Trig Star Sample Test – 2011

Step-by-Step Solutions

Notes to student:

- You will notice that I have added information to the diagrams in order to help visualize the solutions to the problems. You should do this on your test as well.
- Angles need to be expressed in degrees-minutes-seconds on this test. A procedure to do this easily on your calculator is shown below.

Procedure for converting degrees in decimal form to degrees-minutes-seconds.

Example. Convert: $54.82469605°$ to degrees-minutes-seconds.

Step 1: Starting value:	$54.82469605°$
Step 2: Record the integer from the number of degrees:	$54°$
Step 3: Subtract the integer from the decimal form:	$0.82469605°$
Step 4: Multiply the remaining decimal by 60:	x 60
Result: Number of Minutes	$49.481763'$
Step 5: Record the integer from the number of minutes:	$49'$
Step 6: Subtract the integer from the decimal form:	$0.481763'$
Step 7: Multiply the remaining decimal by 60:	x 60
Result: Number of Seconds	$28.90578''$
Step 8: Round the number of seconds	$29''$
Step 9: Combine pieces to get the final answer:	$54°\ 49'\ 29''$

Procedure for converting degrees-minutes-seconds to degrees in decimal form.

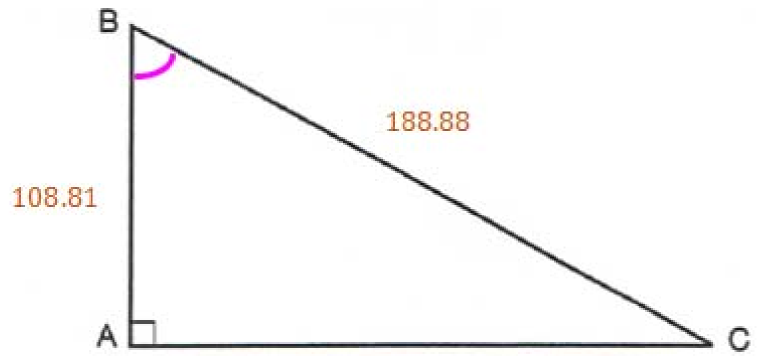
Example. Convert: $54°\ 49'\ 29''$ to degrees in decimal form.

Recall that one degree is 60 minutes and one minute is 60 seconds. Then,

$$54°\ 49'\ 29'' = \left(54 + \frac{49}{60} + \frac{29}{3600}\right)^{\circ} = 54.82472223°$$

Note: this result does not match the starting value of the first example because I rounded the measure of the angle to the nearest second. In general, a conversion of the measure of an angle from "degrees-minutes-seconds form" to "degrees in decimal form" will be accurate to 4 decimal places $\left(\frac{0.5°}{3600} \sim 0.0001°\right)$.

1st Problem on PAGE 1



KNOWN: DISTANCE AB = 108.81 DISTANCE BC = 188.88

FIND: ∠ CBA = ______________ (5 POINTS)

DISTANCE AC = ______________ (5 POINTS)

REQUIRED ANSWER FORMAT
DISTANCES: NEAREST HUNDREDTH
ANGLES: DEGREES–MINUTES–SECONDS TO THE NEAREST SECOND

$AB = 108.81$

$BC = 188.88$

$AC^2 = BC^2 - AB^2 = 188.88^2 - 108.81^2 = 23{,}836.0383$

$\boldsymbol{AC = \sqrt{23{,}836.0383} = 154.39}$

$\cos \measuredangle CBA = \frac{108.81}{188.88} \quad \Rightarrow \quad \measuredangle CBA = \cos^{-1}\frac{108.81}{188.88} = 54.82469605°$

Now, we need to convert this to degrees minutes and seconds.

$\text{Degrees} = 54$

1 minute is 1/60 of a degree. 1 second is 1/60 of a minute.

