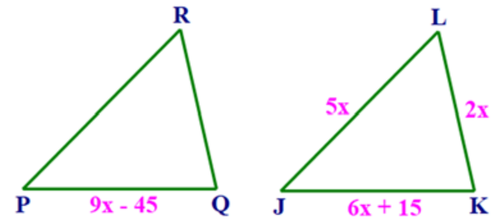
Name______________________________

1) Given $\Delta PQR \cong \Delta JKL$, $PQ = 9x - 45$, $JK = 6x + 15$, $KL = 2x$ and $JL = 5x$, what is the value of x?

It's helpful to draw a picture for this problem.



Notice that congruent segments $\overline{PQ}$ and $\overline{JK}$ have measures $9x - 45$ and $6x + 15$. Then:

$9x - 45 = 6x + 15$

$3x - 45 = 15$

$3x = 60$

$\boldsymbol{x = 20}$

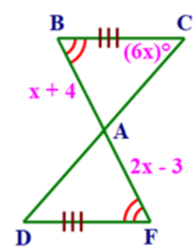
Finally, we need to check the sides of ΔJKL to make sure this results in a viable triangle:

$2x = 40, \quad 5x = 100, \quad 6x + 15 = 135$

Sides of $40, 100, 135$ are viable in a triangle because $40 + 100 > 135$.

Note that if we had calculated $x = 10$, the sides would have been $20, 50, 75$, which is not a viable triangle because $20 + 50 < 75$. This problem would have no solution.

2) State the theorem to prove, $\Delta ABC \cong \Delta AFD$. What is the $m\angle D$?



Our target is to prove $\Delta ABC \cong \Delta AFD$ with one of the SSS, SAS, ASA or AAS triangle congruence theorems. Let's do this as a proof, being careful to use the letters from the diagram in the order required for triangle congruence.

Step	Statement	Reason
1	$\angle B \cong \angle F$ $\overline{BC} \cong \overline{FD}$	Given in the diagram.
2	$\angle BAC \cong \angle FAD$	Vertical angles are congruent.
3	$\Delta ABC \cong \Delta AFD$	**AAS triangle congruence theorem.**

To find x, notice that $\overline{BA} \cong \overline{FA}$ by CPCTC. Then, $BA = FA$, so,

$x + 4 = 2x - 3$

$4 = x - 3$

$7 = x$

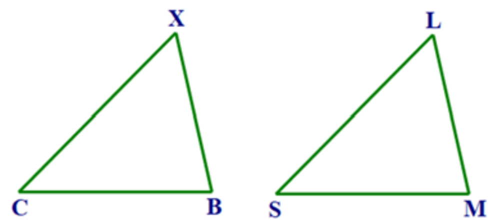
To find $m\angle D$, notice that $\angle C \cong \angle D$ by CPCTC. Then, $m\angle D = m\angle C$, so,

$m\angle D = (6x)° = (6 \cdot 7)° = \mathbf{42°}$

3) Determine which statement is true, given that $\Delta CBX \cong \Delta SML$.

A) $\overline{MB} \cong \overline{SL}$ C) $\angle X \cong \angle S$

B) $\overline{XC} \cong \overline{ML}$ D) $\angle XCB \cong \angle LSM$



It's helpful to draw a picture for this problem. Let's draw two congruent triangles so that relationships between their parts are easy to evaluate.

Consider each statement, looking at the diagram for support.

A) Three of the points in this statement are in one triangle, and one is in the other. FALSE

B) $\overline{XC}$ is the left leg in its triangle, whereas $\overline{ML}$ is the right leg in its triangle. FALSE

C) $\angle X$ is at the top of its triangle, whereas $\angle S$ is on the left of its triangle. FALSE

D) Both $\angle XCB$ and $\angle LSM$ are on the left of their respective triangles. CPCTC TRUE

Answer D is correct.

