BC Calculus Multiple Choice Test

From Calculus Course Description – Effective Fall 2010 (pp. 28-39)

1. A curve is described by the parametric equations $x = t^2 + 2t$ and $y = t^3 + t^2$. An equation of the line tangent to the curve at the point determined by $t = 1$ is

(A) $2x - 3y = 0$
(B) $4x - 5y = 2$
(C) $4x - y = 10$
(D) $5x - 4y = 7$
(E) $5x - y = 13$

Solution: The slope of the tangent line is $\frac{dy}{dx}$ at the point where $t = 1$.

$x = t^2 + 2t$ $\qquad$ $y = t^3 + t^2$

$\frac{dx}{dt} = 2t + 2 = 4$ $\qquad$ $\frac{dy}{dt} = 3t^2 + 2t = 5$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{5}{4}$$

Then, the line has equation: $y = \frac{5}{4}x + b$ or the **form** $\mathbf{5x - 4y = c}$ **Answer D**

2. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

(A) $-\frac{3x + y}{y^2}$
(B) $-\frac{3x + y}{x + y}$
(C) $\frac{1 - 3x - y}{x + y}$
(D) $-\frac{3x}{1 + y}$
(E) $-\frac{3x}{x + y}$

Solution: This problem requires implicit differentiation:

$$3x^2 + 2xy + y^2 = 1$$

Differentiate:

$$6x + (2y + 2xy') + 2yy' = 0$$

$$(6x + 2y) + (2x + 2y)\,y' = 0$$

$$(2x + 2y)\,y' = -(6x + 2y)$$

$$y' = -\frac{(6x + 2y)}{(2x + 2y)}$$

$$\mathbf{y' = -\frac{(3x + y)}{(x + y)}}$$

Answer B

x	$g'(x)$
−1.0	2
−0.5	4
0.0	3
0.5	1
1.0	0
1.5	−3
2.0	−6

3. The table above gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

(A) −6.5
(B) −1.5
(C) 1.5
(D) 2.5
(E) 3

Solution: Euler's method uses the slope at the beginning of each interval (of which there are two), and the length of each interval to estimate the value of a function, as follows:

$$\hat{y} = g(-1) + g'(x_1) \cdot [0.5 - (-1)] + g'(x_2) \cdot [2.0 - (0.5)]$$

$$\hat{y} = -2 + (2) \cdot (1.5) + 1 \cdot (1.5) = \mathbf{2.5}$$

Answer D

4. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ converges?

(A) All x except $x = 0$
(B) $|x| = 3$
(C) $-3 \le x \le 3$
(D) $|x| > 3$
(E) The series diverges for all x.

Solution: Using the root test:

Note that: $\lim_{n\to\infty} \sqrt[n]{n} = 1$

Answer D

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{n \cdot 3^n}{x^n}\right|} = \lim_{n\to\infty} \sqrt[n]{n} \cdot \left|\left(\frac{3}{x}\right)\right| < 1 \quad \Rightarrow \quad \left|\frac{3}{x}\right| < 1 \quad \Rightarrow \quad |x| > 3$$

Testing the endpoints is not needed since there are no answers (A to E) including both this solution and one or both endpoints. However, note that at the endpoints,

$$\sum_{n=1}^{\infty} \frac{n \cdot 3^n}{3^n} = \sum_{n=1}^{\infty} n \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{n \cdot 3^n}{(-3)^n} = \sum_{n=1}^{\infty} (-1)^n n \quad \text{both diverge.}$$

5. If $\frac{d}{dx}f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx}f(h(x)) =$

(A) $g(x^2)$

(B) $2xg(x)$

(C) $g'(x)$

(D) $2xg(x^2)$

(E) $x^2g(x^2)$

Solution: Let $u = h(x) = x^2$

Then, by the chain rule:

$$\frac{d}{dx}\,f(u) = \frac{d}{du}\,f(u) \cdot \frac{du}{dx}$$

Substituting from above, we get:

$$\frac{d}{dx}\,f(u) \;=\; g(u) \cdot \frac{du}{dx} \;=\; g(x^2) \cdot 2x$$

Answer D

6. If F' is a continuous function for all real x, then $\lim_{h \to 0} \frac{1}{h} \int_a^{a+h} F'(x)\,dx$ is

(A) 0

(B) $F(0)$

(C) $F(a)$

(D) $F'(0)$

(E) $F'(a)$

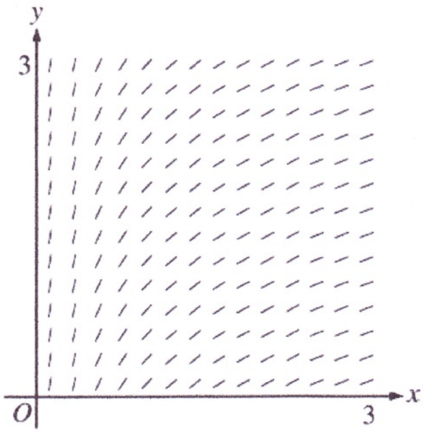
Solution: By the Fundamental Theorem of Calculus,