$\frac{\text{Minutes}}{60} = 0.82469605 \quad \Rightarrow \quad \text{Minutes} = 60 \cdot (0.82469605) = 49.481763'$

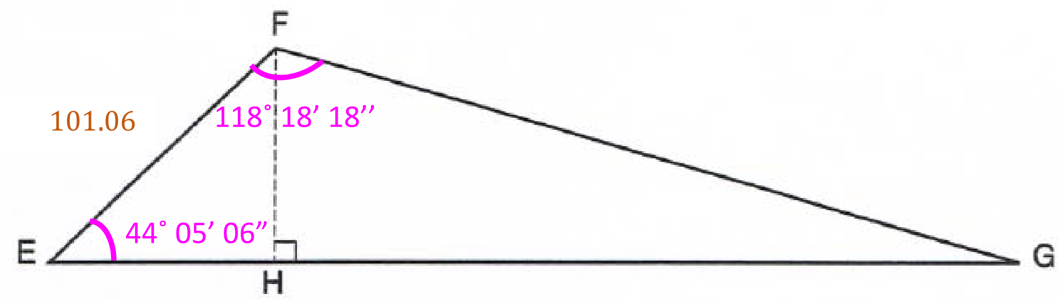
$\text{Minutes} = 49$

$\frac{\text{Seconds}}{60} = 0.481763 \quad \Rightarrow \quad \text{Seconds} = 60 \cdot (0.481763) = 29''$

$\text{Seconds} = 29$

$\boldsymbol{\measuredangle CBA = 54° \, 49' \, 29''}$

2nd Problem on PAGE 1



KNOWN: DISTANCE EF = 101.06 ∡ EFG = 118°18′18″ ∡ FEG = 44°05′06″

FIND:

∡ EGF = ____________ (6 POINTS)

DISTANCE EH = ____________ (6 POINTS)

DISTANCE FH = ____________ (6 POINTS)

DISTANCE FG = ____________ (6 POINTS)

DISTANCE GH = ____________ (6 POINTS)

REQUIRED ANSWER FORMAT
DISTANCES: NEAREST HUNDREDTH
ANGLES: DEGREES–MINUTES–SECONDS TO THE NEAREST SECOND

PAGE TOTAL: ________ POINTS

To get $\measuredangle EGF$, recall that the sum of the measures of the angles in a triangle is 180˚.

$$\measuredangle EGF = \mathbf{180^\circ} - (\measuredangle EFG + \measuredangle FEG) = \mathbf{180^\circ} - (118^\circ\, 18'\, 18'' + 44^\circ\, 05'\, 06'')$$

$$= \mathbf{179^\circ\, 59'\, 60''} - 162^\circ\, 23'\, 24'' = 17^\circ\, 36'\, 36''$$

$$\boldsymbol{\measuredangle EGF = 17^\circ\, 36'\, 36''}$$

To calculate the distances required using our calculators, we need to convert the angle measures from degrees-minutes-seconds to degrees in decimal form.

$$\measuredangle EFG = 118^\circ\, 18'\, 18'' = \left(118 + \frac{18}{60} + \frac{18}{3600}\right)^\circ = 118.305^\circ$$

$$\measuredangle FEG = 44^\circ\, 05'\, 06'' = \left(44 + \frac{5}{60} + \frac{6}{3600}\right)^\circ = 44.085^\circ$$

$$\measuredangle EGF = 17^\circ\, 36'\, 36'' = \left(17 + \frac{36}{60} + \frac{36}{3600}\right)^\circ = 17.610^\circ$$

