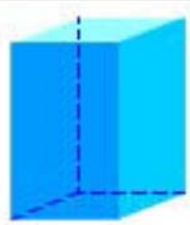
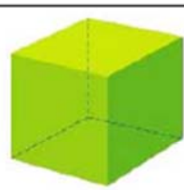
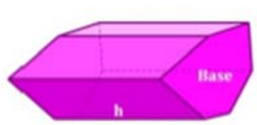
Geometry
Summary of Surface Area and Volume Formulas – 3D Shapes

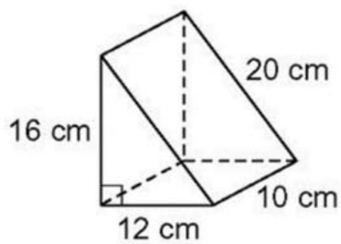
Shape	Figure	Surface Area	Volume
Sphere		$SA = 4\pi r^2$ $r = radius$	$V = \frac{4}{3}\pi r^3$ $r = radius$
Right Cylinder		$SA = 2\pi rh + 2\pi r^2$ $h = height$ $r = radius\ of\ base$	$V = \pi r^2 h$ $h = height$ $r = radius\ of\ base$
Cone		$SA = \pi rl + \pi r^2$ $l = slant\ height$ $r = radius\ of\ base$	$V = \frac{1}{3}\pi r^2 h$ $h = height$ $r = radius\ of\ base$
Square Pyramid		$SA = 2sl + s^2$ $s = base\ side\ length$ $l = slant\ height$	$V = \frac{1}{3}s^2 h$ $s = base\ side\ length$ $h = height$
Rectangular Prism		$SA = 2 \cdot (lw + lh + wh)$ $l = length$ $w = width$ $h = height$	$V = lwh$ $l = length$ $w = width$ $h = height$
Cube		$SA = 6s^2$ $s = side\ length\ (all\ sides)$	$V = s^3$ $s = side\ length\ (all\ sides)$
General Right Prism		$SA = Ph + 2B$ $P = Perimeter\ of\ Base$ $h = height\ (or\ length)$ $B = area\ of\ Base$	$V = Bh$ $B = area\ of\ Base$ $h = height$

Formal Geometry **Name:______________________________**

Ch 11 Review Worksheet

Use exact answers for all problems, unless otherwise indicated.

1) Find the volume.



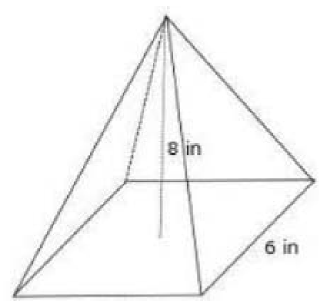
This is a right prism, with a triangle for a base. First, find B, the area of the base.

$$B = \frac{1}{2}(12)(16) = 96$$

The height is the length perpendicular to the base. So, $b = 10$.

Finally, $\boldsymbol{V} = Bh = 96 \cdot 10 = \mathbf{960\ cm^3}$

2) Find the volume of the square pyramid shown, if the perimeter of the base is 24 in.



For a square pyramid, we introduce a factor of $\frac{1}{3}$ into the volume calculation. The same factor is used in the calculation of a cone. The origins of the $\frac{1}{3}$ factor come from Calculus and the fact that we are working in 3 dimensions.

Our base is a square with area: $B = 6^2 = 36$. $h = 8$.

$$\boldsymbol{A} = \frac{1}{3}Bh = \frac{1}{3}(36)(8) = \mathbf{96\ in^3}$$

For #3 – 4: A basketball is shown below. Find the following, in terms of x and π, if the radius of the basketball is represented by $6x$.

3) Surface Area

The surface area of a sphere is: $SA = 4\pi r^2$. In this case, $r = 6x$.

$$\boldsymbol{SA} = 4\pi(6x)^2 = \mathbf{144\pi x^2\ units^2}$$

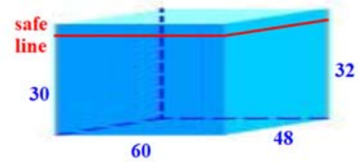
4) Volume

The volume of a sphere is: $V = \frac{4}{3}\pi r^3$. In this case, $r = 6x$.

$$\boldsymbol{V} = \frac{4}{3}\pi(6x)^3 = \mathbf{288\pi x^3\ units^3}$$

Interestingly, in Calculus, you will learn that the formula for the surface area of a sphere is the derivative of the formula for the volume of a sphere.

