

# Trig Star Sample Test – 2011

## Step-by-Step Solutions

Notes to student:

- You will notice that I have added information to the diagrams in order to help visualize the solutions to the problems. You should do this on your test as well.
- Angles need to be expressed in degrees-minutes-seconds on this test. A procedure to do this easily on your calculator is shown below.

### Procedure for converting degrees in decimal form to degrees-minutes-seconds.

Example. Convert:  $54.82469605^\circ$  to degrees-minutes-seconds.

|                                                        |                                               |
|--------------------------------------------------------|-----------------------------------------------|
| Step 1: Starting value:                                | $54.82469605^\circ$ ←                         |
| Step 2: Record the integer from the number of degrees: | $54^\circ$                                    |
| Step 3: Subtract the integer from the decimal form:    | <hr style="width: 100%;"/> $0.82469605^\circ$ |
| Step 4: Multiply the remaining decimal by 60:          | $\times 60$                                   |
| Result: Number of Minutes                              | <hr style="width: 100%;"/> $49.481763'$       |
| Step 5: Record the integer from the number of minutes: | $49'$                                         |
| Step 6: Subtract the integer from the decimal form:    | <hr style="width: 100%;"/> $0.481763'$        |
| Step 7: Multiply the remaining decimal by 60:          | $\times 60$                                   |
| Result: Number of Seconds                              | <hr style="width: 100%;"/> $28.90578''$       |
| Step 8: Round the number of seconds                    | $29''$                                        |
| Step 9: Combine pieces to get the final answer:        | $54^\circ 49' 29''$                           |

### Procedure for converting degrees-minutes-seconds to degrees in decimal form.

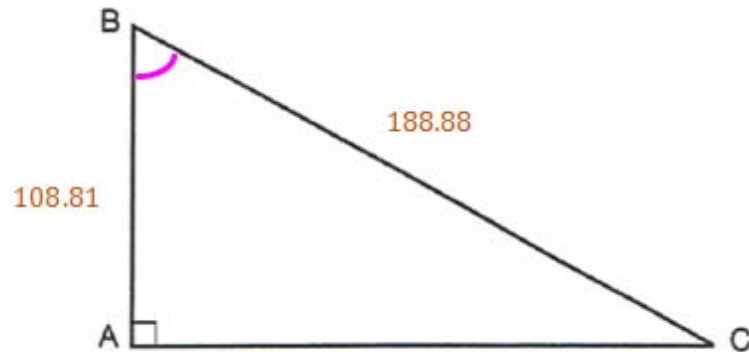
Example. Convert:  $54^\circ 49' 29''$  to degrees in decimal form.

Recall that one degree is 60 minutes and one minute is 60 seconds. Then,

$$54^\circ 49' 29'' = \left( 54 + \frac{49}{60} + \frac{29}{3600} \right)^\circ = 54.82472223^\circ \leftarrow$$

Note: this result does not match the starting value of the first example because I rounded the measure of the angle to the nearest second. In general, a conversion of the measure of an angle from “degrees-minutes-seconds form” to “degrees in decimal form” will be accurate to 4 decimal places  $\left( \frac{0.5^\circ}{3600} \sim 0.0001^\circ \right)$ .

**1<sup>st</sup> Problem on PAGE 1**



KNOWN: DISTANCE AB = 108.81    DISTANCE BC = 188.88  
 FIND:      $\sphericalangle$  CBA = \_\_\_\_\_ (5 POINTS)  
           DISTANCE AC = \_\_\_\_\_ (5 POINTS)

REQUIRED ANSWER FORMAT  
 DISTANCES: NEAREST HUNDREDTH  
 ANGLES: DEGREES-MINUTES-SECONDS  
 TO THE NEAREST SECOND

$AB = 108.81$

$BC = 188.88$

$AC^2 = BC^2 - AB^2 = 188.88^2 - 108.81^2 = 23,836.0383$

$AC = \sqrt{23,836.0383} = 154.39$

$\cos \sphericalangle CBA = \frac{108.81}{188.88} \Rightarrow \sphericalangle CBA = \cos^{-1} \frac{108.81}{188.88} = 54.82469605^\circ$

Now, we need to convert this to degrees minutes and seconds.