$$\int_a^{a+h} F'(x)\;dx \;=\; F(a+h) \;-\; F(a)$$

Then,

$$\lim_{h \to 0} \frac{1}{h} \int_a^{a+h} F'(x)\;dx \;=\; \lim_{h \to 0} \frac{F(a+h) \;-\; F(a)}{h}$$

Which is the very definition of: $F'(a)$ **Answer E**



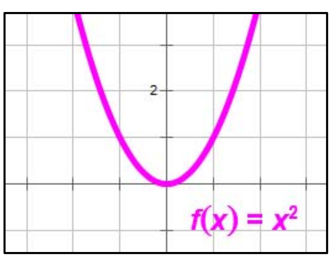
7. The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

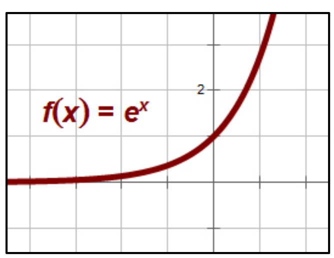
(A) $y = x^2$
(B) $y = e^x$
(C) $y = e^{-x}$
(D) $y = \cos x$
(E) $y = \ln x$

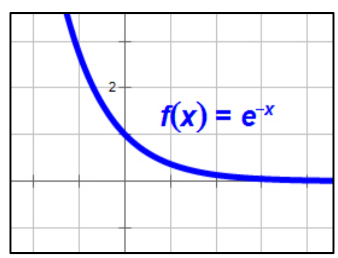
Solution: The Direction Field shown above indicates the slope of the curve at various values of x and y. Notice the following points about the filed:

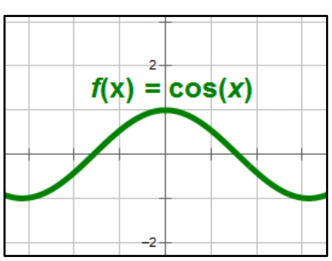
- All segments for a given value of x are the same. Therefore, the direction field is not affected by the value of y, and the function must be a function of x only.
- The direction field is steep and positive for values of x close to 0, and less steep and positive as x increases.

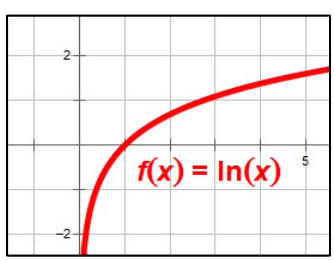
We can look at the shape of the various curves shown to see which ones match these requirements:











The only curve in this set that follows the direction field is:

$$f(x) = \ln x$$

Answer E

8. $\int_0^3 \frac{dx}{(1-x)^2}$ is

(A) $-\frac{3}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

(E) divergent

Solution: Before doing any significant work solving the integral, notice that answer E appears, at first, to be an odd choice. In fact, it is a hint that you should think about the requirements of a function when using the Fundamental Theorem of Calculus.

It is required that $f(x)$ be continuous in order to calculate a value under the Fundamental Theorem. In fact, this function is not continuous at $x = 1$, which can be seen by looking at the denominator.

Answer E

9. Which of the following series converge to 2?

I. $\sum_{n=1}^{\infty} \frac{2n}{n+3}$

II. $\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

III. $\sum_{n=0}^{\infty} \frac{1}{2^n}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

Solution: Look at each summation separately:

I. $\lim_{n\to\infty} \frac{2n}{n+3} = 2 > 0,$ **Series I diverges.**

Clearly, the limits for the terms in II and III are zero, so we can proceed to calculate the values of the sums.

