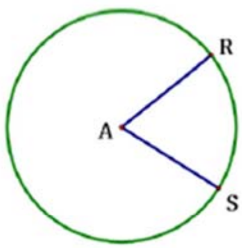
 Name:__________________________

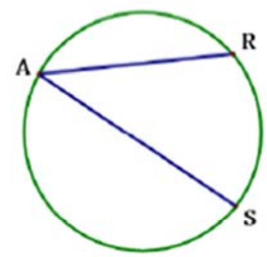
Formulas from the Geometry Handbook:

Central Angle



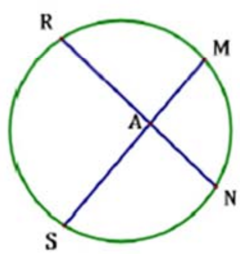
$m\angle A = m\,\widehat{RS}$

Inscribed Angle



$m\angle A = \frac{1}{2}\,m\,\widehat{RS}$

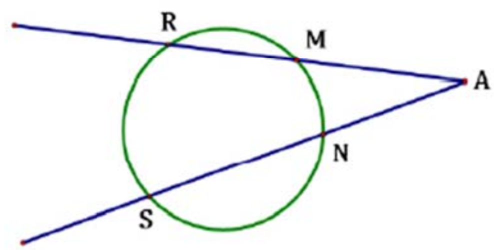
Vertex inside the circle



$m\angle A = \frac{1}{2}(m\,\widehat{RS} + m\,\widehat{MN})$

$RA \cdot AN = SA \cdot AM$

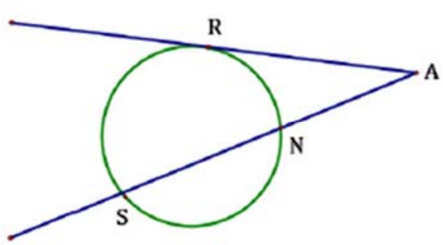
Vertex outside the circle



$m\angle A = \frac{1}{2}(m\,\widehat{RS} - m\,\widehat{MN})$

$AM \cdot AR = AN \cdot AS$

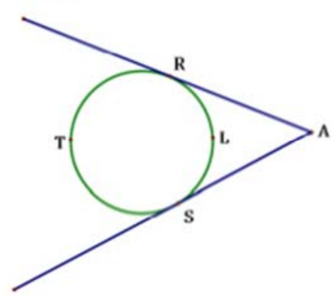
Tangent on one side



$m\angle A = \frac{1}{2}(m\,\widehat{RS} - m\,\widehat{RN})$

$AR^2 = AN \cdot AS$

Tangents on two sides

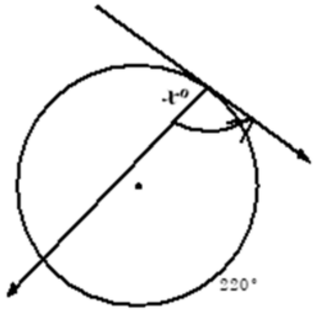


$m\angle A = \frac{1}{2}(m\,\widehat{RTS} - m\,\widehat{RLS})$

$AR = AS$

In problems # 1 – 6, solve for the variable(s).

1.

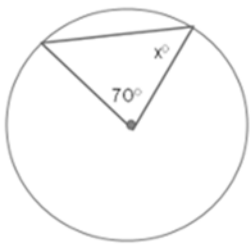


The indicated angle is a tangent-chord angle subtended by an arc of $360° - 220° = 140°$.

The measure of the angle is half the measure of the arc.

$$\boldsymbol{x} = \frac{1}{2} \cdot 140 = \mathbf{70}$$

2.