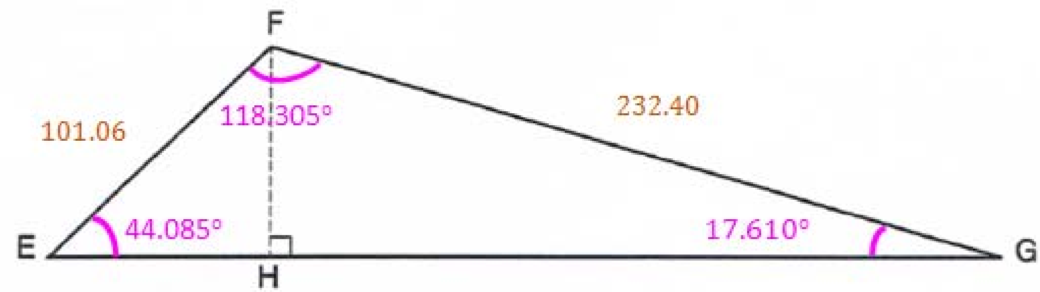
Note: the results of these calculations are exact. If rounding is needed, I recommend that you keep at least 5 decimal places in order to maintain the accuracy required in the test solutions.

The easiest requested distance to calculate is FG, using the Law of Sines.

$$\frac{FG}{\sin \measuredangle FEG} = \frac{EF}{\sin \measuredangle EGF} \quad \Rightarrow \quad \frac{FG}{\sin 44.085^\circ} = \frac{101.06}{\sin 17.610^\circ}$$

$$\boldsymbol{FG = 232.40}$$

2nd Problem on PAGE 1 (continued)



KNOWN: DISTANCE EF = 101.06 ∡EFG = 118°18'18" ∡FEG = 44°05'06"

FIND: ∡EGF = ______________ (6 POINTS)

DISTANCE EH = ______________ (6 POINTS)

DISTANCE FH = ______________ (6 POINTS)

DISTANCE FG = ______________ (6 POINTS)

DISTANCE GH = ______________ (6 POINTS)

REQUIRED ANSWER FORMAT
DISTANCES: NEAREST HUNDREDTH
ANGLES: DEGREES–MINUTES–SECONDS TO THE NEAREST SECOND

PAGE TOTAL: ________ POINTS

Let's calculate FH next.

$$\sin \measuredangle FEG = \frac{FH}{EF} \quad \Rightarrow \quad \sin 44.085° = \frac{FH}{101.06}$$

$$\boldsymbol{FH = 70.31}$$

Let's calculate EH next.

$$\cos \measuredangle FEG = \frac{EH}{EF} \quad \Rightarrow \quad \cos 44.085° = \frac{EH}{101.06}$$

$$\boldsymbol{EH = 72.59}$$

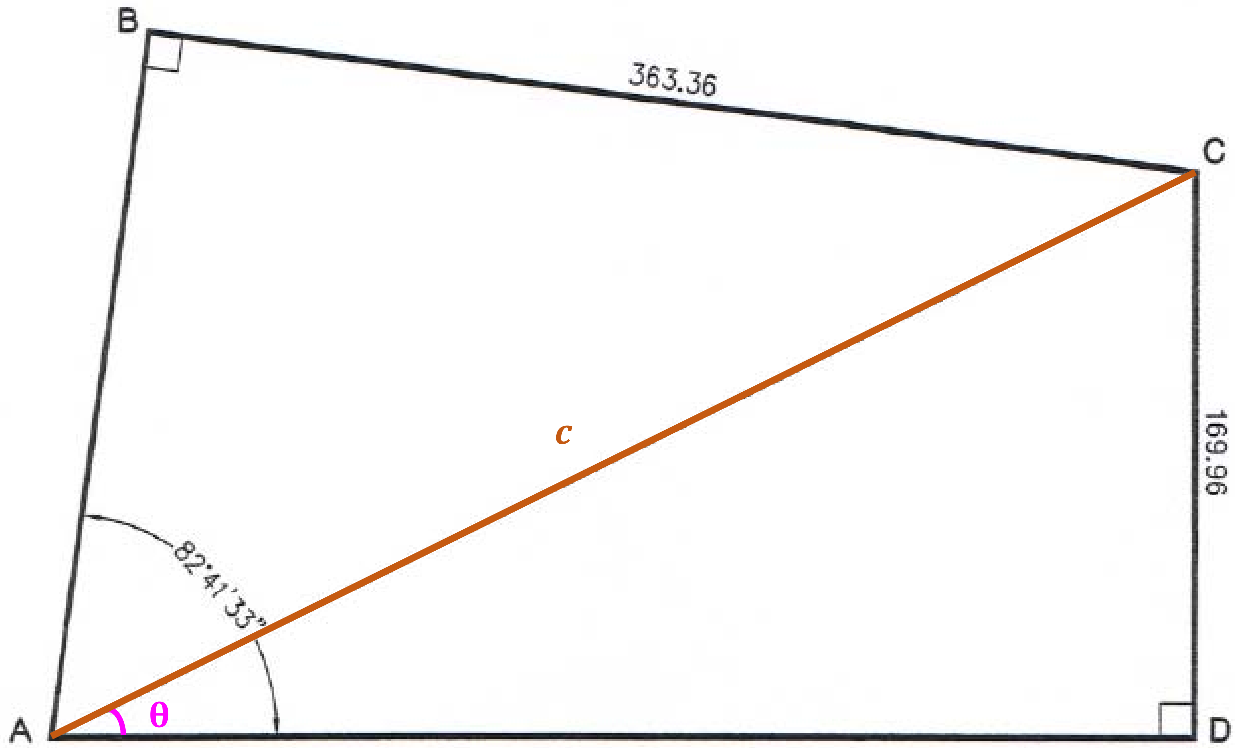
Finally, let's calculate GH.