Degrees = 54

1 minute is 1/60 of a degree. 1 second is 1/60 of a minute.

$\frac{\text{Minutes}}{60} = 0.82469605 \Rightarrow \text{Minutes} = 60 \cdot (0.82469605) = 49.481763'$

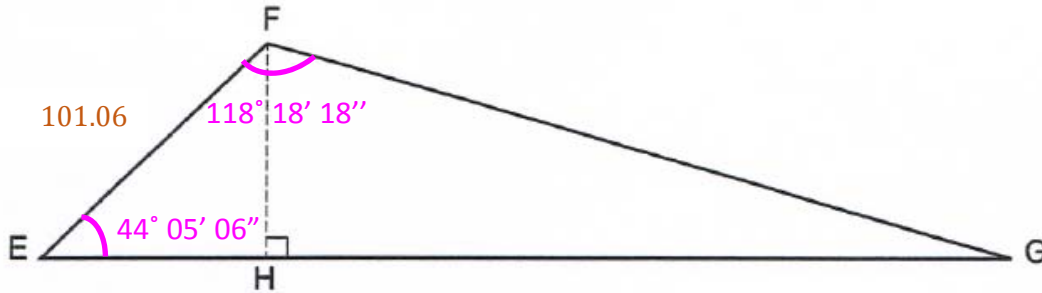
Minutes = 49

$\frac{\text{Seconds}}{60} = 0.481763 \Rightarrow \text{Seconds} = 60 \cdot (0.481763) = 29''$

Seconds = 29

$\sphericalangle CBA = 54^\circ 49' 29''$

2<sup>nd</sup> Problem on PAGE 1



KNOWN: DISTANCE EF = 101.06  $\angle$  EFG = 118°18'18"  $\angle$  FEG = 44°05'06"

FIND:  $\angle$  EGF = \_\_\_\_\_ (6 POINTS)

DISTANCE EH = \_\_\_\_\_ (6 POINTS)

DISTANCE FH = \_\_\_\_\_ (6 POINTS)

DISTANCE FG = \_\_\_\_\_ (6 POINTS)

DISTANCE GH = \_\_\_\_\_ (6 POINTS)

REQUIRED ANSWER FORMAT  
 DISTANCES: NEAREST HUNDREDTH  
 ANGLES: DEGREES-MINUTES-SECONDS  
 TO THE NEAREST SECOND

PAGE TOTAL: \_\_\_\_\_ POINTS

To get  $\angle$ EGF, recall that the sum of the measures of the angles in a triangle is 180°.

$$\begin{aligned} \angle EGF &= 180^\circ - (\angle EFG + \angle FEG) = 180^\circ - (118^\circ 18' 18'' + 44^\circ 05' 06'') \\ &= 179^\circ 59' 60'' - 162^\circ 23' 24'' = 17^\circ 36' 36'' \end{aligned}$$

$$\angle EGF = 17^\circ 36' 36''$$

To calculate the distances required using our calculators, we need to convert the angle measures from degrees-minutes-seconds to degrees in decimal form.

$$\angle EFG = 118^\circ 18' 18'' = \left(118 + \frac{18}{60} + \frac{18}{3600}\right)^\circ = 118.305^\circ$$

$$\angle FEG = 44^\circ 05' 06'' = \left(44 + \frac{5}{60} + \frac{6}{3600}\right)^\circ = 44.085^\circ$$

$$\angle EGF = 17^\circ 36' 36'' = \left(17 + \frac{36}{60} + \frac{36}{3600}\right)^\circ = 17.610^\circ$$

