2 x}$$

14. $\log_5\left(\frac{125}{x}\right)$

$$\log_5\left(\frac{125}{x}\right) = \log_5 125 - \log_5 x = \mathbf{3 - \log_5 x}$$

15. $\log_b(yz^4)$

$$\log_b(yz^4) = \log_b(y) + \log_b(z^4) = \mathbf{\log_b y + 4\log_b z}$$

16. $\log_4\left(\frac{x-6}{x^5}\right)$

$$\log_4\left(\frac{x-6}{x^5}\right) = \log_4(x-6) - \log_4(x^5) = \mathbf{\log_4(x-6) - 5\log_4 x}$$

17. $\log_4(x-8) - \log_4(x-4)$

$$\log_4(x-8) - \log_4(x-4) = \mathbf{\log_4\left(\frac{x-8}{x-4}\right)}$$

18. $3\log_6 x + 5\log_6(x-6)$

$$3\log_6 x + 5\log_6(x-6) = \log_6 x^3 + \log_6(x-6)^5 = \mathbf{\log_6\left[x^3(x-6)^5\right]}$$

19. $4\log_x 2 - \log_x 8$

$$4\log_x 2 - \log_x 8 = \log_x 2^4 - \log_x 2^3 = \log_x \frac{2^4}{2^3} = \mathbf{\log_x 2}$$

Alternatively,

$$4\log_x 2 - \log_x 8 = 4\log_x 2 - \log_x 2^3 = 4\log_x 2 - 3\log_x 2 = \mathbf{\log_x 2}$$

Solve the following equations:

20. $4^{(1+2x)} = 64$

Starting equation:	$4^{(1+2x)} = 64$
Take the "$\log_4$" of both sides:	$1 + 2x = 3$
Subtract 1:	$2x = 2$
Divide by 2:	$\boldsymbol{x = 1}$

21. $e^{(x+8)} = \frac{1}{e^4}$

Starting equation:	$e^{(x+8)} = \frac{1}{e^4}$
Convert exponent on the right:	$e^{(x+8)} = e^{-4}$
Take the "ln" of both sides:	$x + 8 = -4$
Subtract 8:	$\boldsymbol{x = -12}$

Solve the exponential equation. Give the exact answer (no decimals).

22. $5^{(x+7)} = 3$

Starting equation:	$5^{(x+7)} = 3$
Take the "ln" of both sides:	$\ln 5^{(x+7)} = \ln 3$
Simplify:	$(x + 7)\ln 5 = \ln 3$
Divide by $\ln 5$:	$x + 7 = \frac{\ln 3}{\ln 5}$
Subtract 7:	$\boldsymbol{x = \frac{\ln 3}{\ln 5} - 7}$

Note: I tend to use ln in solving problems like this. However, you can use logs with any base. Other useful bases for this problem might be log, $\log_3$ or $\log_5$.

23. $e^{(x+4)} = 2$

Starting equation:	$e^{(x+4)} = 2$
Take the ln of both sides:	$\ln e^{(x+4)} = \ln 2$
Simplify:	$x + 4 = \ln 2$
Subtract 4:	$\boldsymbol{x = \ln 2 - 4}$

24. $\log_3(x-1) = -1$

Starting equation:	$\log_3(x-1) = -1$
Take 3 to the power of both sides:	$3^{\log_3(x-1)} = 3^{-1}$
Simplify:	$x - 1 = \frac{1}{3}$
Add 1:	$\boldsymbol{x = \frac{4}{3}}$

25. 4 + 8 ln (x) = 8

Starting equation:	$4 + 8\ln x = 8$
Subtract 4:	$8\ln x = 4$
Divide by 8:	$\ln x = \frac{1}{2}$
Take e to the power of both sides:	$e^{\ln x} = e^{1/2}$
Simplify:	$\boldsymbol{x = e^{1/2} = \sqrt{e}}$

26. $\log_6 x + \log_6(x-35) = 2$

Starting equation: $\log_6 x + \log_6(x-35) = 2$

Combine log terms: $\log_6[x \cdot (x-35)] = 2$

Take 6 to the power of both sides: $6^{\log_6[x \cdot (x-35)]} = 6^2$

Simplify: $x \cdot (x-35) = 36$

Distribute x: $x^2 - 35x = 36$

Subtract 36: $x^2 - 35x - 36 = 0$

Factor: $(x-36)(x+1) = 0$

Determine solutions for x: $x = \{36, -1\}$

Note: To test the solutions you derive, use the original equation or a simplified form of the original equation.

