Trigonometry – Key Relationships and Formulas

Function Relationships

$\sin\theta = \dfrac{1}{\csc\theta}$ $\qquad \csc\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ $\qquad \sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Opposite Angle Formulas

$\sin(-\theta) = -\sin(\theta)$

$\cos(-\theta) = \cos(\theta)$

$\tan(-\theta) = -\tan(\theta)$

$\cot(-\theta) = -\cot(\theta)$

$\sec(-\theta) = \sec(\theta)$

$\csc(-\theta) = -\csc(\theta)$

Cofunction Formulas (in Quadrant I)

$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ $\qquad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$ $\qquad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$ $\qquad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$

Angle Addition Formulas

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Half Angle Formulas

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

$\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

Double Angle Formulas

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Product-to-Sum Formulas

$\sin A\cdot\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$\cos A\cdot\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$\sin A\cdot\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\cos A\cdot\sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

Triple Angle Formulas

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Power Reducing Formulas

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$

Sum-to-Product Formulas

$\sin A + \sin B = 2\cdot\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2\cdot\sin\left(\frac{A-B}{2}\right)\cdot\cos\left(\frac{A+B}{2}\right)$

$\cos A + \cos B = 2\cdot\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = -2\cdot\sin\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

Arc Length

$S = r\theta$

Law of Sines

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Law of Tangents

$\dfrac{a-b}{a+b} = \dfrac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left[\frac{1}{2}(A+B)\right]}$

mathguy.us

Mollweide's Formulas

$\dfrac{a+b}{c} = \dfrac{\cos\left[\frac{1}{2}(A-B)\right]}{\sin\left(\frac{1}{2}C\right)}$

$\dfrac{a-b}{c} = \dfrac{\sin\left[\frac{1}{2}(A-B)\right]}{\cos\left(\frac{1}{2}C\right)}$

Euler's Formula

$e^{i\theta} = \cos\theta + i\sin\theta = \operatorname{cis}\theta$

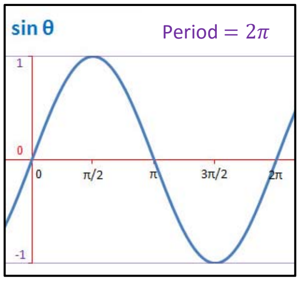
Polar Multiplication and Division

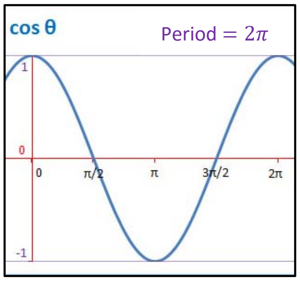
Let: $a = r_1\operatorname{cis}\theta$ $\qquad b = r_2\operatorname{cis}\varphi$

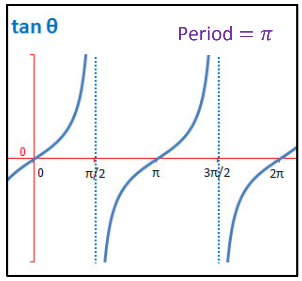
$a\cdot b = r_1 r_2\operatorname{cis}(\theta+\varphi)$ $\qquad \dfrac{a}{b} = \dfrac{r_1}{r_2}\operatorname{cis}(\theta-\varphi)$

DeMoivre's Formula

$(r\operatorname{cis}\theta)^n = r^n\operatorname{cis}(n\theta)$







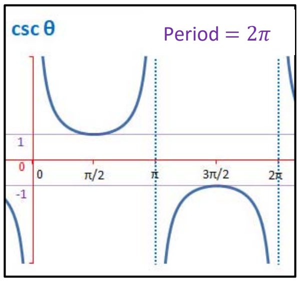
$\boldsymbol{y = A \cdot f(Bx - C) + D}$

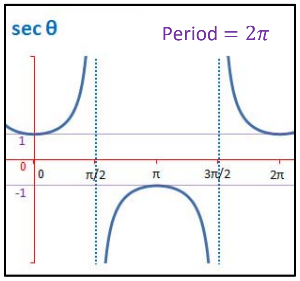
Amplitude: $|A|$

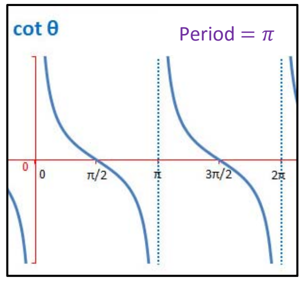
Period: $\dfrac{parent\ "f"\ period}{B}$

