Formal Geometry | Algebra Review Wk Name________________________________

1) Solve: $5-3(2x-1)=6-2x$

Start: $5-3(2x-1)=6-2x$

Distribute: $5-6x+3=6-2x$

Collect terms: $8-6x=6-2x$

Add $6x$: $8=6+4x$

Subtract 6: $2=4x$

Divide by 4: $\frac{1}{2}=x$

2) Solve: $2x+5(x-7)\geq 3x+4$

Start: $2x+5(x-7)\geq 3x+4$

Distribute: $2x+5x-35\geq 3x+4$

Collect terms: $7x-35\geq 3x+4$

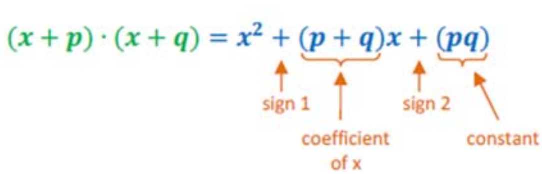
Subtract $3x$: $4x-35\geq 4$

Add 35: $4x\geq 39$

Divide by 4: $x\geq \frac{39}{4}$

3) Factor: $x^2+5x-24$

To factor a trinomial with a lead coefficient of 1, consider the following (see p. 68 in the Algebra Handbook available on www.mathguy.us).



For this problem, $p+q=5$, and $p\cdot q=-24$.

Thinking about possibilities we conclude that p and q must be -3 and 8. Notice that the sign of the middle term goes with the larger of p and q.

Then, $x^2+5x-24=(x-3)(x+8)$

4) Factor: $a^2 - 7a + 10$

For this problem, $p + q = -7$, and $p \cdot q = 10$.

p and q must both be negative because the coefficient of a in the original expression is negative, but the constant term is positive.

Thinking about possibilities we conclude that p and q must be -2 and -5. Then,

$$a^2 - 7a + 10 = \boldsymbol{(a-2)(a-5)}$$

5) Factor: $5x^2 - 6x + 1$

To factor a trinomial with a lead coefficient other than 1, consider the steps shown on p. 69 in the Algebra Handbook. *The AC Method works allows you to focus on a singular solution.*

Alternatively, the factored form must be: $(mx + p)(nx + q)$. So, $(m \cdot n)$ is the coefficient of x^2 and $(p \cdot q)$ is the constant term. If there are not many possibilities for m, n, p, q, we can try various combinations of them to see if the correct coefficient of the x term results when multiplying $(mx + p)(nx + q)$.

For this problem, $m \cdot n = 5$ and $p \cdot q = 1$.

Thinking through the possibilities with $5x^2 - 6x + 1$, we settle on:

$$5x^2 - 6x + 1 = \boldsymbol{(5x-1)(x-1)}.$$

6) Factor: $6x^2 + 7x - 3$

The lead coefficient is not 1, so let's use the AC Method with this problem. Note that the name of the method reflects the multiplication of a and c of $ax^2 + bx + c$ in the process:

$6x^2 + 7x - 3$

↘ ↙

-18

Now, we want values that multiply to -18, and add to 7.

Thinking through the possibilities, we come up with 9 and -2. These become the coefficients of two middle terms that replace $+7x$.

$$6x^2 + 7x - 3 = 6x^2 + 9x - 2x - 3$$

Next, group terms in pairs. Be careful to distribute the negative in the second pair:

$$(6x^2 + 9x) - (2x + 3)$$

Factor each pair of terms separately and collect terms.

$$(6x^2 + 9x) - (2x + 3) = 3x(2x + 3) - 1(2x + 3)$$

$$= \boldsymbol{(3x-1)(2x+3)}$$

7) Factor: $4x^2 - 6x - 40$

First, factor out the greatest common factor: $4x^2 - 6x - 40 = 2(2x^2 - 3x - 20)$

The lead coefficient of the remaining trinomial is not 1, so let's use the AC Method with this problem.

$2x^2 - 3x - 20$

-40

Now, we want values that multiply to -40, and add to -3.