Test the solutions of x: $x = 36$: $\log_6 36 + \log_6(36-35) = 2$ ✓

$x = -1$: $\log_6(-1) + \log_6(-1-35) = 2$ ✗

These terms are both Invalid because negative numbers are not in the domain of the log function.

Final solution: $\mathbf{x = 36}$

27. $\log_6(5x-5) = \log_6(3x+7)$

Starting equation: $\log_6(5x-5) = \log_6(3x+7)$

Take 6 to the power of both sides: $6^{\log_6(5x-5)} = 6^{\log_6(3x+7)}$

Simplify: $5x - 5 = 3x + 7$

Add 5: $5x = 3x + 12$

Subtract $3x$: $2x = 12$

Divide by 2: $x = 6$

Test the solution of x: $\log_6(5 \cdot 6 - 5) = \log_6(3 \cdot 6 + 7)$ ✓

Final solution: $\mathbf{x = 6}$

28. $\log_{14}(x+5) = 1 - \log_{14} x$

Starting equation: $\log_{14}(x+5) = 1 - \log_{14} x$

Add $\log_{14} x$ to both sides: $\log_{14}(x+5) + \log_{14} x = 1$ ← Use this equation to test your solutions below.

Combine log terms: $\log_{14}[(x+5)\cdot x] = 1$

Take 14 to the power of both sides: $14^{\log_{14}[(x+5)\cdot x]} = 14^1$

Simplify: $(x+5)\cdot x = 14$

Multiply terms: $x^2 + 5x = 14$

Subtract 14: $x^2 + 5x - 14 = 0$

Factor: $(x-2)(x+7) = 0$

Determine solutions for x: $x = \{2, -7\}$

Test the solutions of x: $x = 2$: $\log_{14}(2+5) + \log_{14} 2 = 1$ ✓

$x = -7$: $\log_{14}(-7+5) + \log_{14}(-7) = 1 = 2$ ✗

These terms are both Invalid because negative numbers are not in the domain of the log function.

Final solution: $\mathbf{x = 2}$

29. $4^{(x+9)} = 8^{(x-2)}$

Starting equation: $4^{(x+9)} = 8^{(x-2)}$

Convert 4 and 8 to powers of 2: $(2^2)^{(x+9)} = (2^3)^{(x-2)}$

Simplify: $2^{2x+18} = 2^{3x-6}$

Equate the exponents: $2x + 18 = 3x - 6$

Subtract $2x$: $18 = x - 6$

Add 6: $24 = x$

Note: You can only equate exponents when the bases are the same.

Final solution: $\mathbf{x = 24}$

Use the parent function to obtain the graph of the following:

For an exponential function of the form: $f(x) = b^{x-h} + k$, start with the parent function: $f(x) = b^x$, and shift the function h units right (shift left if h is negative) and k units up (shift down if k is negative). The horizontal asymptote is at $y = k$. One way to do all of this is to:

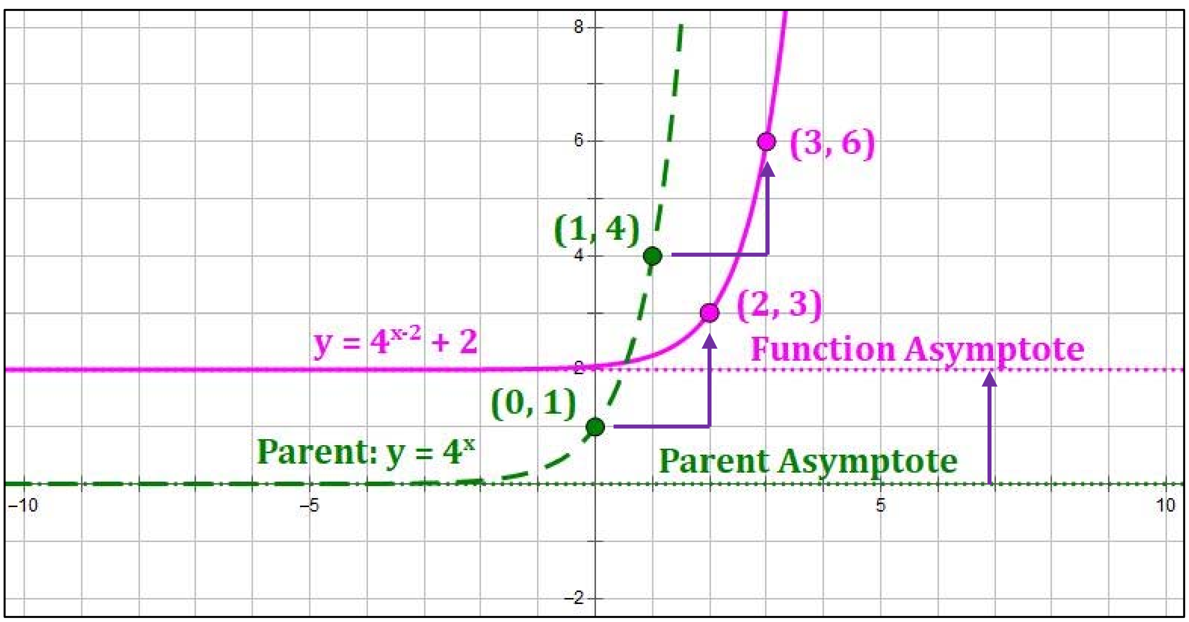
- Identify two points and any asymptotes on the parent function graph.
- Shift the two points and any asymptotes to set up the desired graph.
- Graph the desired function.
- Note: in the graphs that follow, all shifts are shown with purple arrows.

30. $g(x) = 4^{(x-2)} + 2$

The parent function is $y = 4^x$

Shift the parent function 2 units right and 2 units up.

Shift the horizontal asymptote of the parent, $y = 0$, 2 units up to become $y = 2$.

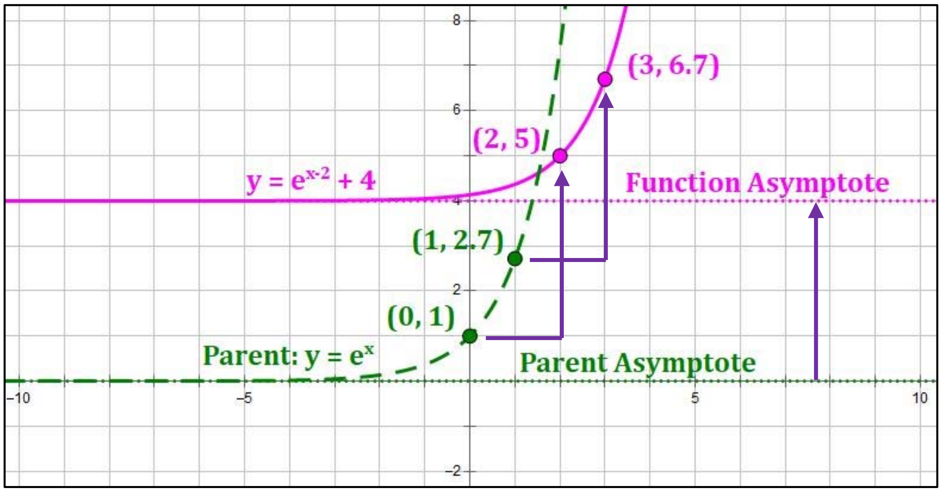


31. $g(x) = e^{(x-2)} + 4$

The parent function is $y = e^x$

Shift the parent function 2 units right and 4 units up.