$$\measuredangle GFH = \mathbf{90°} - \measuredangle EGF = \mathbf{90°} - 17.61° = 72.39°$$

$$\tan \measuredangle GFH = \frac{GH}{FH} \quad \Rightarrow \quad \tan 72.39° = \frac{GH}{70.31}$$

$$\boldsymbol{GH = 221.51}$$

Problem on PAGE 2



Let's add a couple of labels to make life easier. Label AC as c and $\measuredangle CAD$ as θ.

Nothing easy here. Notice that $\overline{AC}$ is in both triangles, so let's see what we can do with it.

We will work with $\measuredangle BAD$ because it is the only angle shown. Let's start by converting the measure of $\measuredangle BAD$ to degrees so we can use it on our calculators.

$$\measuredangle BAD = 82°\,41'33'' = \left(82 + \frac{41}{60} + \frac{33}{3600}\right)° = 82.6925°$$

Let's break $\measuredangle BAD$ into two angles, $\measuredangle CAD$ and $\measuredangle BAC$, which share side $\overline{AC}$.

Then, $\sin \measuredangle CAD = \dfrac{169.96}{c}$ and $\sin \measuredangle BAC = \dfrac{363.36}{c}$. Now it's time for some Algebra.

$$c = \frac{169.96}{\sin \measuredangle CAD} \text{ and } c = \frac{363.36}{\sin \measuredangle BAC}.$$

Next, since we have two expressions equal to c, we can set them equal to each other.

$$\frac{169.96}{\sin \measuredangle CAD} = \frac{363.36}{\sin \measuredangle BAC} \quad \text{so,} \quad \frac{\sin \measuredangle BAC}{\sin \measuredangle CAD} = \frac{363.36}{169.96}$$

Substituting $\measuredangle CAD = \theta$ and $\measuredangle BAC = 82.6925° - \theta$ gives: $\dfrac{\sin(82.6925° - \theta)}{\sin\theta} = \dfrac{363.36}{169.96}$

Problem on PAGE 2 (continued)

Next recall the difference of angles formula: $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

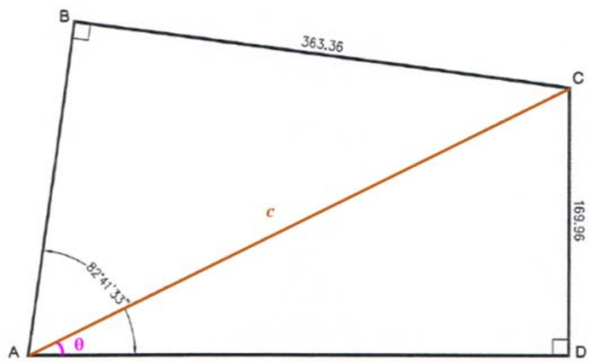
$$\frac{\sin(82.6925°-\theta)}{\sin\theta}=\frac{\sin(82.6925°)\cos\theta-\cos(82.6925°)\sin\theta}{\sin\theta}=\frac{363.36}{169.96}$$