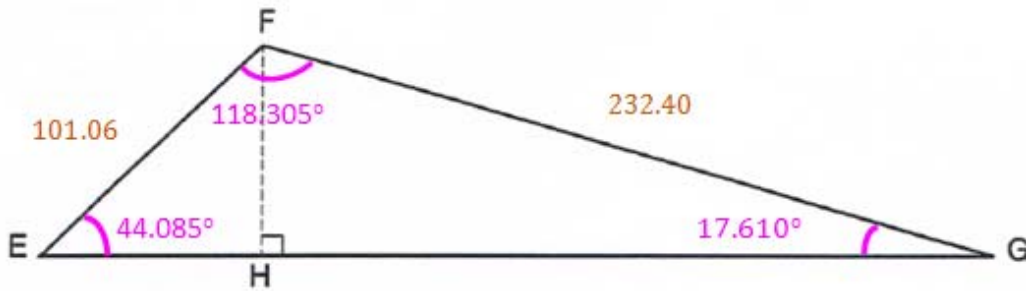
**Note:** the results of these calculations are exact. If rounding is needed, I recommend that you keep at least 5 decimal places in order to maintain the accuracy required in the test solutions.

The easiest requested distance to calculate is FG, using the Law of Sines.

$$\frac{FG}{\sin \angle FEG} = \frac{EF}{\sin \angle EGF} \Rightarrow \frac{FG}{\sin 44.085^\circ} = \frac{101.06}{\sin 17.610^\circ}$$

$$FG = 232.40$$

2<sup>nd</sup> Problem on PAGE 1 (continued)



KNOWN: DISTANCE EF = 101.06  $\angle$  EFG = 118°18'18"  $\angle$  FEG = 44°05'06"

- FIND:
- $\angle$  EGF = \_\_\_\_\_ (6 POINTS)
  - DISTANCE EH = \_\_\_\_\_ (6 POINTS)
  - DISTANCE FH = \_\_\_\_\_ (6 POINTS)
  - DISTANCE FG = \_\_\_\_\_ (6 POINTS)
  - DISTANCE GH = \_\_\_\_\_ (6 POINTS)

REQUIRED ANSWER FORMAT  
 DISTANCES: NEAREST HUNDREDTH  
 ANGLES: DEGREES-MINUTES-SECONDS  
 TO THE NEAREST SECOND

PAGE TOTAL: \_\_\_\_\_ POINTS

Let's calculate  $FH$  next.

$$\sin \angle FEG = \frac{FH}{EF} \Rightarrow \sin 44.085^\circ = \frac{FH}{101.06}$$

$$FH = 70.31$$

Let's calculate  $EH$  next.

$$\cos \angle FEG = \frac{EH}{EF} \Rightarrow \cos 44.085^\circ = \frac{EH}{101.06}$$

$$EH = 72.59$$

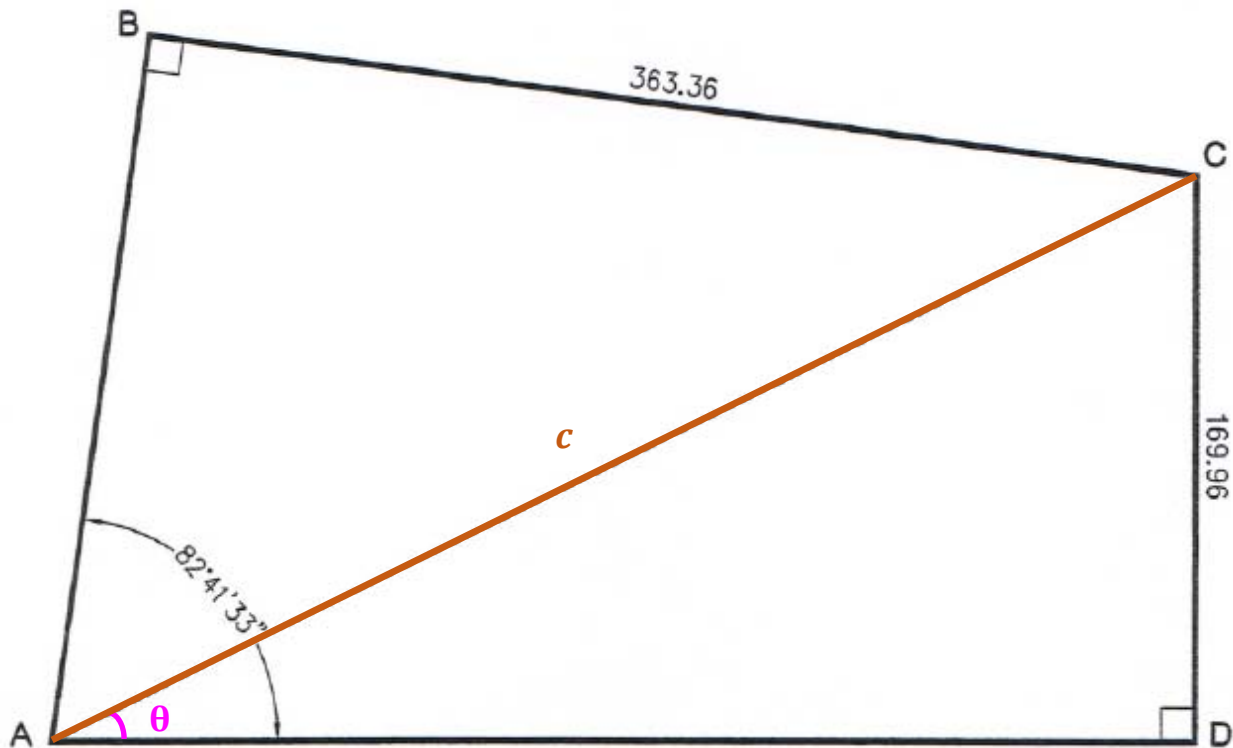
Finally, let's calculate  $GH$ .