II. $\sum_{n=1}^{\infty} \frac{-8}{(-3)^n} = -8 \sum_{n=1}^{\infty} \frac{1}{(-3)^n}$ Set this $= x$. Then,

$$x = -8 \cdot \left(-\frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \ ...\right)$$

$$\frac{1}{3}x = -8 \cdot \left(\quad -\frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \ ...\right)$$

$$\frac{4}{3}x = -8 \cdot \left(-\frac{1}{3}\right)$$ which simplifies to: $\boldsymbol{x = 2}$

III. $\sum_{n=0}^{\infty} \frac{1}{2^n}$ Set this $= x$. Then,

$$x = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ ...$$

$$-\frac{1}{2}x = \quad -\frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \ ...$$

$$\frac{1}{2}x = 1$$ which simplifies to: $\boldsymbol{x = 2}$

Answer E

10. If the function f given by $f(x) = x^3$ has an average value of 9 on the closed interval $[0, k]$, then $k =$

(A) 3

(B) $3^{1/2}$

(C) $18^{1/3}$

(D) $36^{1/4}$

(E) $36^{1/3}$

Solution: This one is very straightforward:

$$9 = \frac{\int_0^k x^3\,dx}{k} \quad \longrightarrow \quad 9k = \int_0^k x^3\,dx$$

Then,

$$9k = \frac{1}{4}k^4$$

$$36 = k^3$$

$$36^{1/3} = k$$

Answer E

11. Which of the following integrals gives the length of the graph $y = \sin(\sqrt{x})$ between $x = a$ and $x = b$, where $0 < a < b$?

(A) $\int_a^b \sqrt{x + \cos^2(\sqrt{x})}\,dx$

(B) $\int_a^b \sqrt{1 + \cos^2(\sqrt{x})}\,dx$

(C) $\int_a^b \sqrt{\sin^2(\sqrt{x}) + \frac{1}{4x}\cos^2(\sqrt{x})}\,dx$

(D) $\int_a^b \sqrt{1 + \frac{1}{4x}\cos^2(\sqrt{x})}\,dx$

(E) $\int_a^b \sqrt{\frac{1 + \cos^2(\sqrt{x})}{4x}}\,dx$

Solution: The formula for arc length, when y is given as a function of x, is:

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

Then,

$$\frac{dy}{dx} = \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)^2 = \cos^2(\sqrt{x}) \cdot \frac{1}{4x}$$

Answer D

12. Which of the following integrals represents the area enclosed by the smaller loop of the graph of $r = 1 + 2\sin\theta$?

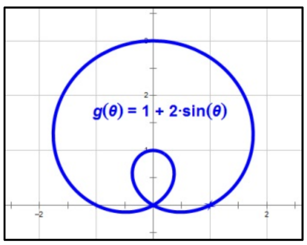
(A) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6}(1+2\sin\theta)^2\,d\theta$

(B) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6}(1+2\sin\theta)\,d\theta$

(C) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6}(1+2\sin\theta)^2\,d\theta$

(D) $\int_{-\pi/6}^{7\pi/6}(1+2\sin\theta)^2\,d\theta$

(E) $\int_{7\pi/6}^{-\pi/6}(1+2\sin\theta)\,d\theta$



Solution: See the illustration in blue below.

The limits of integration for the short loop must occur when $\sin\theta$ is negative, since that produces smaller values of r. This occurs in Quadrants III and IV or, in terms of the answers given, from $7\pi/6$ to $11\pi/6$.

The formula for area is:

$$A = \frac{1}{2}\int_a^b r^2\,d\theta$$

$$\boldsymbol{A = \frac{1}{2}\int_{7\pi/6}^{11\pi/6}(1+2\sin\theta)^2\,d\theta}$$

Answer A

13. The third-degree Taylor polynomial about $x = 0$ of $\ln(1-x)$ is

(A) $-x - \frac{x^2}{2} - \frac{x^3}{3}$

(B) $1 - x + \frac{x^2}{2}$

(C) $x - \frac{x^2}{2} + \frac{x^3}{3}$

(D) $-1 + x - \frac{x^2}{2}$

(E) $-x + \frac{x^2}{2} - \frac{x^3}{3}$

Solution: **Method 1** is to recall that:

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ \ldots$$

And substitute $(-x)$ in for x to get:

$$\ln(1-x) = (-x) - \frac{(-x)^2}{2} + \frac{(-x)^3}{3} - \ \ldots$$

$$\boldsymbol{\ln(1+x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ \ldots}$$

Answer A

Solution: **Method 2** is to develop the polynomial directly:

$\mathrm{f(x)} = \ln(1-x)$ $\qquad$ $\mathrm{f}(0) = 0$

$\mathrm{f}'(\mathrm{x}) = -(1-x)^{-1}$ $\qquad$ $\mathrm{f}'(0) = -1$

$\mathrm{f}''(\mathrm{x}) = -(1-x)^{-2}$ $\qquad$ $\mathrm{f}''(0) = -1$

$\mathrm{f}'''(\mathrm{x}) = -2(1-x)^{-3}$ $\qquad$ $\mathrm{f}'''(0) = -2$

The Taylor Polynomial for $\mathrm{f(x)} = \ln(1-x)$, then is:

$$P(x) = \sum_{i=0}^{3}\frac{f^{(i)}(0)}{i!}\,x^i \;=\; 0 + \frac{-1}{1}x + \frac{-1}{2}x^2 + \frac{-2}{6}x^3 = \boldsymbol{-x - \frac{1}{2}x^2 - \frac{1}{3}x^3}$$

14. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$

(B) $e^{\tan x} + 5$

(C) $5e^{\tan x}$

(D) $\tan x + 5$

(E) $\tan x + 5e^x$

Solution: Separate the variables to get:

$$\frac{1}{y}\, dy = \sec^2 x\ dx$$

Then, Integrate:

$$\int \frac{1}{y}\, dy = \int \sec^2 x\ dx$$

$$\ln y = \tan x + C$$

Exponentiate both sides:

$$y = e^{\tan x + C} = e^C \cdot e^{\tan x} = k \cdot e^{\tan x}$$

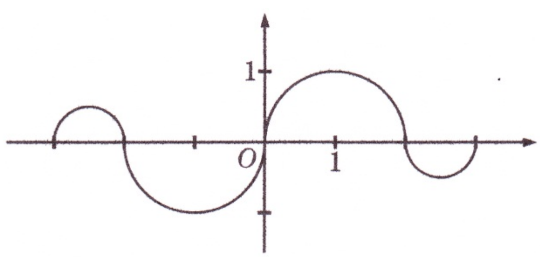
Then, when $x = 0$ and $y = 5$,

$$5 = k \cdot e^0, \text{ so } k = 5$$

And finally,

$$y = 5\, e^{\tan x}$$

Answer C



Graph of f

15. The graph of the function f above consists of four semicircles. If $g(x) = \int_0^x f(t)\, dt$, where is $g(x)$ nonnegative?

(A) $[-3, 3]$
(B) $[-3, -2] \cup [0, 2]$ only
(C) $[0, 3]$ only
(D) $[0, 2]$ only
(E) $[-3, -2] \cup [0, 3]$ only

Solution: A solution to this can be developed based on logic.

- If $0 < x \leq 3$, then $\int_0^x f(t)\, dt$ represents the net area under the curve from 0 to x, which is always positive.
- If $-3 \leq x < 0$, then $\int_x^0 f(t)\, dt$ represents the net area under the curve from x to 0, which is always negative.

  Then $\int_x^0 f(t)\, dt < 0 \quad \forall\, x \in [-3, 0)$

  And so, $\int_0^x f(t)\, dt > 0 \quad \forall\, x \in [-3, 0)$

- At $x = 0$, the area is zero.

Conclusion: $\int_0^x f(t)\, dt > 0 \quad \forall\, x \in [-3, 3]$ **Answer A**

16. If f is differentiable at $x = a$, which of the following could be false?

(A) f is continuous at $x = a$.

(B) $\lim_{x \to a} f(x)$ exists.

(C) $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists.

(D) $f'(a)$ is defined.

(E) $f''(a)$ is defined.

Solution: Let's take the statements one at a time:

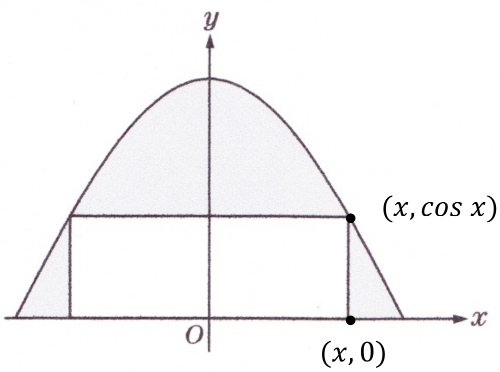
- **A.** In order for a function $f(x)$ to be differentiable at a point, it must also be continuous at that point. TRUE
- **B.** A function $f(x)$ is differentiable at a point a if $f'(a)$ exists. If $f'(a)$ exists, then the limit must exist. TRUE
- **C.** This expression is the definition of $f'(a)$ which, as stated above, must exist if $f(x)$ is differentiable at $x = a$. TRUE
- **D.** Again, $f'(a)$ must exist if $f(x)$ is differentiable at $x = a$. TRUE
- **E.** **If $f(x)$ is differentiable at $x = a$, then $f''(a)$ may or may not exist. See the example below for one that does not exist. FALSE**

Answer E

Example: consider the function $y = x^{1.5}$. We have:

$f(x) = x^{1.5}$	$f(0) = 0$
$f'(x) = 1.5\,x^{0.5}$	$f'(0) = 0$
$f''(x) = 0.75\,x^{-0.5}$	$f''(0)$ does not exist

So, $\boldsymbol{f(x) = x^{1.5}}$ is differentiable at $x = 0$, but $f''(0)$ is not defined.