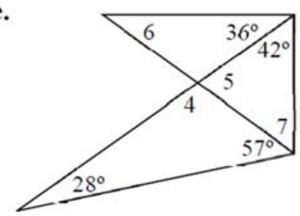
For #4 – 7, find the measure of each numbered angle.

4) $m\angle 4$

5) $m\angle 5$

6) $m\angle 6$

7) $m\angle 7$



4) The bottom triangle in the diagram has two known angles, so we can calculate the third, knowing that the sum of the interior angles of a triangle is $180°$.

$$m\angle 4 = 180° - 28° - 57° = \mathbf{95°}$$

5) Angles 4 and 5 form a linear pair, so they add to $180°$. From above, $m\angle 4 = 95°$.

$$m\angle 5 = 180° - 95° = \mathbf{85°}$$

Alternatively, $\angle 5$ is an external angle to the triangle containing $\angle 5$. Therefore, it is equal to the sum of the measures of the two non-adjacent angles in that triangle.

$$m\angle 5 = 57° + 28° = \mathbf{85°}$$

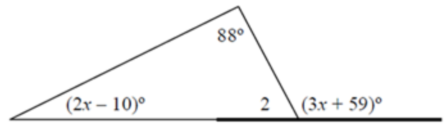
6) In the top triangle in the diagram, we have $\angle 6$, an angle of $36°$, and an angle of $95°$. The last of these is because the remaining angle is vertical with $\angle 4$, so it has the same measure. The sum of these interior angles must be $180°$.

$$m\angle 6 = 180° - 36° - 95° = \mathbf{49°}$$

7) The triangle on the right in the above diagram contains angles 5 and 7, and an angle of $42°$. From above, we know $m\angle 5 = 85°$. We can calculate $m\angle 7$, knowing that the sum of the interior angles of a triangle is $180°$.

$$m\angle 7 = 180° - 42° - 85° = \mathbf{53}°$$

8) Solve for x and the measure of $\angle 2$.



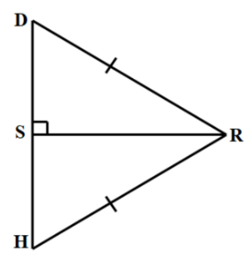
The angle external to $\angle 2$ has a measure of $(3x + 59)°$. The exterior angle theorem says that the measure of an exterior angle is equal to the sum of the measures of the two non-adjacent angles in the triangle.

$$3x + 59 = (2x - 10) + 88$$
$$x + 59 = 78$$
$$x = \mathbf{19}$$

Angle 2 and its exterior angle form a linear pair, so they add to $180°$.

$$m\angle 2 = 180° - (3x + 59)°$$
$$= 180° - (3 \cdot 19 + 59)°$$
$$= 180° - 116° = \mathbf{64}°$$

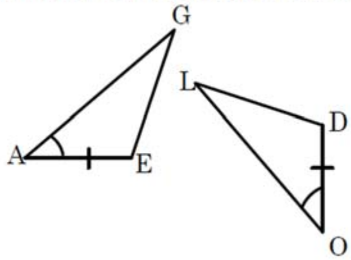
9) Are the following triangles congruent? If so, write the congruence statement and state the theorem used to prove the congruent triangles.



Step	Statement	Reason
1	a) $\angle DSR$ is a right angle. b) $\angle DSR$ and $\angle HSR$ form a linear pair. c) $\overline{DR} \cong \overline{HR}$.	Given in the diagram.
2	$\overline{SR} \cong \overline{SR}$	Reflexive property of congruence.
3	$\angle HSR$ is a right angle.	$\angle DSR$ and $\angle HSR$ form a linear pair and $\angle DSR$ is a right angle.
4	ΔDSR and ΔHSR are right triangles.	Both triangles contain right angles.
5	**$\Delta DSR \cong \Delta HSR$** **Yes, the Δs are $\cong$.**	**Hypotenuse ($\overline{DR} \cong \overline{HR}$) leg ($\overline{SR} \cong \overline{SR}$) congruence theorem.**

10) In the figure, $\angle GAE \cong \angle LOD$ *and* $\overline{AE} \cong \overline{DO}$. What information is needed to prove that $\Delta AGE \cong \Delta OLD$ by SAS ?