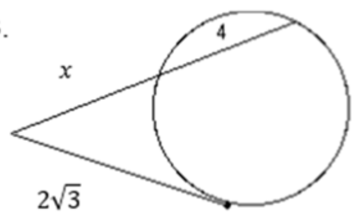


The central angle of $70°$ becomes part of an isosceles triangle with the other two angles of the triangle being congruent, with measure x. Therefore,

$$2x + 70 = 180$$

$$\boldsymbol{x} = \frac{1}{2} \cdot (180 - 70) = \mathbf{55}$$

3.



The exterior angle in this problem is a tangent-chord angle.

$$\left(2\sqrt{3}\right)^2 = x(x+4)$$

$$12 = x^2 + 4x$$

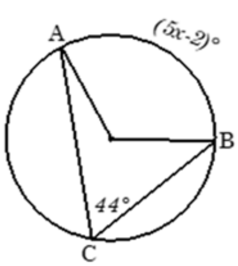
$$x^2 + 4x - 12 = 0$$

$$(x+6)(x-2) = 0$$

$$x = -6, 2$$

The solution $x = -6$ must be discarded because it would result in a negative length in the diagram. The solution to this problem, then, is $\boldsymbol{x} = \mathbf{2}$.

4.



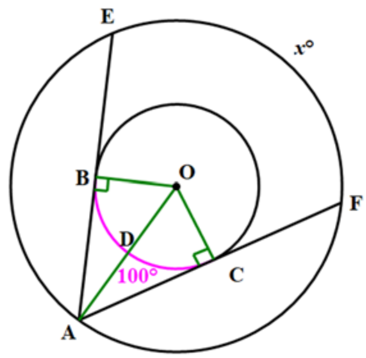
The measure of an arc is double the measure of a subtended inscribed angle. (It is also equal to the measure of a subtended central angle.)

$$5x - 2 = 2 \cdot 44$$

$$5x = 90$$

$$\boldsymbol{x} = \mathbf{18}$$

5.



What we are given in this problem is shown in black and magenta (the magenta shows the arc given in the problem). In order to solve the problem, we can add the green segments to form two triangles.

Each triangle is formed from a segment connecting the center (O) to a chord of the outer circle. The connection is made at the point of tangency of the chord to the inner circle. Right angles are formed at the points of tangency.

The measure of $\widehat{BDC}$ is given to be $100°$, so $m\angle BOC = 100°$ ($\angle BOC$ is a central angle subtended by the $100°$ arc).

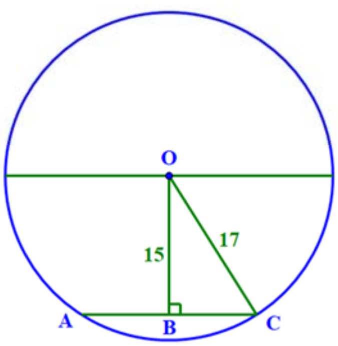
Then, $m\angle BOA = m\angle COA = 100° \div 2 = 50°$. (each is half of $\angle BOC$)

$$m\angle BAO = m\angle CAO = 180° - 90° - 50° = 40°.$$

$$m\angle BAC = m\angle BAO + m\angle CAO = 40° + 40° = 80°$$

$\boldsymbol{x} = 2 \cdot 80 = \mathbf{160}$ because $\angle BAC$ is subtended by arc $\widehat{EF}$.

6. Find the length of a chord that is 15 cm from the center of a circle with a diameter of 34 cm.



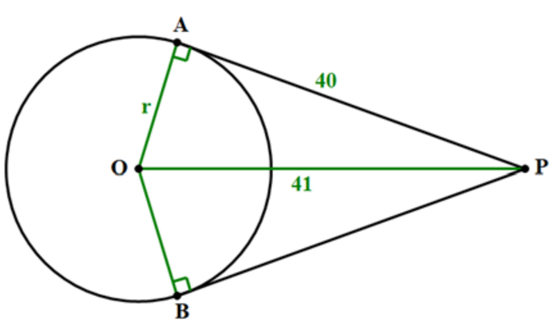
The figure to the left diagrams this problem. All radii of the circle are 17 cm in length. The distance from the center to the chord ($\overline{\mathrm{AC}}$) is 15 cm, and $\overline{\mathrm{AC}}$ is perpendicular to the segment drawn from the center to the chord, $\overline{\mathrm{OB}}$.