17. A rectangle with one side on the x-axis has its upper vertices on the graph of $y = \cos x$, as shown in the figure above. What is the minimum area of the shaded region?

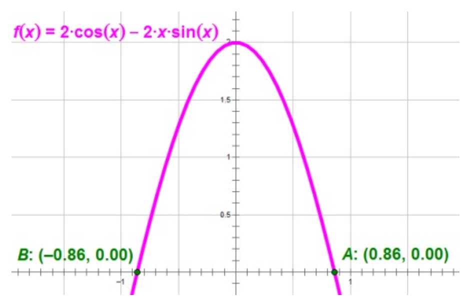
(A) 0.799
(B) 0.878
(C) 1.140
(D) 1.439
(E) 2.000

Solution: The shaded area under the curve is minimized when the area inside the rectangle is maximized. Add the two points shown above. Then, the area of the rectangle is:

$A_{rect} = (2x)\cos x$ The area of the rectangle is maximized when:

$$\frac{d}{dx}(2x)\cos x = 0$$

$$2\cos x - 2x\sin x = 0$$



Based on the graph at right (you need to use a graphing calculator or program for this), we get:

$x = 0.86$

At $x = 0.86$, $\cos x = 0.6524$, and so,

$A_{rect} = (2 \cdot 0.86) \cdot 0.6524 = \mathbf{1.122}$

Then, the area under the curve $y = \cos x$ is:

$$A_{under:\ y=\cos x} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx = \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) = 1 - (-1) = \mathbf{2}$$

And, finally,

$A_{shaded} = \mathbf{2 - 1.122 = 0.878}$ **Answer B**

18. A solid has a rectangular base that lies in the first quadrant and is b x- and y-axes and the lines $x = 2$ and $y = 1$. The height of the solid point (x, y) is $1 + 3x$. Which of the following is a Riemann sum appr the volume of the solid?

(A) $\sum_{i=1}^{n} \frac{1}{n}\left(1+\frac{3i}{n}\right)$

(B) $2\sum_{i=1}^{n} \frac{1}{n}\left(1+\frac{3i}{n}\right)$

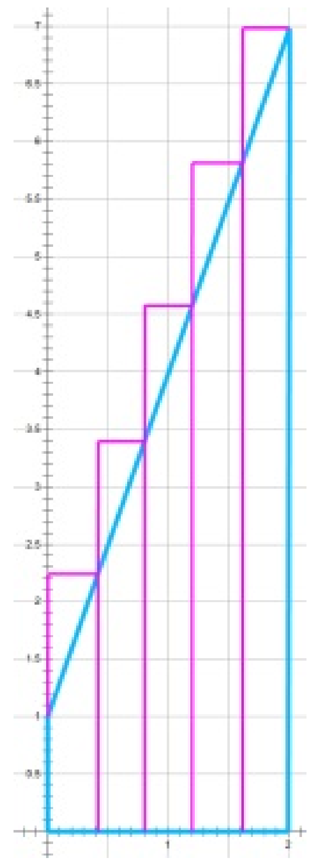
(C) $2\sum_{i=1}^{n} \frac{i}{n}\left(1+\frac{3i}{n}\right)$

(D) $\sum_{i=1}^{n} \frac{2}{n}\left(1+\frac{6i}{n}\right)$

(E) $\sum_{i=1}^{n} \frac{2i}{n}\left(1+\frac{6i}{n}\right)$

Solution: The bounding area includes values of y between 0 and 1, and the value of the function (e.g., z=f(x)) does not depend on y. Since the y-interval is 1, this problem simplifies to finding the Riemann sum for the area of a trapezoid bounded by the x-axis, the lines $x = 0$ and $x = 1$, and the function $z = 1 + 3x$.

See figure at right.