5) A container for mixing cement is shaped like a rectangular prism with a length of 60 inches, a width of 48 inches, and a height of 32 inches. The container can be safely filled within 2 inches of the top. Nicole makes enough cement to fill the container one time in order to fulfill an order for 48 *cubic feet* of cement. How many cubic feet of cement will she have left after filling the order?



It is safe to fill the container to a volume of 60 x 48 x 30 in^3.

Converting this to feet, we get 5 x 4 x 2½ ft^3.

So, the safe volume is: $V = 5 \cdot 4 \cdot 2.5 = 50\ \text{ft}^3$

Since Nicole uses $48\ \text{ft}^3$, she has $50 - 48 = \mathbf{2\ ft^3}$ **left.**

6) Find the radius of a sphere whose volume is 420 m^3. Round to the nearest tenth.

The volume of a sphere is: $V = \frac{4}{3}\pi r^3$. In this case, $V = 420.$

$$420 = \frac{4}{3}\pi r^3$$

$$\sqrt[3]{\frac{315}{\pi}} = \boldsymbol{r = 4.6\ \text{m}}$$

7) A sphere has a volume of 972π ft^3. Find the radius.

The volume of a sphere is: $V = \frac{4}{3}\pi r^3$. In this case, $V = 972\pi.$

$$972\pi = \frac{4}{3}\pi r^3$$

$$\sqrt[3]{729} = \boldsymbol{r = 9\ \text{ft}}$$

8) A paperweight is made of pure iron and is in the shape of a cone. The cone has diameter of 4 cm and is 6 cm tall. If the iron costs \$3.50 per gram, how much is the paperweight worth? Hint: iron weighs approximately 7.8 grams per cubic centimeter.

The volume of a cone is: $V = \frac{1}{3}\pi r^2 h$. In this case, $r = 4 \div 2 = 2, h = 6.$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 2^2 \cdot 6 = 8\pi\ \text{cm}^3$$

$$\textbf{Value} = 8\pi\ \text{cm}^3 \cdot \left(7.8\ \frac{\text{grams}}{\text{cm}^3}\right) \cdot \left(3.5\ \frac{\$}{\text{gram}}\right) = \$\mathbf{686.12}$$

Notice that the units cancel in this equation. This process is called dimensional analysis, and you will learn about it in Chemistry (if you have not learned about it already).

9) If two similar solids have ratio of volumes of 16 : 54, and the surface area of the larger solid is 108π in^2, find the surface area of the smaller solid.

If a linear ratio between similar objects is k, then:

Linear measure : area : volume have relative ratios of $k : k^2 : k^3$. To get from a volume ratio to a surface area ratio, we need to take the cube root (to get from volume to linear) and square the result (to get from linear to area). Alternatively, we could take the 2/3 power of the volume relativities to get the same answer.

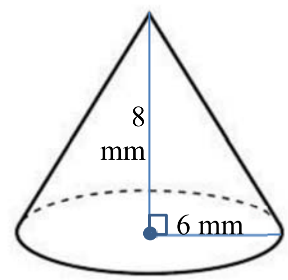
$$\text{Area ratio} = \left(\sqrt[3]{\frac{16}{54}}\right)^2 = \left(\sqrt[3]{\frac{8}{27}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Let x be the surface area of the smaller solid.

$$\frac{4}{9} = \frac{\text{small surface area}}{\text{large surface area}} = \frac{x}{108\pi}$$

$$x = \frac{4 \cdot 108\pi}{9} = \mathbf{48\pi \ in^2}$$

10) Find the surface area of the cone **shown**, in terms of π.



The surface area of a cone is weird, so it's a good idea to memorize it.

$$SA = \pi r l + \pi r^2 = \text{area of side} + \text{area of base.}$$

Given the cone to the right, we can calculate the slant height as:

$$l = \sqrt{6^2 + 8^2} = 10 \text{ mm}$$

We are also given that $r = 6$. Then,

$$\boldsymbol{SA} = (\pi \cdot 6 \cdot 10) + (\pi \cdot 6^2) = \mathbf{96\pi \ mm^2}$$

11) Find the volume of the cone from #10, in terms of π.