$$\mathbf{AC} = 2 \cdot \mathrm{AB}$$

$$= 2 \cdot \sqrt{17^2 - 15^2}$$

$$= 2 \cdot 8 = \mathbf{16}$$

7. $\overline{PA}$ *and* $\overline{PB}$ are tangent to circle O at A and B. PA = 40 and PO = 41. Find PB and the radius of the circle.



Once again, the original figure is provided in black. Additions to solve this problem are in green.

Tangents to a circle from an external point are congruent, so $\boldsymbol{PB} = PA = \mathbf{40.}$

There are right angles at the points of tangency. Pythagoras will help us get the radius.

$$\boldsymbol{r} = \sqrt{41^2 - 40^2} = \mathbf{9}$$

8. A square with an area of 100 in^2 is circumscribed about a circle. Find the exact circumference of the circle.

If a square has an area of 100 in^2, it must have a side length of:

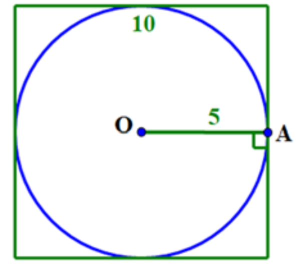
$s = \sqrt{100} = 10.$

The radius of the circle is the length of $\overline{OA}$.

$OA = \frac{1}{2} \cdot 10 = 5$ because $\overline{OA}$ is half the length of a side.

Finally, the circumference requested is:

$$C = 2\pi r = 2\pi \cdot 5 = \mathbf{10\pi} \textbf{ in.}$$

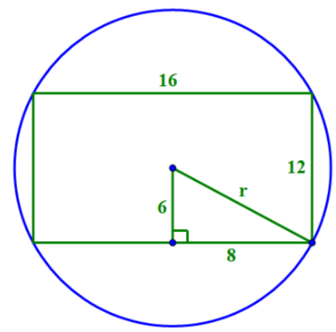


9. A 16 by 12 rectangle is inscribed in a circle. Find the radius of the circle.

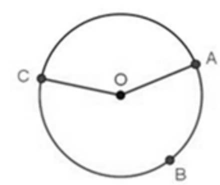
The radius of the circle that circumscribes the rectangle is equal to the length of the hypotenuse of the triangle shown in the diagram. The length of the sides of the triangle are half of the lengths of the sides of the rectangle. So,

$$r = \sqrt{6^2 + 8^2} = \mathbf{10}$$

There are no units identified in this problem.



10. Find the length of arc AC if $m\angle AOC = 110$ and the diameter of the circle is 12 m. Round your answer to the nearest hundredth of a meter.



The circumference of the circle is: $C = \pi d = 12\pi$ m.

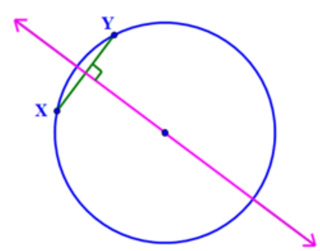
The minor arc $\widehat{AC}$ has the same degree measure as the central angle, $m\angle AOC = 110°$, out of a total of $360°$ around the whole circle. Then,

$$m\widehat{AC} = \frac{110}{360} \cdot 12\pi \approx \mathbf{11.52 \text{ m}}$$

11.) Points X and Y are two different points on a circle. Point M is located so that length XM=YM. Which of the following could be true.