The violet rectangles at right represent the component parts of the Riemann sum. Each rectangle has the following dimensions:

$$\text{width} = \frac{2}{n}$$

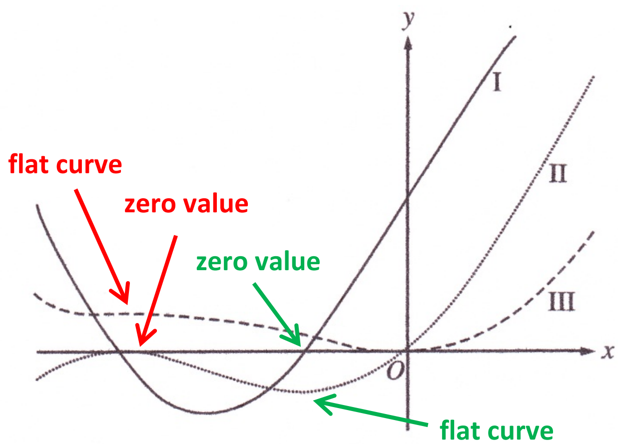
$$\text{height} = f\left(\frac{2i}{n}\right) = 1 + 3\left(\frac{2i}{n}\right) = 1 + \frac{6i}{n}$$

$$\text{Area of 1 rectangle} = \frac{2}{n} \cdot \left(1 + \frac{6i}{n}\right)$$

$$\textbf{Riemann Sum} = \sum_{i=1}^{n} \frac{2}{n} \cdot \left(1 + \frac{6i}{n}\right)$$

Answer D

Note: A Riemann Sum can take any value of $f(x)$ in the interval. So, for example, in the first interval of the figure shown above, $f(x)$ could be anywhere from $f(0) = 1$ to $f(1/5) = 2.2$. The selection of the value of $f(x)$ at its maximum value in each interval seems like a particularly poor choice when n is small. However, when a limit is taken and n becomes arbitrarily large, the selection of a particular value of $f(x)$ in the interval becomes immaterial because the values of $f(x)$ in each interval converge to a single value.



19. Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	I	III	II
(C)	II	I	III
(D)	II	III	I
(E)	III	II	I

Solution: This problem boils down to determining functions that are derivatives of other functions, and then using the following relationships to identify each curve:

$f'(x)$ is the slope of $f(x)$

$f''(x)$ is the slope of $f'(x)$

Key points are identified above in green and red. Notice that:

- When the value of Curve II is zero, Curve III is flat. Therefore, Curve II must be the derivative of Curve III.
- When the value of Curve I is zero, Curve II is flat. Therefore, Curve I must be the derivative of Curve II.

Combining these, we get:

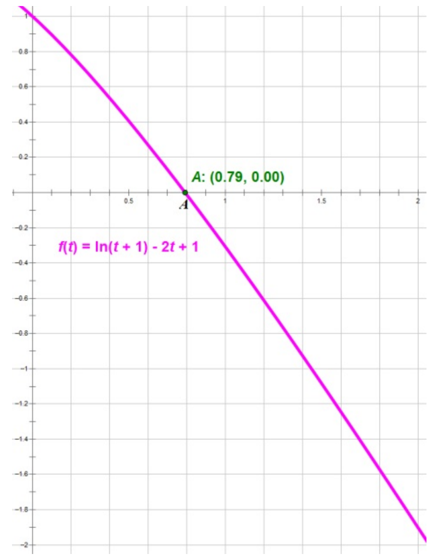
- $f(x) = Curve\ \text{III}$
- $f'(x) = Curve\ \text{II}$
- $f''(x) = Curve\ \text{I}$

Answer E

20. A particle moves along the x-axis so that at any time t $v(t) = \ln(t+1) - 2t + 1$. The total distance traveled by $t = 2$ is

(A) 0.667
(B) 0.704
(C) 1.540
(D) 2.667
(E) 2.901

Solution: Since the question asks for total distance, we first need to find where $v(t)$ is positive and where $v(t)$ is negative for $0 < t < 2$. We will need separate integrals for each change of direction.



Method 1 (the calculus way):

The function changes from positive to negative at roughly $x = .79$. So,

$$D = \left|\int_0^{0.79} v(t)\ dt\right| + \left|\int_{0.79}^{2} v(t)\ dt\right|$$

where,

$$\int v(t)\ dt = \int [\ln(t+1) - 2t + 1]\ dt$$

$$= (t+1)\cdot\ln(t+1) - (t+1) - t^2 + t$$

$$= (t+1)\cdot\ln(t+1) - t^2 \ - 1$$

Note: the 1 can be ignored because it is a constant.