The volume of a cone is: $V = \frac{1}{3}\pi r^2 h$. In this case, $r = 6, h = 8$.

$$\boldsymbol{V} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 6^2 \cdot 8 = \mathbf{96\pi \ mm^3}$$

12. What is the ratio of the volumes of two cubes with edges of lengths 4 inches and 8 inches?

A. $1:8$
B. $16:64$
C. $125:512$
D. $625:4096$

$$\text{Volume ratio} = (\text{Linear ratio})^3$$

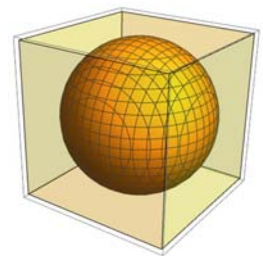
$$\text{Volume ratio} = \left(\frac{4}{8}\right)^3 = \left(\frac{1}{2}\right)^3 = \mathbf{\frac{1}{8}} \ \boldsymbol{or} \ \mathbf{1:8}$$

Answer A

13. Two similar cones have radii of 3 cm and 8 cm. Find the ratio of their volumes, small to large.

$$\text{Volume ratio} = (\text{Linear ratio})^3$$
$$\text{Volume ratio} = \left(\frac{3}{8}\right)^3 = \mathbf{\frac{27}{512}} \ \textbf{or } \mathbf{27:512}$$

14. A sphere is inscribed inside a cube with a volume of 64 cm^3. Find the surface area of the sphere.



For a cube: $V = s^3 \quad \rightarrow \quad 64 = s^3 \quad \rightarrow \quad s = 4$

The radius of the sphere is half of a side of the cube: $r = 2$.

$$\boldsymbol{SA} = 4\pi r^2 = 4\pi \cdot 2^2 = \mathbf{16\pi\ cm^2}$$

15. A paperweight is made of pure silver in the shape of a cylinder. The cylinder has diameter of 6 cm and is 10 cm tall. If the silver weighs 10.5 grams per cubic centimeter, to the nearest hundredth of a gram, how much does the paperweight weigh?

For a cylinder, $V = \pi r^2 h$. In this case, $r = 6 \div 2 = 3, h = 10$.

$$V = \pi r^2 h = \pi \cdot 3^2 \cdot 10 = 90\pi \text{ cm}^3$$
$$\textbf{Value} = 90\pi \text{ cm}^3 \cdot 10.5 \frac{\text{grams}}{\text{cm}^3} = 945\pi\ g \approx \mathbf{2,968.81\ g}$$

For #16 – 18: Convert between ratios

A manufacturer is designing two geometrically similar cylinders to be made out of plastic and painted. She will need to have surface area calculations for how much paint is needed to cover the outside; she also needs a volume calculation know how much plastic to order.

The height of the green cylinder is 7 centimeters. The height of the blue cylinder is 10 centimeters.

16. What is the ratio of surface areas (small to large)?

$$\textbf{ratio} = \left(\frac{7}{10}\right)^2 = \mathbf{\frac{49}{100}} \ \textbf{or } \mathbf{49:100}$$

17. What is the ratio of volumes (small to large?)

$$\textbf{ratio} = \left(\frac{7}{10}\right)^3 = \mathbf{\frac{343}{1000}} \ \textbf{or } \mathbf{343:1000}$$

18. If the blue cylinder has a volume of 36 cubic centimeters, what is the volume of the green cylinder? Retain every digit of the decimal.

Using the ratios calculated in Problem 17, remembering the blue cylinder is larger,

$$\frac{343}{1000} = \frac{x}{36} \quad \rightarrow \quad \boldsymbol{x = 12.348\ \mathrm{cm}^3}$$

19. The volume of a sphere is $\frac{500}{3}\pi$ inches cubed. What is its radius?

The volume of a sphere is: $V = \frac{4}{3}\pi r^3$. In this case, $V = \frac{500}{3}\pi$.

$$\frac{500}{3}\pi = \frac{4}{3}\pi r^3$$

$$\sqrt[3]{125} = \boldsymbol{r = 5\ \mathrm{in}}$$