Shift the horizontal asymptote of the parent, $y = 0$, 4 units up to become $y = 4$.



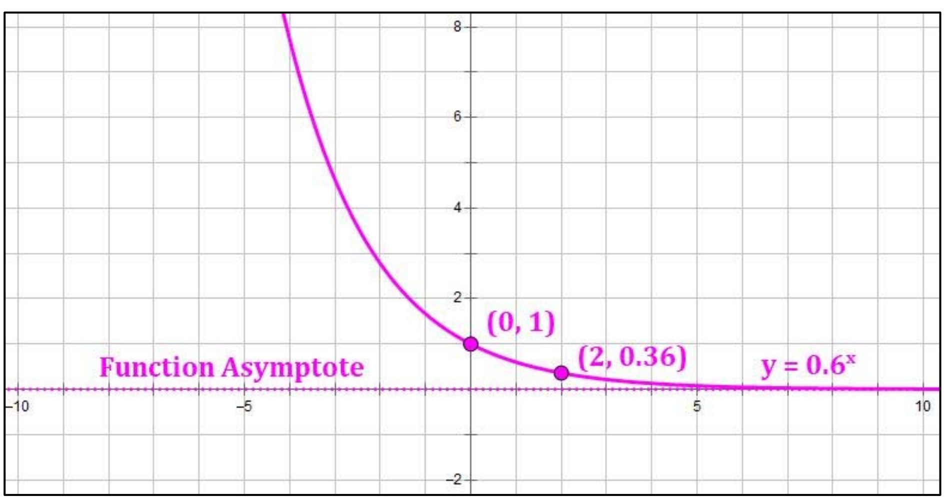
32. $f(x) = \left(\frac{3}{5}\right)^x$

The parent function is $y = \left(\frac{3}{5}\right)^x = 0.6^x$

There are no shifts.

Plot two points that are easy to identify on the curve: $(0, 1)$ and $(2, 0.36)$.

Run a decreasing exponential curve (since $\frac{3}{5} < 1$) through the points.

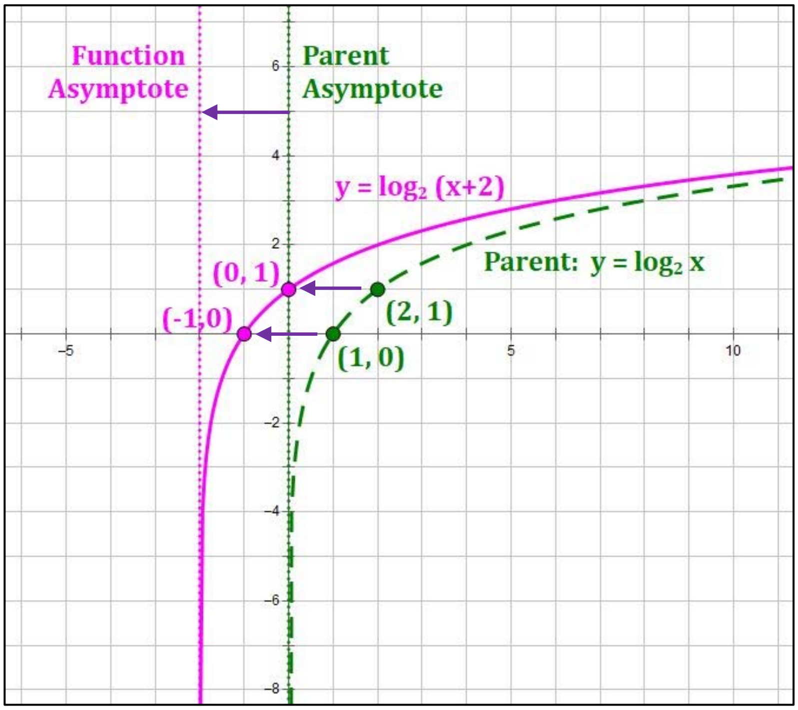


33. $f(x) = \log_2(x+2)$

The parent function is $y = \log_2 x$

Shift the parent function 2 units left. There is no vertical shift. $f(x) = \log_2(x-(-2)) + 0$

Shift the vertical asymptote of the parent, $x = 0$, 2 units left to become $x = -2$.