This reduces to:

$$\frac{\sin(82.6925°)}{\tan\theta}-\cos(82.6925°)=\frac{363.36}{169.96}$$

Solve for $\tan\theta$ to get:

$$\tan\theta=.437894 \qquad \Rightarrow \qquad \theta=23.6483°$$



Now we can identify all of the angles we need:

$$\measuredangle CAD=\theta=23.6483°$$

$$\measuredangle BAC=82.6925°-23.6483°=59.0442°$$

$$\measuredangle ACD=90°-23.6483°=66.3517°$$

$$\measuredangle ACB=90°-59.0442°=40.9558°$$

With the angles known, we can calculate the required sides easily.

Let's calculate AB.

$$\tan\measuredangle BAC=\frac{363.36}{AB}$$

$$\tan 59.0442°=\frac{363.36}{AB}$$

$$\boldsymbol{AB=217.95}$$

Let's calculate AD.

$$\tan\measuredangle CAD=\frac{169.96}{AD}$$

$$\tan 23.6483°=\frac{169.96}{AD}$$

$$\boldsymbol{AD=388.13}$$

Let's calculate AC.

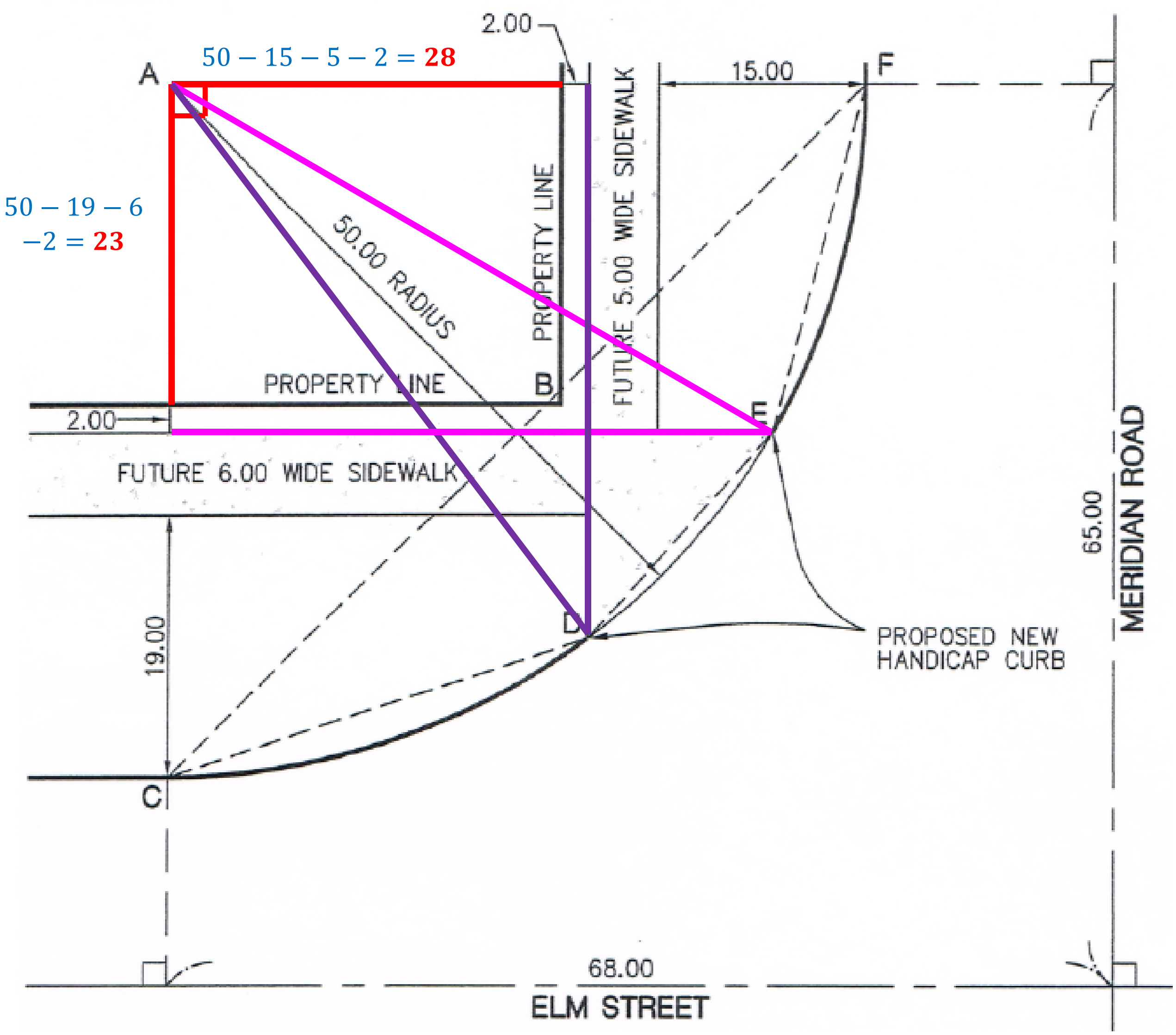
$$\sin\measuredangle BAC=\frac{363.36}{AC}$$