I. M is the center of the circle
II. M is on arc XY
III. M is outside of the circle

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, III



The locus of all points equidistant to X and Y is the magenta line shown in the diagram, which is the perpendicular bisector of chord $\overline{XY}$. The center of the circle, a point on arc $\widehat{XY}$, and points outside the circle are all on the magenta line. Therefore, all of the statements (I, II and III) **could** be true. **Answer E**

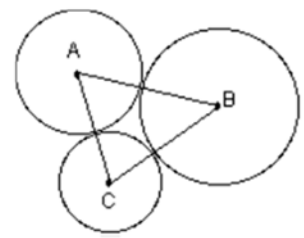
12. Find the area of the shaded region if the length of the arc along the shaded region is 12π cm. Exact answers only.

The length of the arc is $\frac{360-120}{360} = \frac{2}{3}$ of the circumference of the circle.

$$C = 12\pi \div \frac{2}{3} = 18\pi = 2\pi r \quad \rightarrow \quad r = 9$$

$$A_{region} = \frac{2}{3} \cdot A_{circle} = \frac{2}{3} \cdot \pi r^2 = \frac{2}{3} \cdot \pi \cdot 9^2 = \mathbf{54\pi\ cm^2}$$

13. Given Tangent circles A, B, C, with AB=9, BC = 14, AC = 11. Find the radii of the 3 circles.



Let the lower-case letter associated with each circle represent its radius. Then,

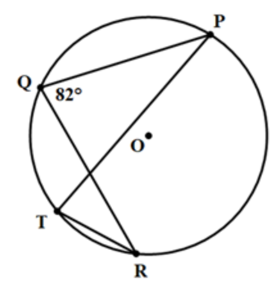
$$a + b = 9, \quad b + c = 14, \quad a + c = 11$$

Solve.

$$\begin{aligned} a + c &= 11 \\ -a - b &= -9 \\ \hline c - b &= 2 \end{aligned} \quad \rightarrow \quad \begin{aligned} -b + c &= 2 \\ b + c &= 14 \\ \hline 2c &= 16 \end{aligned} \quad \rightarrow \quad \boldsymbol{c = 8}$$

With $c = 8$, we get $\boldsymbol{b = 6}$, $\boldsymbol{a = 3}$ from the starting equations.

14. Use circle *O*. Given $m\angle RQP = 82°$, what is $m\angle RTP$?

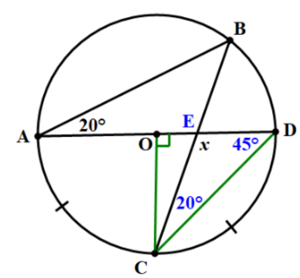


$\angle RQP$ and $\angle RTP$ are subtended by the same arc, $\widehat{PR}$, so they have the same measure.

$$\boldsymbol{m\angle RTP} = m\angle RQP = \mathbf{82°}$$

15. *Given*: $\angle BAD = 20°$, $\widehat{AC} \cong \widehat{CD}$

Find: $m\angle x$.



Start with the diagram in black as shown in the problem, and add the green lines and **blue** labels. Here's how the labels were determined.

$\angle BAD$ and $\angle BCD$ are subtended by the same arc, $\widehat{BD}$, so they have the same measure. $\boldsymbol{m\angle BCD} = m\angle BAD = \mathbf{20°}$.

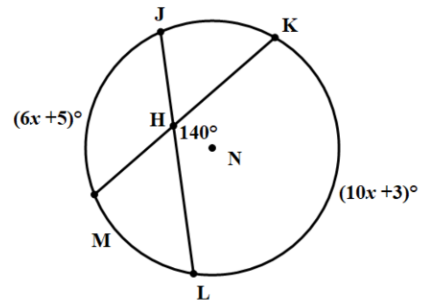
$\widehat{ACD} = 180°$ because $\overline{AD}$ is a diameter.

$\widehat{ACD}$ is composed of congruent arcs $\widehat{AC}$ and $\widehat{CD}$, so $\widehat{AC}$ and $\widehat{CD}$ must each be $90°$.

Then, $m\angle COD = 90°$, making ΔCOD a right isosceles triangle, so $\boldsymbol{m\angle CDO} = \mathbf{45°}$.