A. $\overline{GE} \cong \overline{LD}$
B. $\overline{AG} \cong \overline{OL}$
C. $\angle AGE \cong \angle OLD$
D. $\angle AEG \cong \angle ODL$

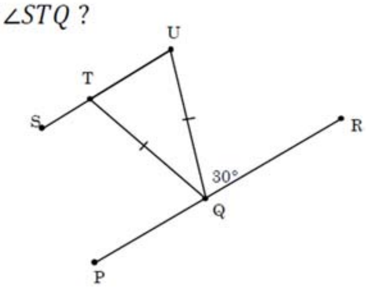


We have congruence for one pair of sides and one pair of angles in the diagram. We need congruence of legs on the other side of the congruent angles. That would be $\overline{\boldsymbol{AG}} \cong \overline{\boldsymbol{OL}}$.

Answer B

11) In the figure, $\overline{PR} \parallel \overline{SU}$ and $\overline{QT} \cong \overline{QU}$. What is the measure of $\angle STQ$?

A. 30°
B. 120°
C. 150°
D. 165°



Since $\overline{PR} \parallel \overline{SU}$, $\angle RQU$ and $\angle QUT$ are alternate interior angles and are, therefore, congruent. So, $m\angle QUT = 30°$.

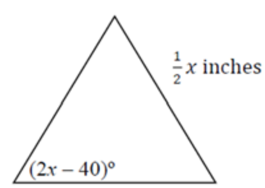
Next, $\angle QUT \cong \angle QTU$ because they are angles opposite congruent legs in isosceles ΔQUT. So $m\angle QTU = 30°$.

Finally, $\angle QTU$ and $\angle STQ$ form a linear pair, so the sum of their measures must be 180°.

$$m\angle STQ = 180° - m\angle QTU = 180° - 30° = \mathbf{150°}$$

Answer C

12) Find the perimeter of the equilateral triangle shown below.



In an equilateral triangle, all interior angles measure 60°. Then,

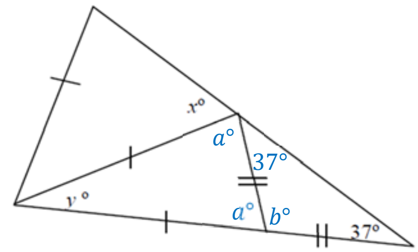
$$2x - 40 = 60$$
$$2x = 100$$
$$x = 50$$

The perimeter of an equilateral triangle is three times the length of one side. Then,

$$P = 3\left(\frac{1}{2}x\right) = 3\left(\frac{1}{2} \cdot 50\right) = \textbf{75 inches}$$

Always remember to identify units in your solution when they are given in the problem!

13 Find the measure of x and y.



This problem becomes easier if we label a few more angles. Angles opposite congruent sides in isosceles triangles are congruent, which helps with our labeling.

In the triangle on the right, the sum of the interior angles must be 180°, so,

$b = 180 - 37 - 37 = 106.$

The adjacent angles marked $a°$ and $b°$ form a linear pair, so,

$a = 180 - 106 = 74.$

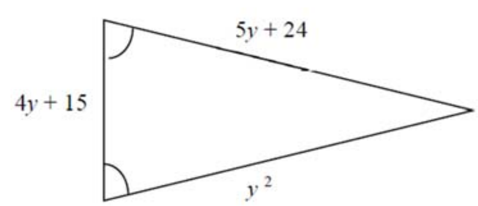
The center triangle has two angles of $a°$ and one angle of $y°$, which must add to 180°, so,

$\boldsymbol{y} = 180 - 74 - 74 = \mathbf{32}.$

Finally, along the top right, angles marked 37°, $a°$, and $x°$ must add to 180° in order to form a straight angle, so,

$\boldsymbol{x} = 180 - 37 - 74 = \mathbf{69}.$

14) Find y and the perimeter of the triangle.