$$\sin 59.0442°=\frac{363.36}{AC}$$

$$\boldsymbol{AC=423.71}$$

Problem on Page 3

A LOCAL SURVEYOR HAS BEEN ASKED TO STAKE OUT POINTS D AND E WHERE A FUTURE SIDEWALK WILL MEET A NEW HANDICAP CURB AT THE CORNER OF MERIDIAN ROAD AND ELM STREET. THE SURVEYOR ALREADY KNOWS THE LOCATION OF POINTS B, C AND F BUT MUST MAKE SOME CALCULATIONS TO ESTABLISH AND VERIFY THE POINTS NEEDED.



Since the radius is 50.00, we can calculate some additional distances not shown on the diagram (shown in red). Note that because of all the right angles shown, $\measuredangle A$ is also a right angle.

Two calculations are very easy: CF and $m\,\widehat{CF}$

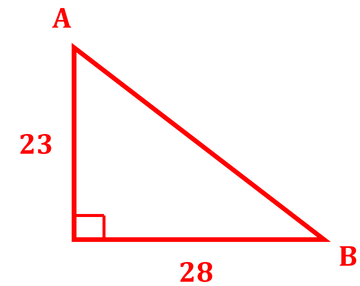
CF is the hypotenuse of ΔACF, which is isosceles with two sides equal to the radius ($r = 50$).

Therefore, $\boldsymbol{CF} = 50 \cdot \sqrt{2} = \mathbf{70.71}$

Arc $\widehat{CF}$ is one-fourth of the circumference of a circle of radius 50.

Therefore, $\boldsymbol{m\,\widehat{CF}} = \frac{1}{4} \cdot 2\pi r = \frac{1}{4} \cdot 2\pi \cdot 50 = 25\pi = \mathbf{78.54}$

Problem on Page 3 (continued)

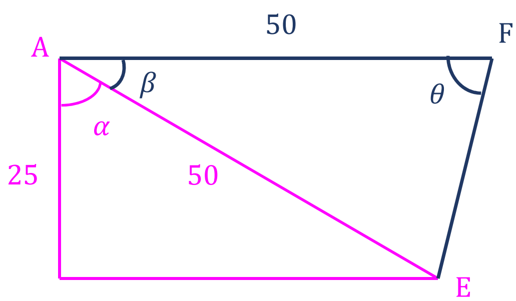


AB is relatively easy also. It is the hypotenuse of a triangle with sides **23** and **28**, both of which were added to the diagram above (in red). Each of these was calculated by subtracting the distances shown from the radius of 50.