$$\angle GFH = 90^\circ - \angle EGF = 90^\circ - 17.61^\circ = 72.39^\circ$$

$$\tan \angle GFH = \frac{GH}{FH} \Rightarrow \tan 72.39^\circ = \frac{GH}{70.31}$$

$$GH = 221.51$$

## Problem on PAGE 2



Let's add a couple of labels to make life easier. Label  $AC$  as  $c$  and  $\angle CAD$  as  $\theta$ .

Nothing easy here. Notice that  $\overline{AC}$  is in both triangles, so let's see what we can do with it.

We will work with  $\angle BAD$  because it is the only angle shown. Let's start by converting the measure of  $\angle BAD$  to degrees so we can use it on our calculators.

$$\angle BAD = 82^\circ 41' 33'' = \left(82 + \frac{41}{60} + \frac{33}{3600}\right)^\circ = 82.6925^\circ$$

Let's break  $\angle BAD$  into two angles,  $\angle CAD$  and  $\angle BAC$ , which share side  $\overline{AC}$ .

Then,  $\sin \angle CAD = \frac{169.96}{c}$  and  $\sin \angle BAC = \frac{363.36}{c}$ . Now it's time for some Algebra.

$$c = \frac{169.96}{\sin \angle CAD} \text{ and } c = \frac{363.36}{\sin \angle BAC}.$$

Next, since we have two expressions equal to  $c$ , we can set them equal to each other.

$$\frac{169.96}{\sin \angle CAD} = \frac{363.36}{\sin \angle BAC} \quad \text{so,} \quad \frac{\sin \angle BAC}{\sin \angle CAD} = \frac{363.36}{169.96}$$

Substituting  $\angle CAD = \theta$  and  $\angle BAC = 82.6925^\circ - \theta$  gives:  $\frac{\sin(82.6925^\circ - \theta)}{\sin \theta} = \frac{363.36}{169.96}$

**Problem on PAGE 2 (continued)**

Next recall the difference of angles formula:  $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$$\frac{\sin(82.6925^\circ - \theta)}{\sin \theta} = \frac{\sin(82.6925^\circ) \cos \theta - \cos(82.6925^\circ) \sin \theta}{\sin \theta} = \frac{363.36}{169.96}$$

This reduces to:

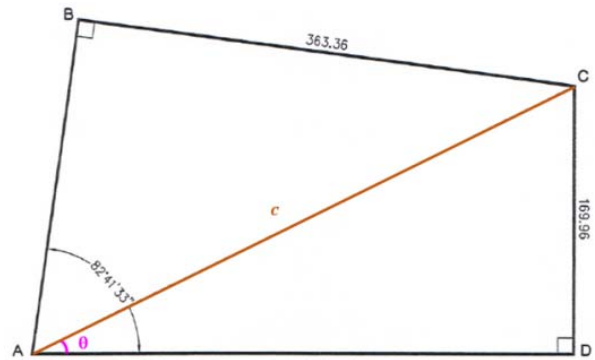
$$\frac{\sin(82.6925^\circ)}{\tan \theta} - \cos(82.6925^\circ) = \frac{363.36}{169.96}$$