Use the compound interest formulas $A = P\left(1+\frac{r}{n}\right)^{nt}$ *and* $A = Pe^{rt}$ to solve:

34. Find the accumulated value of an investment of \$900 at 12% compounded quarterly for 6 years.

We are given: $P = \$900$ $r = 0.12$ $n = 4$ (quarterly compounding)

$t = 6$ years $\cdot$ 4 quarters $= 24$ periods

$$A = P\left(1+\frac{r}{n}\right)^{n\cdot t} = 900\cdot\left(1+\frac{0.12}{4}\right)^{24} = 900\cdot(1.03)^{24} = \$1,829.51$$

35. Find the accumulated value of an investment of \$4000 at 7% compounded continuously for 5 years.

We are given: $P = \$4,000$ $r = 0.07$ continuous compounding (use Pert formula)

$t = 5$ years

$$A = Pe^{rt} = 4000\cdot(e^{5\cdot 0.07}) = 4000\cdot(e^{0.35}) = \$5,676.27$$

Use Newton's Law of Cooling $T = C + (T_0 - C)e^{kt}$, to solve the problem.

36. Mostacolli baked at 375°F is taken out of the oven into a kitchen that is 68°F. After 5 minutes, the temperature of the mostacolli is 327°F. What will its temperature be 19 minutes after it was taken out of the oven? Round your answer to the nearest degree.

There are two steps to problems like this:

1) Find the value of k based on the change in temperature over the first 5 minutes.
2) Find the final temperature, T, after 19 minutes.

What are the variables?

$\boldsymbol{T}$ is the temperature of the object (mostacolli) at any point in time. $\boldsymbol{T}$ changes as $\boldsymbol{t}$ (time) changes.

$\boldsymbol{C}$ is the current temperature of the environment (e.g., the room).

$\boldsymbol{T_0}$ is the starting temperature of the object (mostacolli).

$\boldsymbol{k}$ is the rate of change in the temperature of the object (mostacolli) over time.

$\boldsymbol{t}$ is the time elapsed since the event (removal from the oven) occurred. Initially, $\boldsymbol{t} = 0$.

Step 1: Find the value of k.

We are given: $T_0 = 375$ $C = 68$ $t = 5$ $T = 327$

Starting equation: $T = C + (T_0 - C) \cdot e^{k \cdot t}$

Substitute known values: $327 = 68 + (375 - 68) \cdot e^{k \cdot 5}$

Subtract 68 from both sides: $259 = (375 - 68) \cdot e^{k \cdot 5}$

Simplify: $259 = (307) \cdot e^{5k}$

Divide by 307: $0.843648 = e^{5k}$

Take natural logs: $\ln(0.843648) = \ln(e^{5k})$

Simplify: $-0.17002 = 5k$

Divide by 5 to obtain k: $k = -0.0340039$

Step 2: Find the final temperature, T, after 19 minutes.

We are given: $T_0 = 375$ $C = 68$ $t = 19$ $k = -0.0340039$

Starting equation: $T = C + (T_0 - C) \cdot e^{k \cdot t}$

Substitute known values: $T = 68 + (375 - 68) \cdot e^{(-0.0340039) \cdot 19}$

Calculate the final value of T: $\boldsymbol{T = 229° \, F}$

Solve the following:

37. The half-life of silicon-32 is 710 years. If 50 grams is present now, how much will be present in 200 years? (Round your answer to three decimal places.)

There are two steps to problems like this:

1) Find the value of k based on the half-life of 710 years.
2) Find how much would be left after 200 years.

What are the variables?

The formula for exponential decay is: $A = A_0 e^{kt}$, where:

- A is the amount of substance left at time t.
- A_0 is the starting amount of the substance.
- k is the annual rate of decay.
- t is the number of years.