$$\boldsymbol{AB = \sqrt{23^2 + 28^2} = 36.24}$$

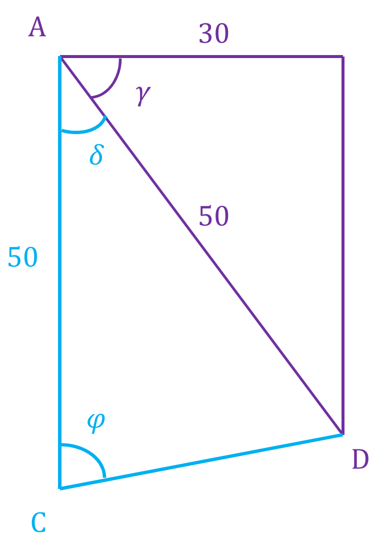
To calculate the distances of the three chords, CD, DE and EF, draw in segments AD and AE, as shown in the diagram on the previous page.

To find segment EF, consider the following sub-drawing of the diagram.



- Calculate $\alpha = \cos^{-1}\left(\frac{25}{50}\right) = 60°$
- Calculate $\beta = 90° - 60° = 30°$
- ΔAEF is isosceles, so $\measuredangle AEF = \measuredangle AFE$
- Calculate $\theta = \frac{1}{2}(180° - 30°) = 75°$
- Calculate EF from the Law of Sines
  $$\frac{EF}{\sin\beta} = \frac{AE}{\sin\theta} \quad \Rightarrow \quad \frac{EF}{\sin 30°} = \frac{50}{\sin 75°}$$
- $\boldsymbol{EF = 25.88}$

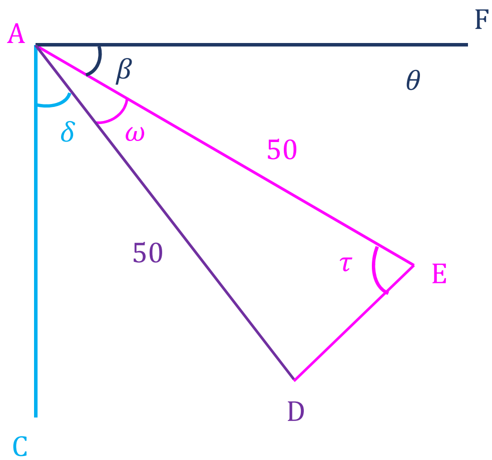
To find segment CD, consider the following sub-drawing of the diagram.



- Calculate $\gamma = \cos^{-1}\left(\frac{30}{50}\right) = 53.13°$
- Calculate $\delta = 90° - 53.1301° = 36.87°$
- ΔADC is isosceles, so $\measuredangle ADC = \measuredangle ACD$
- Calculate $\varphi = \frac{1}{2}(180° - 36.87°) = 71.565°$
- Calculate CD from the Law of Sines
  $$\frac{CD}{\sin\delta} = \frac{AD}{\sin\varphi} \quad \Rightarrow \quad \frac{CD}{\sin 36.87°} = \frac{50}{\sin 71.565°}$$
- $\boldsymbol{CD = 31.62}$

Problem on Page 3 (continued)

To find segment DE, consider the following sub-drawing of the diagram.



- From above, $\beta = 30°$
- From above, $\delta = 36.87°$
- Then, $\omega = 90° - 30° - 36.87° = 23.13°$
- ΔADE is isosceles, so $\measuredangle ADE = \measuredangle AED$
- Calculate $\tau = \frac{1}{2}(180° - 23.13°) = 78.435°$
- Calculate DE from the Law of Sines

$$\frac{DE}{\sin\omega} = \frac{AD}{\sin\tau} \quad \Rightarrow \quad \frac{DE}{\sin 23.13°} = \frac{50}{\sin 78.435°}$$

- $\boldsymbol{DE = 20.05}$