Then,

$$D = \left|\left((t+1)\cdot\ln(t+1) - t^2\right)\Big|_0^{0.79}\right| + \left|\left((t+1)\cdot\ln(t+1) - t^2\right)\Big|_{0.79}^{2}\right|$$

$$= |\ (1.79\ \ln 1.79 - 0.79^2) - (0)\ | + |\ (3\ln 3 - 4) - \ (1.79\ \ln 1.79 - 0.79^2)\ |$$

$$= |1.042 - .624| + |(3.296 - 4) - \ (1.042 - .624)\ |$$

$$\boldsymbol{D} = .418 + 1.122 = \mathbf{1.540}$$

Answer C

Method 2 (the geometry – and easy – way):

Notice that the function is pretty close to a straight line with:

$f(0) = 1$ $\qquad$ $f(0.79) \sim 0$ $\qquad$ $f(2) \sim 1.9$

Then the two areas between the x-axis and the curve are triangles, so:

$$\boldsymbol{D} = \left(\frac{1}{2}\cdot 0.79\cdot 1\right) + \left(\frac{1}{2}\cdot 1.21\cdot 1.9\right) = \mathbf{1.545}$$

which is very close to answer C and no other answer – with very little work!

21. If the function f is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

(A) -3.268
(B) -1.585
(C) 1.732
(D) 6.585
(E) 11.585

Solution: Approximate the solution by creating the following table:

$x = 1$	$g(1)$	$g'(1) = f(1) = \sqrt{3} \sim 1.732$
$x = 2$		$g'(2) = f(2) = \sqrt{10} \sim 3.162$
$x = 3$	$g(3) = 5$	$g'(3) = f(3) = \sqrt{29} \sim 5.385$

Then, approximate the change from $x = 1$ to $x = 3$ based on the average slope in each of the two intervals (1 to 2 and 2 to 3), as follows:

$$g(2) \sim g(1) + \frac{1.732 + 3.162}{2} = g(1) + 2.447$$

$$g(3) \sim g(2) + \frac{3.162 + 5.385}{2} = (g(1) + 2.447) + 4.274$$

Since $g(3) = 5$, we get:

$$5 \sim g(1) + 6.721$$

And, finally:

$$\boldsymbol{g(1) \sim (-1.721)}$$

which is close to answer B and no other answer.

Answer B

22. Let g be the function given by $g(x) = \int_1^x 100\left(t^2 - 3t + 2\right)e^{-t^2}\,dt$.

Which of the following statements about g must be true?

I. g is increasing on $(1, 2)$.

II. g is increasing on $(2, 3)$.

III. $g(3) > 0$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

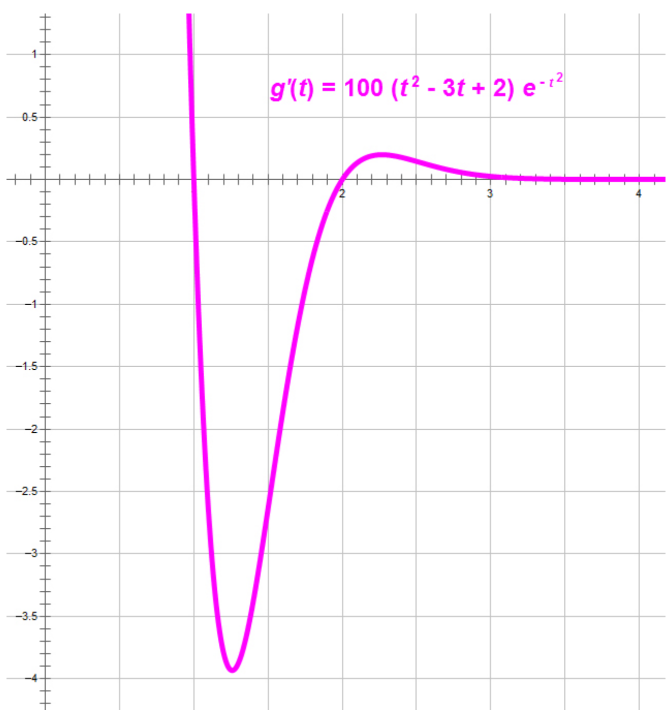
Solution: Look at the graph of $g'(x)$ below and evaluate each of the three statements.