Phase Shift: $\dfrac{C}{B}$ →

Vertical Shift: D







Harmonic Motion

$d = a\cos\omega t$ or

$d = a\sin\omega t$

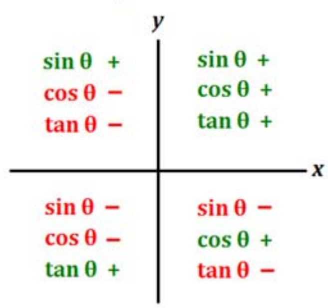
$f = \dfrac{1}{\text{period}} = \dfrac{\omega}{2\pi}$

$\omega = 2\pi f, \quad \omega > 0$

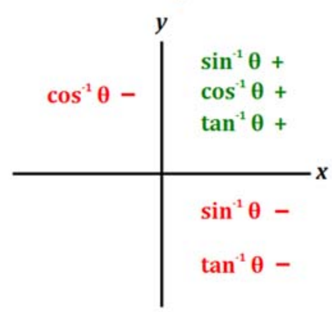
Trig Functions of Special Angles (Unit Circle)

θ Rad	$\theta°$	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0^o	0	1	0
$\pi/6$	30^o	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
$\pi/4$	45^o	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	60^o	$\sqrt{3}/2$	1/2	$\sqrt{3}$
$\pi/2$	90^o	1	0	*undefined*

Signs of Trig Functions by Quadrant



Locations of Principal Values of Inverse Trig Functions



Rectangular/Polar Conversion

Rectangular	Polar
(x, y)	(r, θ)
$x = r\cos\theta$ $y = r\sin\theta$	$r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
$a + bi$	$r\,(\cos\theta + i\sin\theta)$ or $r\ cis\theta$
$a = r\cos\theta$ $b = r\sin\theta$	$r = \sqrt{a^2 + b^2}$ $\theta = \tan^{-1}\left(\frac{b}{a}\right)$
$a\mathbf{i} + b\mathbf{j}$	$\lVert\mathbf{v}\rVert\ \angle\theta$
$a = \lVert\mathbf{v}\rVert\cos\theta$ $b = \lVert\mathbf{v}\rVert\sin\theta$	$\lVert\mathbf{v}\rVert = \sqrt{a^2 + b^2}$ $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Triangle Area

$$A = \frac{1}{2}bh$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}P = \frac{1}{2}(a + b + c)$$

$$A = \frac{1}{2}\left(\frac{a^2\sin B\sin C}{\sin A}\right)$$

$$A = \frac{1}{2}\,ab\sin C$$

$$A = \frac{1}{2}\left|\,\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\,\right|$$

$$A = \lVert\mathbf{u}\rVert\ \lVert\mathbf{v}\rVert\ \sin\theta$$

Vector Properties

$0 + \mathbf{u} = \mathbf{u} + 0 = \mathbf{u}$

$\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = 0$

$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$

$m(n\mathbf{u}) = (mn)\mathbf{u}$

$m(\mathbf{u} + \mathbf{v}) = m\mathbf{u} + m\mathbf{v}$

$(m + n)\mathbf{u} = m\mathbf{u} + n\mathbf{u}$

$1(\mathbf{v}) = \mathbf{v}$

$\lVert m\mathbf{v}\rVert = |m|\ \lVert\mathbf{v}\rVert$

Unit Vector: $\dfrac{\mathbf{v}}{\lVert\mathbf{v}\rVert}$

Vector Dot Product

$\mathbf{u} \circ \mathbf{v} = (u_1 \cdot v_1) + (u_2 \cdot v_2)$

$\mathbf{u} \circ (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \circ \mathbf{v}) + (\mathbf{u} \circ \mathbf{w})$

Vector Cross Product

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} = u_1 v_2 - u_2 v_1$

$\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$

Angle between Vectors

$\cos\theta = \dfrac{\mathbf{u} \circ \mathbf{v}}{\lVert\mathbf{u}\rVert\ \lVert\mathbf{v}\rVert}$ $\quad$ $\sin\theta = \dfrac{\lVert\mathbf{u} \times \mathbf{v}\rVert}{\lVert\mathbf{u}\rVert\ \lVert\mathbf{v}\rVert}$

$\perp$ iff $\mathbf{u} \circ \mathbf{v} = 0$ $\quad$ $\parallel$ iff $\mathbf{u} \times \mathbf{v} = 0$