Finally, in ΔCED, $\boldsymbol{m\angle x} = 180° - \boldsymbol{m\angle BCD} - \boldsymbol{m\angle CDO} = 180° - \mathbf{20°} - \mathbf{45°} = \mathbf{115°}$

16. In circle N, m arc JM = $(6x + 5)°$, m arc KL = $(10x + 3)°$ and $m\angle KHL = 140°$. Find x.



H is a vertex inside the circle, so we have the relationship:

$$140° = \frac{1}{2}(m\,\widehat{JM} + m\,\widehat{KL})$$

$$140 = \frac{1}{2}[(6x + 5) + (10x + 3)]$$

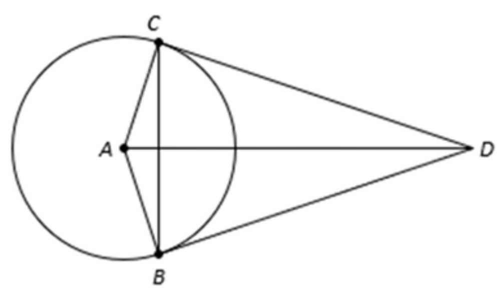
$$140 = \frac{1}{2}(16x + 8)$$

$$140 = 8x + 4$$

$$136 = 8x$$

$$\mathbf{17 = x}$$

17. Two tangents are drawn from point D to circle A.



What conclusion is guaranteed by this diagram?

(A) $AD = BD$

(B) $AC = DC$

(C) $\frac{1}{2}\, m\, arc\ \mathrm{BC} = m\angle\mathrm{BDC}$

(D) ΔABD is a right triangle.

Let's consider each of these possibilities.

(A) Tangency makes ΔABD a right triangle, with $\overline{AD}$ its hypotenuse and $\overline{BD}$ a leg. A leg and a hypotenuse in a right triangle can never be equal, so this is FALSE.

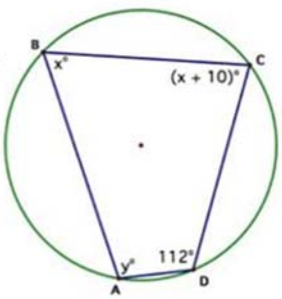
(B) $\overline{AC}$ and $\overline{DC}$ are two legs of right triangle ΔACD, with $AC = r$, the radius of the circle. There is no requirement on D which would make $DC = r$, so this is FALSE.

(C) $m\angle BDC = \frac{1}{2}[m(\text{major } BC \text{ arc}) - m(\text{minor } BC \text{ arc})]$. The formula shown in (C) is missing one of the arcs, and "arc BC" is ambiguous. It could refer to either the major BC arc or the minor BC arc. A third point is required to name these arcs. So, this is very FALSE in so many ways.

(D) As mentioned in (A) above, tangency makes ΔABD a right triangle. So this is **TRUE.**

Answer D

18. Find x and y.



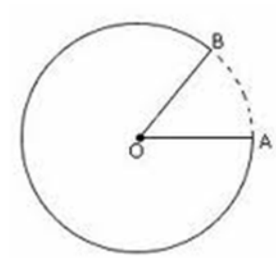
Opposite angles in a quadrilateral inscribed in a circle add to 180°.

$x + 112 = 180 \quad \rightarrow \quad \boldsymbol{x = 68}$

$m\angle C = (x + 10)° = (68 + 10)° = 78°$

$y + 78 = 180 \quad \rightarrow \quad \boldsymbol{y = 102}$

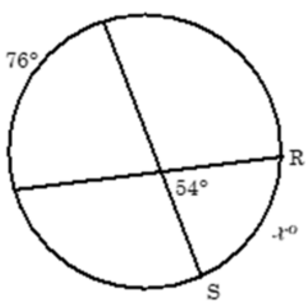
19. The dotted arc has a length of 4.2 in. Find the radius of the circle to the nearest hundredth, if $m\angle BOA = 35°$.