Statement I. $g'(x) < 0$ on $(1, 2)$, so $g(x)$ decreases in this interval. FALSE

Statement II. $g'(x) > 0$ on $(2, 3)$, so $g(x)$ increases in this interval. TRUE

Statement III. $g'(3) > 0$ according to the graph, but Statement III states that $g(3) > 0$, so don't fall for this trick. $g(3)$ can be thought of as the accumulated area under the curve from $x = 1$ to $x = 3$, which is clearly less than 0 because of the large negative area between $x = 1$ and $x = 2$. FALSE

Answer B



23. For a series S, let

$$S = 1 - \frac{1}{9} + \frac{1}{2} - \frac{1}{25} + \frac{1}{4} - \frac{1}{49} + \frac{1}{8} - \frac{1}{81} + \frac{1}{16} - \frac{1}{121} + \cdots + a_n + \cdots,$$

$$\text{where } a_n = \begin{cases} \dfrac{1}{2^{(n-1)/2}} & \text{if } n \text{ is odd} \\ \dfrac{-1}{(n+1)^2} & \text{if } n \text{ is even.} \end{cases}$$

Which of the following statements are true?

I. S converges because the terms of S alternate and $\lim_{n\to\infty} a_n = 0$.

II. S diverges because it is not true that $|a_{n+1}| < |a_n|$ for all n.

III. S converges although it is not true that $|a_{n+1}| < |a_n|$ for all n.

(A) None
(B) I only
(C) II only
(D) III only
(E) I and III only

Solution: Consider each of the three statements.

Statement I. The Alternating Series Test also requires that $|a_{n+1}| \le |a_n|\ \forall\, n$. That is, the absolute values of the terms must be monotonically decreasing. Therefore, the Alternating Series Test does not apply. FALSE

Statement II. Looking at the two series separately, we can see that each converges. Since S is the sum of the two, it must also converge. S is equal to the sum of sums of the two individual series. FALSE

Statement III. As stated above, the two series converge, so S converges. TRUE **Answer D**

Convergence of the Individual Series

note: for S_2, you only need use only one of the three convergence tests presented

- $S_1 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ is a geometric series with $r = \frac{1}{2}$. It converges by the ratio test since $|r| < 1$. Also note that, in Problem 9 above, we showed that $S_1 = 2$.
- $S_2 = -(\frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81} + \cdots)$ can be compared to S_1, and since the absolute value of each term in S_2 is less than the absolute value of the corresponding term in S_1, the series converges.

  Alternatively, note that S_2 is a sub-series of a p-series with $p = 2$, so it converges.

  If you don't like either of the two tests above, you can use the integral test as follows:

  $\int_1^\infty (2x+1)^{-2}\,dx = -\frac{1}{4x+2}\Big|_1^\infty = \frac{1}{6}$ Since the integral converges, so does the series.

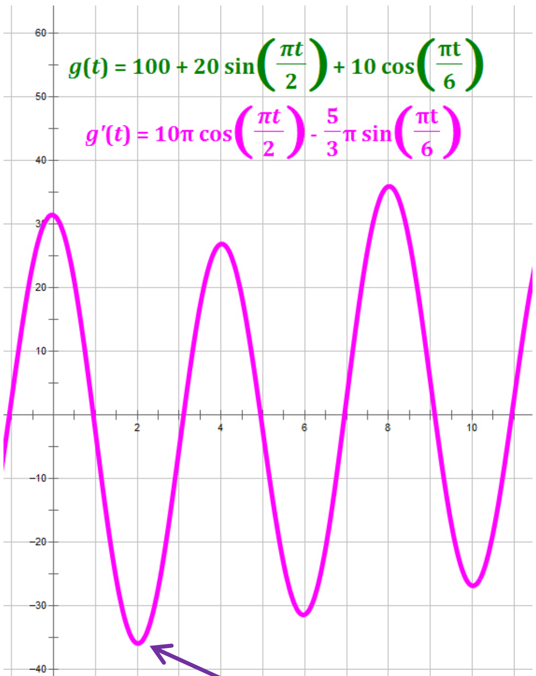
24. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$.
For $0 \le t \le 8$, g is decreasing most rapidly when $t =$

(A) 0.949
(B) 2.017
(C) 3.106
(D) 5.965
(E) 8.000

Solution: We want the point at which $g'(t)$ is at a minimum on the interval $0 \le t \le 8$.

Based on the graph below, this clearly occurs at $x \sim 2$. The only answer near 2 is B.

Answer B



Answers to Calculus BC Multiple-Choice Questions

Part A		*Part B*	
†1.	D	15.	A
2.	B	16.	E
†3.	D	17.*	B
†4.	D	18.	D
5.	D	19.	E
6.	E	20.*	C
7.	E	21.*	B
†8.	E	22.*	B
†9.	E	†23.	D
10.	E	24.*	B
†11.	D		
†12.	A		
†13.	A		
14.	C		