Step 1: Determine the value of k

We are given: $t = 710$, $\dfrac{A}{A_0} = \dfrac{1}{2}$ (because we are given a "half-life")

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\dfrac{A}{A_0} = e^{kt}$

Substitute in values: $\frac{1}{2} = e^{710\,k}$

Take natural logs: $\ln\frac{1}{2} = 710\,k$

Divide by 710: $k = \dfrac{\ln\frac{1}{2}}{710} = -0.00097626$

Step 2: Find how much is left after 200 years

We are given: are given: $t = 200$, $A_0 = 50$ grams $k = -0.00097626$

Starting equation: $A = A_0 e^{kt}$

Substitute in values: $A = 50 \cdot e^{(-0.00097626)\cdot 200} =$ **41.131 grams**

38. The logistic growth function $f(t) = \frac{87{,}000}{1+1449e^{-1.2t}}$ models the number of people who have become ill with a particular infection t weeks after its initial outbreak in a particular community. How many people were ill after 9 weeks?

This is a simple substitution problem. Substitute 9 for t and calculate the solution.

$$f(9) = \frac{87{,}000}{1 + 1{,}449 \cdot e^{(-1.2)\cdot 9}} = \frac{87{,}000}{1 + 1{,}449 \cdot e^{-10.8}} = \mathbf{84{,}502}$$

39. The population of a particular country was 30 million in 1984; in 1989 it was 37 million. The exponential growth function $A = 30e^{kt}$ describes the population of this country t years after 1984. Use the fact that 5 years after 1984 the population increased by 7 million to find k to three decimal places.

There is only one step to this problem:

1) Find the value of k based on the population change from 1984 to 1989.

What are the variables?

The formula for exponential decay is: $A = A_0e^{kt}$, where:

- A is the population at time t.
- A_0 is the starting population.
- k is the annual rate of growth.
- t is the number of years.

Step 1: Determine the value of k

We are given: $A = 37$, $A_0 = 30$, $t = 5$

Starting equation: $A = A_0e^{kt}$

Substitute in values: $37 = 30e^{k\cdot 5}$

Divide by 30: $\frac{37}{30} = e^{5k}$

Take natural logs: $\ln\left(\frac{37}{30}\right) = 5k$

Divide by 5: $k = \frac{\ln\left(\frac{37}{30}\right)}{5} = \mathbf{0.0419441}$

Round to 3 decimal places: $\mathbf{k = 0.042}$

40. A fossilized leaf contains 18% of its normal amount of carbon 14. How old is the fossil (to the nearest year)? Use 5600 years as the half-life of carbon 14.

There are two steps to problems like this:

1) Find the value of k based on the half-life of 5600 years.
2) Find how old the fossil is when there is 18% left.

What are the variables?

The formula for exponential decay is: $A = A_0 e^{kt}$, where:

- A is the amount of substance left at time t.
- A_0 is the starting amount of the substance.
- k is the annual rate of decay.
- t is the number of years.

Step 1: Determine the value of k.

We are given: $t = 5600$, $\frac{A}{A_0} = \frac{1}{2}$ (because we are given a "half-life")

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\frac{A}{A_0} = e^{kt}$

Substitute in values: $\frac{1}{2} = e^{5600\,k}$

Take natural logs: $\ln\frac{1}{2} = 5600\,k$

Divide by 5600: $k = \frac{\ln\frac{1}{2}}{5600} = -0.000123776$

Step 2: Find how old the fossil is when there is 18% left.

We are given: are given: $\frac{A}{A_0} = 18\%$ left, $k = -0.000123776$

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\frac{A}{A_0} = e^{kt}$

Substitute in values: $0.18 = e^{(-0.000123776)\cdot t}$

Take natural logs: $\ln(0.18) = -0.000123776 \cdot t$

Divide by (-0.000123776): $t = \frac{\ln(0.18)}{-0.000123776} = \mathbf{13,854}$ **years old**