Solve for  $\tan \theta$  to get:

$$\tan \theta = .437894 \quad \Rightarrow \quad \theta = 23.6483^\circ$$

Now we can identify all of the angles we need:

$$\begin{aligned} \sphericalangle CAD &= \theta = 23.6483^\circ \\ \sphericalangle BAC &= 82.6925^\circ - 23.6483^\circ = 59.0442^\circ \\ \sphericalangle ACD &= 90^\circ - 23.6483^\circ = 66.3517^\circ \\ \sphericalangle ACB &= 90^\circ - 59.0442^\circ = 40.9558^\circ \end{aligned}$$



With the angles known, we can calculate the required sides easily.

Let's calculate  $AB$ .

$$\begin{aligned} \tan \sphericalangle BAC &= \frac{363.36}{AB} \\ \tan 59.0442^\circ &= \frac{363.36}{AB} \\ \mathbf{AB} &= \mathbf{217.95} \end{aligned}$$

Let's calculate  $AD$ .

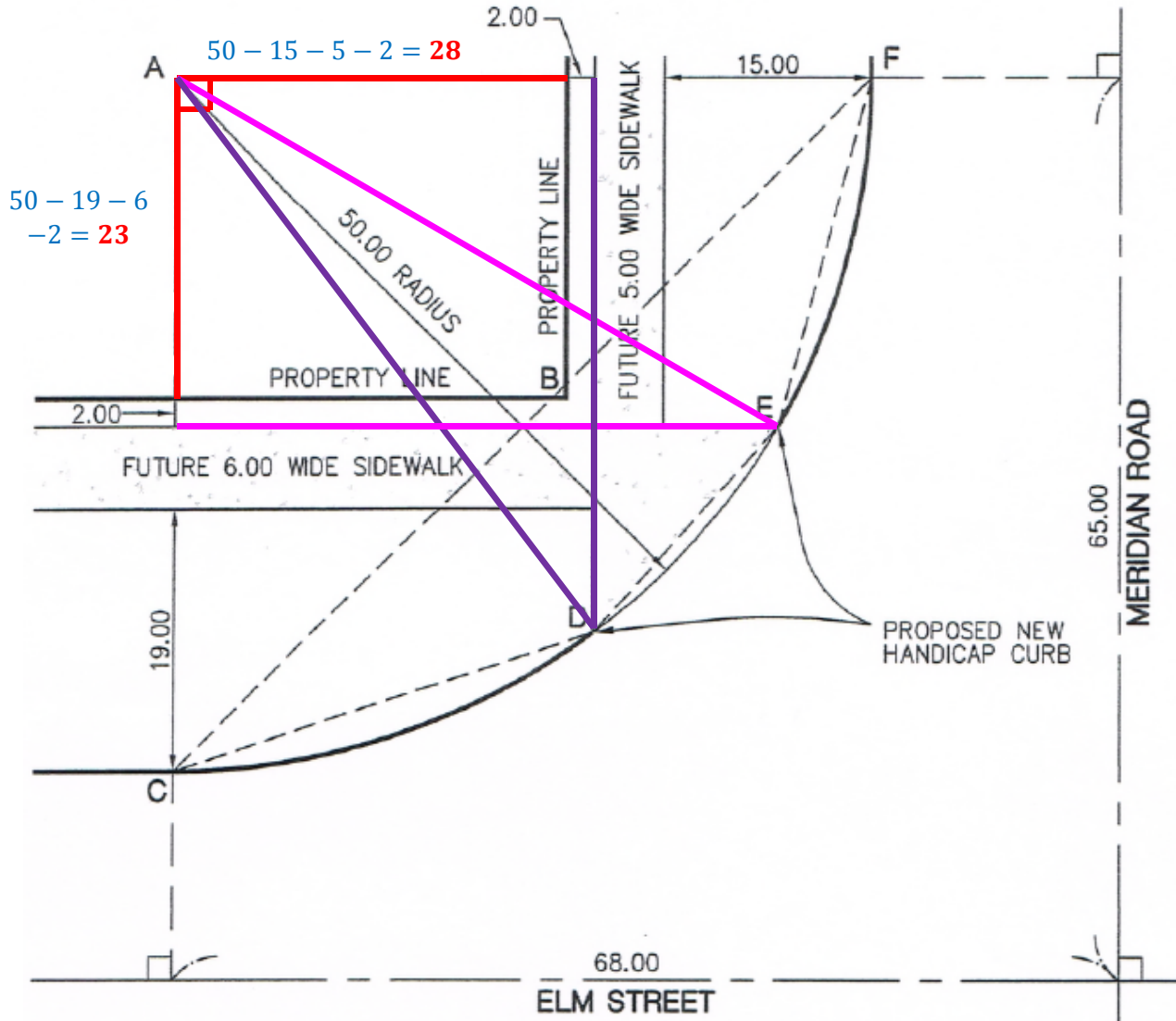
$$\begin{aligned} \tan \sphericalangle CAD &= \frac{169.96}{AD} \\ \tan 23.6483^\circ &= \frac{169.96}{AD} \\ \mathbf{AD} &= \mathbf{388.13} \end{aligned}$$