Legs opposite congruent angles in isosceles triangles are congruent.

$y^2 = 5y + 24$

$y^2 - 5y - 24 = 0$

$(y - 8)(y + 3) = 0$

$\boldsymbol{y} = \mathbf{8}, \mathbf{-3}$ (2 possibilities)

If we plug each of these values into the lengths of the sides shown in the diagram, we always get positive numbers, so there are two cases. If we had gotten a length that was negative for either $y = 8$ or $y = -3$, we would have had to discard that solution.

The perimeter of the triangle is: $P = y^2 + (4y + 15) + (5y + 24) = y^2 + 9y + 39.$

Case 1 ($y = 8$): $P = y^2 + 9y + 39 = 8^2 + 9 \cdot 8 + 39 = \mathbf{175}.$ (we are not given units)

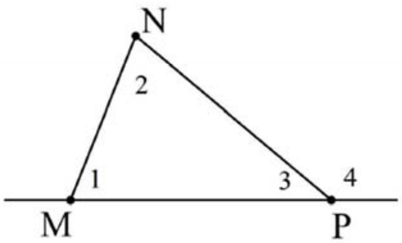
Sides of this triangle are $64, 64, 47$, which gives a viable triangle.

Case 2 ($y = -3$): $P = y^2 + 9y + 39 = (-3)^2 + 9 \cdot (-3) + 39 = \mathbf{21}.$

Sides of this triangle are $9, 9, 3$, which gives a viable triangle.

15)

Given ΔMNP, Anna is proving $m\angle 1 + m\angle 2 = m\angle 4$. Which statement should be part of her proof?



A. $m\angle 1 = m\angle 2$

B. $m\angle 1 = m\angle 3$

C. $m\angle 1 + m\angle 3 = 180°$

D. $m\angle 3 + m\angle 4 = 180°$

Interestingly, the external angle theorem says that $m\angle 1 + m\angle 2 = m\angle 4$. Assuming that Anna has not yet learned that theorem, she would need to include in her proof:

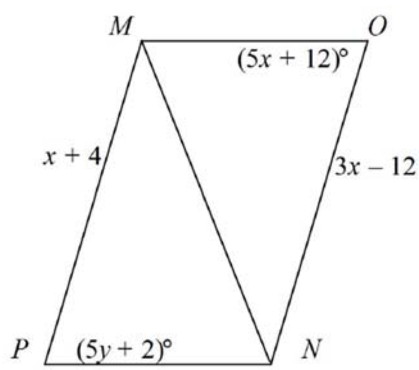
$m\angle 1 + m\angle 2 + m\angle 3 = 180°$ (interior angles in a triangle add to $180°$), and

$m\angle 4 + m\angle 3 = 180°$ ($\angle 4$ and $\angle 3$ form a linear pair).

Comparing these two statements with the possible answers, we see that the second statement is the same as **Answer D**, which is the correct answer.

It appears that Anna is well on her way to proving the external angle theorem!

16) In the figure, $\Delta MON \cong \Delta NPM$. Find the values of x and y.