Thinking through the possibilities, we come up with -8 and 5. These become the coefficients of two middle terms that replace $-3x$.

$$2(2x^2 - 3x - 20) = 2(2x^2 - 8x + 5x - 20)$$

Next, group terms in pairs:

$$2[(2x^2 - 8x) + (5x - 20)]$$

Factor each pair of terms separately and collect terms.

$$2[(2x^2 - 8x) + (5x - 20)] = 2[2x(x - 4) + 5(x - 4)]$$
$$= \mathbf{2(2x + 5)(x - 4)}$$

8) Multiply: $(x - 4)^2$

$(x - 4)^2 = (x - 4)(x - 4)$

First: $x \cdot x = x^2$

Outside: $x \cdot (-4) = -4x$

Inside: $(-4) \cdot x = -4x$

Last: $(-4) \cdot (-4) = 16$

Now, add the resulting terms: $x^2 - 4x - 4x + 16 = \mathbf{x^2 - 8x + 16}$

9) Multiply: $(7x + 2)^2$

$(7x + 2)^2 = (7x + 2)(7x + 2)$

First: $7x \cdot 7x = 49x^2$

Outside: $7x \cdot (2) = 14x$

Inside: $(2) \cdot 7x = 14x$

Last: $(2) \cdot (2) = 4$

Now, add the resulting terms: $49x^2 + 14x + 14x + 4 = \mathbf{49x^2 + 28x + 4}$

10) Solve by factoring: $2x^2 + 3x - 35 = 0$

The lead coefficient is not 1, so let's use the AC Method with this problem.

$$2x^2 + 3x - 35 = 0$$

$$-70$$

Now, we want values that multiply to -70, and add to 3.

Thinking through the possibilities, we come up with 10 and -7. These become the coefficients of two middle terms that replace $+3x$.

$$2x^2 + 3x - 35 = 2x^2 + 10x - 7x - 35$$

Next, group terms in pairs. Be careful to distribute the negative in the second pair:

$$(2x^2 + 10x) - (7x + 35)$$

Factor each pair of terms separately and collect terms.

$$(2x^2 + 10x) - (7x + 35) = 2x(x + 5) - 7(x + 5)$$

$$= (2x - 7)(x + 5)$$

Finally, set each term equal to zero.

$2x - 7 = 0$ $\qquad$ $x + 5 = 0$

$2x = 7$ $\qquad$ $\boldsymbol{x = -5}$

$\boldsymbol{x = \frac{7}{2}}$

11) Solve by factoring: $x^2 - 28 = -3x$

First, get all terms on one side (preferably with a positive lead coefficient).

$$x^2 - 28 = -3x$$

$$x^2 + 3x - 28 = 0$$

$$(x + 7)(x - 4) = 0$$

Finally, set each term equal to zero.

$x + 7 = 0$ $\qquad$ $x - 4 = 0$

$\boldsymbol{x = -7}$ $\qquad$ $\boldsymbol{x = 4}$

12) Solve: $5x^2 - 2x - 1 = 0$
(what if it doesn't factor; how can we solve a quadratic?)

Use the quadratic formula if the quadratic function is in the form $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this problem, $a = 5, b = -2, c = -1$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(-1)(5)}}{2(5)} = \frac{2 \pm \sqrt{4 + 20}}{10} = \frac{2 \pm \sqrt{24}}{10} = \frac{2 \pm \sqrt{4} \cdot \sqrt{6}}{10} = \frac{2 \pm 2\sqrt{6}}{10}$$

$$= \frac{2(1 \pm \sqrt{6})}{2 \cdot 5} = \mathbf{\frac{1 \pm \sqrt{6}}{5}}$$

Note: It is possible to determine if a quadratic equation can be factored by evaluating the discriminant, which we identify with the Capital Greek letter delta, "Δ". The discriminant is the portion of the quadratic formula that is under the radical: $\Delta = b^2 - 4ac$. Then,

- If Δ **IS** a perfect square (i.e., $0, 1, 4, 9, 16, 25, \ldots$), the quadratic **CAN** be factored.
- If Δ **IS NOT** a perfect square, the quadratic **CANNOT** be factored.

This concept will be useful to the student countless times in their mathematical career.

13) Solve for x in terms of y:

$3y + 2x = -4$

We want to isolate x to determine what it is in terms of y.

Start: $3y + 2x = -4$

Subtract $3y$: $2x = -3y - 4$

Divide by 2: $\mathbf{x = -\frac{3}{2}y - 2}$

14) Solve for (x, y): $\begin{cases} 3x - 5y = 8 \\ -3x + 2y = 1 \end{cases}$

$3x - 5y = 8 \longrightarrow (\cdot 1) \longrightarrow$ $3x - 5y = 8$

$-3x + 2y = 1 \longrightarrow (\cdot 1) \longrightarrow$ $+\ -3x + 2y = 1$

$-3y = 9$

$y = -3$ → $3x - 5(-3) = 8$

$3x + 15 = 8$

$3x = -7$

$x = -\frac{7}{3}$

The solution to this set of simultaneous equations, then, is $\left(-\frac{7}{3}, -3\right)$