$$4.2 = \frac{35}{360}C = \frac{35}{360} \cdot 2\pi r = 0.610865 \cdot r$$

$$\boldsymbol{r} = \frac{4.2}{0.610865} \approx \boldsymbol{6.88\ \text{in}}$$

20. Find x.

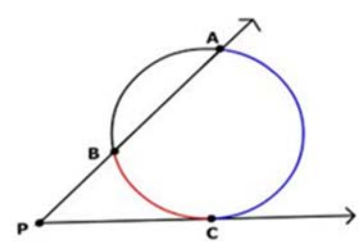


$$54 = \frac{1}{2}(x + 76)$$

$$108 = x + 76$$

$$\boldsymbol{32 = x}$$

21. Find the measure of arc AB if $m\angle P = 62$ and the measure of arc AC = 151.



$$62° = \frac{1}{2}(m\ \widehat{AC} - m\ \widehat{BC})°$$

$$62° = \frac{1}{2}(151 - m\ \widehat{BC})°$$

$$124° = (151 - m\ \widehat{BC})°$$

$$m\ \widehat{BC} = 27°$$

$$m\ \widehat{AB} = 360° - m\ \widehat{BC} - m\ \widehat{AC}$$

$$\boldsymbol{m\ \widehat{AB}} = 360° - 27° - 151° = \boldsymbol{182°}$$

22. A child's bicycle tire travels a distance of 450 inches after eight rotations of the tire. Find the area of the circular surface of the tire, rounded to the nearest square inch.

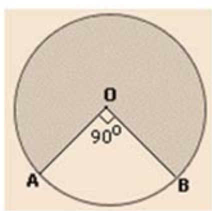
One rotation of a tire is the circumference of the wheel. $C = 2\pi r$.

For 8 rotations, the distance $= 8 \cdot 2\pi r = 16\pi r$. Then,

$450 = 16\pi r$, so, $r = \dfrac{450}{16\pi}$

$\boldsymbol{Area} = \pi r^2 = \pi\left(\dfrac{450}{16\pi}\right)^2 \approx \mathbf{252\ in^2}$

23. Find the area of the shaded region if the circumference of the circle is 10π cm (exact answer only.)

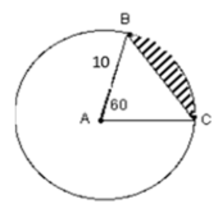


$C = 2\pi r = 10\pi \quad \rightarrow \quad r = 5$

$A_{full\ circle} = \pi r^2 = \pi \cdot 5^2 = 25\pi$

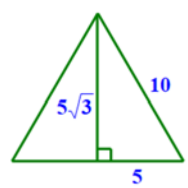
$\boldsymbol{A_{region}} = \dfrac{360 - 90}{360} \cdot 25\pi = \dfrac{3}{4} \cdot 25\pi = \mathbf{\dfrac{75\pi}{4}\ cm^2}$

24. Find the area of segment $\overline{BC}$ in the diagram to the right. Exact answer only.



Shaded area = sector area – triangle area.

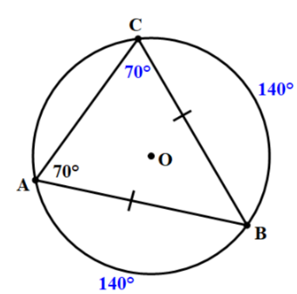
Sector area $= \dfrac{60}{360} \cdot \pi \cdot 10^2 = \dfrac{50\pi}{3}$.



Triangle area $= \dfrac{1}{2} bh = \dfrac{10 \cdot 5\sqrt{3}}{2} = 25\sqrt{3}$

Shaded area $= \mathbf{\dfrac{50\pi}{3} - 25\sqrt{3}}$ **units²**

25. Given that $\overline{AB} \cong \overline{BC}$, and $m\angle A = 70°$. Find the measure of major arc $\widehat{ABC}$.