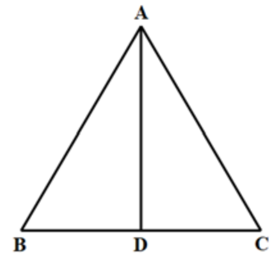


Looking carefully at the letter order in the congruence, we see that $PM = ON$ and $m\angle O = m\angle P$. Then,

$$ON = PM \quad \rightarrow \quad 3x - 12 = x + 4$$
$$2x - 12 = 4$$
$$2x = 16$$
$$\boldsymbol{x = 8}$$

$$m\angle O = m\angle P \quad \rightarrow \quad 5x + 12 = 5y + 2$$
$$5 \cdot 8 + 12 = 5y + 2$$
$$52 = 5y + 2$$
$$50 = 5y$$
$$\boldsymbol{10 = y}$$

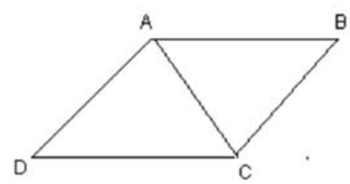
17) **Given:** $\overline{AD} \perp \overline{BC}$; $\overline{AD}$ bisects $<BAC$
Prove: $\angle B \cong \angle C$



It looks like we want to head toward $\Delta ADB \cong \Delta ADC$, and use CPCTC.

Step	Statement	Reason
1	$\overline{AD} \perp \overline{BC}$. $\overline{AD}$ bisects $\angle BAC$.	Given.
2	$\angle ADB$ is a right angle. $\angle ADC$ is a right angle.	$\overline{AD} \perp \overline{BC}$. Perpendicular lines form right angles.
3	$\angle ADB \cong \angle ADC$.	All right angles are congruent.
4	$\overline{AD} \cong \overline{AD}$.	Reflexive property of congruence.
5	$\angle BAD \cong \angle CAD$.	$\overline{AD}$ bisects $\angle BAC$.
6	$\Delta ADB \cong \Delta ADC$	ASA congruence theorem.
7	$\angle B \cong \angle C$	CPCTC.

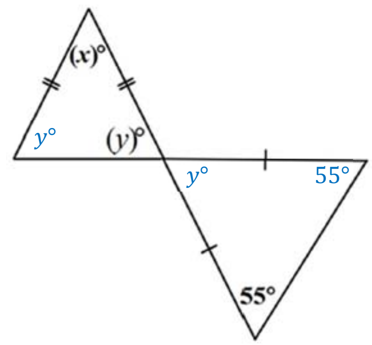
18) **Given:** $\overline{AD} \parallel \overline{CB}, \overline{AB} \parallel \overline{CD}$
Prove: $\angle B \cong \angle D$



With parallel lines, we will typically look for alternate interior angles or corresponding angles to prove things. Also, this looks like another situation where we have congruent triangles and can use CPCTC.

Step	Statement	Reason
1	$\overline{AD} \parallel \overline{CB}$. $\overline{AB} \parallel \overline{CD}$.	Given.
2	$\angle BAC \cong \angle DCA$.	Alternate interior angles of $\overline{AB} \parallel \overline{CD}$, with $\overline{AC}$ a transversal.
3	$\angle BCA \cong \angle DAC$.	Alternate interior angles of $\overline{AD} \parallel \overline{CB}$, with $\overline{AC}$ a transversal.
4	$\overline{AC} \cong \overline{AC}$.	Reflexive property of congruence.
5	$\Delta BAC \cong \Delta DCA$	ASA congruence theorem.
6	$\angle B \cong \angle D$	CPCTC.

19.) Find x and y using isosceles triangle properties.



This problem becomes easier if we label a few more angles. Angles opposite congruent sides in isosceles triangles are congruent, which helps with our labeling.

Angles opposite congruent legs in isosceles triangles are congruent.

In the top left triangle, the base angles are congruent, so they are both labeled $y°$.

In the bottom right triangle, the base angles are congruent, so they are both labeled $55°$. Also, we see congruent vertical angles between the two triangles, so they are both labeled $y°$.

Then, we can add up the angles in each triangle to get $180°$. In the bottom right triangle,

$y + 55 + 55 = 180$

$y + 110 = 180$

$\boldsymbol{y = 70}$

In the top left triangle,

$x + y + y = 180$

$x + 70 + 70 = 180$

$x + 140 = 180$

$\boldsymbol{x = 40}$