Let's calculate  $AC$ .

$$\begin{aligned} \sin \sphericalangle BAC &= \frac{363.36}{AC} \\ \sin 59.0442^\circ &= \frac{363.36}{AC} \\ \mathbf{AC} &= \mathbf{423.71} \end{aligned}$$

**Problem on Page 3**

A LOCAL SURVEYOR HAS BEEN ASKED TO STAKE OUT POINTS D AND E WHERE A FUTURE SIDEWALK WILL MEET A NEW HANDICAP CURB AT THE CORNER OF MERIDIAN ROAD AND ELM STREET. THE SURVEYOR ALREADY KNOWS THE LOCATION OF POINTS B, C AND F BUT MUST MAKE SOME CALCULATIONS TO ESTABLISH AND VERIFY THE POINTS NEEDED.



Since the radius is 50.00, we can calculate some additional distances not shown on the diagram (shown in red). Note that because of all the right angles shown,  $\triangle ACF$  is also a right angle.

Two calculations are very easy:  $CF$  and  $m\widehat{CF}$

$CF$  is the hypotenuse of  $\triangle ACF$ , which is isosceles with two sides equal to the radius ( $r = 50$ ).

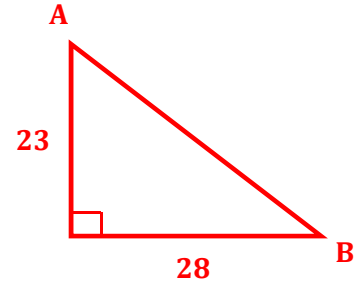
Therefore,  $CF = 50 \cdot \sqrt{2} = 70.71$

Arc  $\widehat{CF}$  is one-fourth of the circumference of a circle of radius 50.

Therefore,  $m\widehat{CF} = \frac{1}{4} \cdot 2\pi r = \frac{1}{4} \cdot 2\pi \cdot 50 = 25\pi = 78.54$

**Problem on Page 3 (continued)**

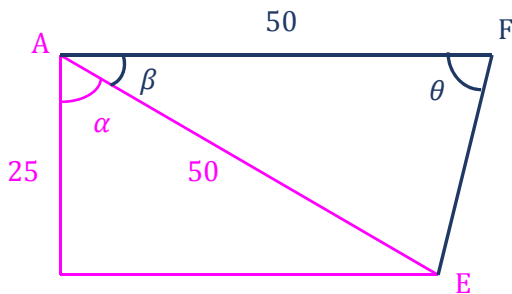
$AB$  is relatively easy also. It is the hypotenuse of a triangle with sides **23** and **28**, both of which were added to the diagram above (in red). Each of these was calculated by subtracting the distances shown from the radius of 50.



$$AB = \sqrt{23^2 + 28^2} = 36.24$$

To calculate the distances of the three chords,  $CD$ ,  $DE$  and  $EF$ , draw in segments  $AD$  and  $AE$ , as shown in the diagram on the previous page.