In the diagram, information in black is given, and information in **blue** is added based on what we know. Here's how we got the blue measures:

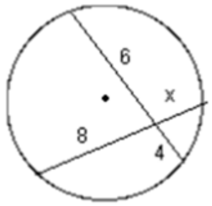
$\boldsymbol{m\angle C} = m\angle A = \mathbf{70°}$ because the two angles are opposite congruent sides in a triangle.

$\boldsymbol{m\ \widehat{BC}} = \boldsymbol{m\ \widehat{AB}} = 2 \cdot \mathbf{70°} = \mathbf{140°}$ because the arcs subtend angles of 70°.

$\boldsymbol{m\ \widehat{ABC}} = m\ \widehat{AB} + m\ \widehat{BC} = 140° + 140° = \mathbf{280°}$

In problems #26 – 29, solve for the variable. Simplify any radical answers.

26.



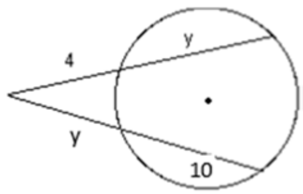
This diagram has a vertex inside the circle, so

$8x = 6 \cdot 4$

$8x = 24$

$\boldsymbol{x = 3}$

27.



This diagram has a vertex outside the circle, so

$4(4 + y) = y(10 + y)$

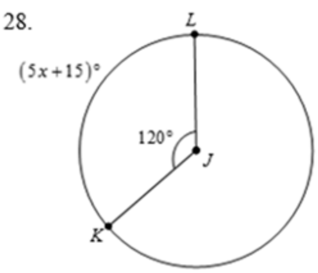
$16 + 4y = 10y + y^2$

$y^2 + 6y - 16 = 0$

$(y + 8)(y - 2) = 0$

$y = -8, 2$. However, $y = -8$ gives negative lengths in the diagram, so it is discarded. $\boldsymbol{y = 2}$

28.



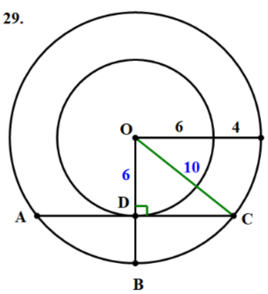
The arc and its subtended central angle have the same measure.

$5x + 15 = 120$

$5x = 105$

$\boldsymbol{x = 21}$

29.



We want to find the value of $x = \text{AC}$.

In the diagram, information in black is given, and information in green and blue is added based on what we know. Here's how we got the blue measures:

$\text{OC} = 6 + 4 = 10$ (sum of the two widths given)

$\text{OD} = 6$ (radius of the inner circle)

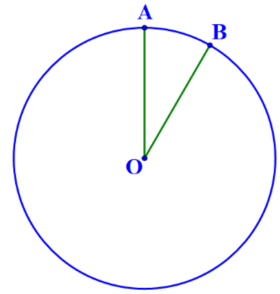
$\text{DC} = \sqrt{10^2 - 6^2} = 8$

$\text{AD} = \text{DC}$ (symmetry within the circle)

$\textbf{AC} = \text{AD} + \text{DC} = 8 + 8 = \textbf{16}$

30) A circular spinner is divided into 12 equal sections. If the area of one sector is 28 cm^2, then find the length of the curved outer portion of one section, rounded to the nearest tenth of a cm.

One sector of the spinner is shown to the right. The area of the sector is one-twelfth of the area of the circle:



$$A_{\text{sector}} = \frac{1}{12}\pi r^2 = 28$$

$$r = \sqrt{\frac{28 \cdot 12}{\pi}} \approx 10.342 \text{ cm}$$

The length of the arc, then, is one-twelfth of the circumference of the circle.

$$\boldsymbol{m\,\widehat{AB}} = \frac{1}{12}C = \frac{1}{12}(2\pi r) = \frac{1}{12}2\pi \cdot 10.342 = \mathbf{5.4 \text{ cm}}$$