15) Solve for (x, y): $\begin{cases} x + 2y = 3 \\ 2x + 3y = 3 \end{cases}$

$$\begin{aligned} x + 2y &= 3 \longrightarrow (\cdot -2) \longrightarrow & -2x - 4y &= -6 \\ 2x + 3y &= 3 \longrightarrow (\cdot \ \ 1) \longrightarrow & + \ \ 2x + 3y &= \ \ 3 \\ & & -y &= -3 \\ & & y &= 3 \end{aligned}$$

$$\begin{aligned} 2x + 3(3) &= 3 \\ 2x + 9 &= 3 \\ 2x &= -6 \\ x &= -3 \end{aligned}$$

The solution to this set of simultaneous equations, then, is $\mathbf{(-3, 3)}$

16) Solve for (x, y): $\begin{cases} 2x - 3(y + 1) = 8 \\ 3(x + 2) + 5y = -6 \end{cases}$

Let's put both of these equations in standard linear form.

$$\begin{aligned} 2x - 3(y + 1) &= 8 \\ 2x - 3y - 3 &= 8 \\ 2x - 3y &= 11 \end{aligned} \qquad \begin{aligned} 3(x + 2) + 5y &= -6 \\ 3x + 6 + 5y &= -6 \\ 3x + 5y &= -12 \end{aligned}$$

Then, use the above technique to solve resulting equations.

$$\begin{aligned} 2x - 3y &= \ \ 11 \longrightarrow (\cdot -3) \longrightarrow & -6x + \ \ 9y &= -33 \\ 3x + 5y &= -12 \longrightarrow (\cdot \ \ 2) \longrightarrow & + \ \ 6x + 10y &= -24 \\ & & 19y &= -57 \\ & & y &= -3 \end{aligned}$$

$$\begin{aligned} 2x - 3(-3) &= 11 \\ 2x + 9 &= 11 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

The solution to this set of simultaneous equations, then, is $\mathbf{(1, -3)}$

For #17 – 20 simplify each expression completely.

17) $\sqrt{24x^3y^8}$

$$\begin{aligned} \sqrt{24x^3y^8} &= \sqrt{24} \cdot \sqrt{x^3} \cdot \sqrt{y^8} \\ &= \sqrt{4} \cdot \sqrt{6} \cdot x\sqrt{x} \cdot y^4 \\ &= 2\sqrt{6} \cdot x\sqrt{x} \cdot y^4 \\ &= \mathbf{2xy^4\sqrt{6x}} \end{aligned}$$

18) $(5\sqrt{2})^2$

$$(5\sqrt{2})^2 = 5^2 \cdot (\sqrt{2})^2 = 25 \cdot 2 = \mathbf{50}$$

Alternatively,

$$(5\sqrt{2}) \cdot (5\sqrt{2}) = 5 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{2} = 25 \cdot 2 = \mathbf{50}$$

19) $(-2\sqrt{12})(5\sqrt{3})$

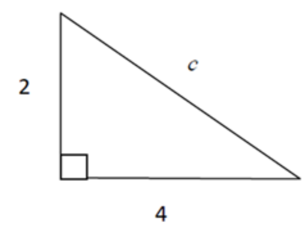
$$(-2\sqrt{12})\cdot(5\sqrt{3}) = -2\cdot 5\cdot\sqrt{12}\cdot\sqrt{3} = -10\cdot\sqrt{36} = -10\cdot 6 = \mathbf{-60}$$

20) $\frac{\sqrt{18}}{\sqrt{15}}$

$$\frac{\sqrt{18}}{\sqrt{15}} = \frac{\sqrt{18}}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{18}\cdot\sqrt{15}}{15}$$

$$= \frac{\sqrt{3}\cdot\sqrt{6}\cdot\sqrt{3}\cdot\sqrt{5}}{15} = \frac{(\sqrt{3}\cdot\sqrt{3})\cdot(\sqrt{6}\cdot\sqrt{5})}{15} = \frac{3\cdot\sqrt{30}}{15} = \frac{\sqrt{\mathbf{30}}}{\mathbf{5}}$$

21) Use the Pythagorean Theorem ($a^2 + b^2 = c^2$) to solve for the missing hypotenuse in the right triangle shown.



When using the Pythagorean Theorem, a and b refer to the lengths of the legs, and c refers to the length of the hypotenuse.

$a^2 + b^2 = c^2$

$2^2 + 4^2 = c^2$

$4 + 16 = c^2$

$20 = c^2$

$\sqrt{20} = \sqrt{c^2}$

$\sqrt{20} = \sqrt{4}\cdot\sqrt{5} = \mathbf{2\sqrt{5} = c}$