To find segment  $EF$ , consider the following sub-drawing of the diagram.

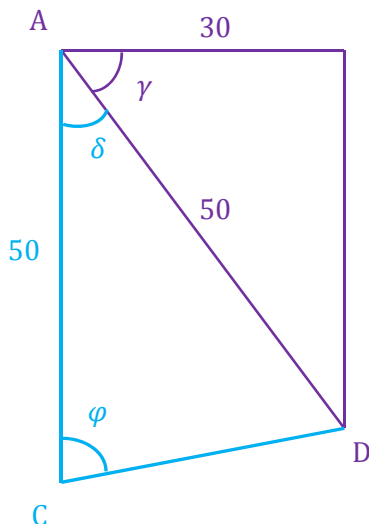


- Calculate  $\alpha = \cos^{-1}\left(\frac{25}{50}\right) = 60^\circ$
- Calculate  $\beta = 90^\circ - 60^\circ = 30^\circ$
- $\triangle AEF$  is isosceles, so  $\angle AEF = \angle AFE$
- Calculate  $\theta = \frac{1}{2}(180^\circ - 30^\circ) = 75^\circ$
- Calculate  $EF$  from the Law of Sines

$$\frac{EF}{\sin \beta} = \frac{AE}{\sin \theta} \Rightarrow \frac{EF}{\sin 30^\circ} = \frac{50}{\sin 75^\circ}$$

- $EF = 25.88$

To find segment  $CD$ , consider the following sub-drawing of the diagram.



- Calculate  $\gamma = \cos^{-1}\left(\frac{30}{50}\right) = 53.13^\circ$
- Calculate  $\delta = 90^\circ - 53.1301^\circ = 36.87^\circ$
- $\triangle ADC$  is isosceles, so  $\angle ADC = \angle ACD$
- Calculate  $\phi = \frac{1}{2}(180^\circ - 36.87^\circ) = 71.565^\circ$
- Calculate  $CD$  from the Law of Sines

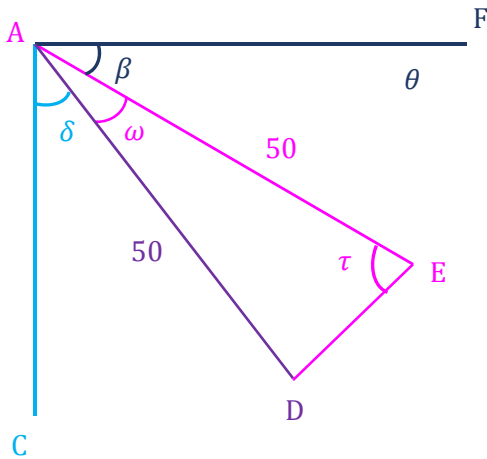
$$\frac{CD}{\sin \delta} = \frac{AD}{\sin \phi} \Rightarrow \frac{CD}{\sin 36.87^\circ} = \frac{50}{\sin 71.565^\circ}$$

- $CD = 31.62$



**Problem on Page 3 (continued)**

To find segment  $DE$ , consider the following sub-drawing of the diagram.



- From above,  $\beta = 30^\circ$
  - From above,  $\delta = 36.87^\circ$
  - Then,  $\omega = 90^\circ - 30^\circ - 36.87^\circ = 23.13^\circ$
  - $\triangle ADE$  is isosceles, so  $\angle ADE = \angle AED$
  - Calculate  $\tau = \frac{1}{2}(180^\circ - 23.13^\circ) = 78.435^\circ$
  - Calculate  $DE$  from the Law of Sines
- $$\frac{DE}{\sin \omega} = \frac{AD}{\sin \tau} \Rightarrow \frac{DE}{\sin 23.13^\circ} = \frac{50}{\sin 78.435^\circ}$$
- $DE = 20.05$