Trigonometry (Chapter 5) – Sample Test #2

Complete the Identity

1) $\sec x - \frac{1}{\sec x}$

$= \frac{1}{\cos x} - \cos x$

$= \frac{1}{\cos x} - \cos x \cdot \frac{\cos x}{\cos x}$

$= \frac{1-\cos^2 x}{\cos x}$

$= \frac{\sin^2 x}{\cos x}$

$= \sin x \cdot \frac{\sin x}{\cos x}$

$= \mathbf{\sin x \cdot \tan x}$

2) $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$= \frac{\sin x}{\sin x} \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}$

$= \frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x}$

$= \frac{1}{\sin x \cdot \cos x}$

$= \frac{1}{\sin x} \cdot \frac{1}{\cos x}$

$= \mathbf{\csc x \cdot \sec x}$

3) $\frac{(\sin x + \cos x)^2}{1 + 2\sin x \cdot \cos x}$

$= \frac{\sin^2 x + 2\sin x \cdot \cos x + \cos^2 x}{1 + 2\sin x \cdot \cos x}$

$= \frac{1 + 2\sin x \cdot \cos x}{1 + 2\sin x \cdot \cos x}$

$= \mathbf{1}$

4) $\tan x \cdot (\cot x - \cos x)$

$= \frac{\sin x}{\cos x} \cdot \left(\frac{\cos x}{\sin x} - \frac{\cos x}{1}\right)$

$= \left(\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x}\right) - \left(\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1}\right)$

$= \mathbf{1 - \sin x}$

5) $\sin^2 x + \sin^2 x \cdot \cot^2 x$

$= \sin^2 x + \left(\frac{\sin^2 x}{1} \cdot \frac{\cos^2 x}{\sin^2 x}\right)$

$= \sin^2 x + \cos^2 x$

$= \mathbf{1}$

6) $1 - \frac{\sin^2 x}{1 + \cos x}$

$= 1 - \frac{1 - \cos^2 x}{1 + \cos x}$

$= 1 - \frac{(1 - \cos x) \cdot (1 + \cos x)}{1 + \cos x}$

$= 1 - (1 - \cos x)$

$= \mathbf{\cos x}$

7) $\frac{\csc x \cdot \cot x}{\sec x}$

$= \frac{\cos x}{\sin x} \cdot \cot x$

$= \cot x \cdot \cot x$

$= \mathbf{\cot^2 x}$

8) Deleted

9) $\tan x \cdot \cot x$

$= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x}$

$= \mathbf{1}$

10) $\sin^4 x - \cos^4 x$

$= (\sin^2 x + \cos^2 x) \cdot (\sin^2 x - \cos^2 x)$

$= 1 \cdot (1 - \cos^2 x - \cos^2 x)$

$= \mathbf{1 - 2\cos^2 x}$

11) $\frac{\cos^2 x - \sin^2 x}{1 - \tan^2 x}$

$= \frac{\cos^2 x - \sin^2 x}{\frac{\cos^2 x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}}$

$= \frac{\cos^2 x - \sin^2 x}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$

$= \frac{\cos^2 x - \sin^2 x}{1} \cdot \frac{\cos^2 x}{\cos^2 x - \sin^2 x}$

$= \mathbf{\cos^2 x}$

12) $\frac{1 - \sin x}{\cos x}$

$\frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$\mathbf{\sec x - \tan x}$

Verify the Identity

13) $\cot\theta \cdot \sec\theta = \csc\theta$

$\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta}$

$\frac{1}{\sin\theta}$

$\mathbf{\csc\theta \quad = \quad \csc\theta}$

14) $(1 + \tan^2 u) \cdot (1 - \sin^2 u) = 1$

$\left(\frac{\cos^2 u}{\cos^2 u} + \frac{\sin^2 u}{\cos^2 u}\right) \cdot \cos^2 u$

$\left(\frac{\cos^2 u + \sin^2 u}{\cos^2 u}\right) \cdot \cos^2 u$

$\left(\frac{1}{\cos^2 u}\right) \cdot \cos^2 u$

$\mathbf{1 \quad = \quad 1}$

15) $1 + \sec^2 x \cdot \sin^2 x = \sec^2 x$

$1 + \frac{1}{\cos^2 x} \cdot \sin^2 x$

$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$

$\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$\frac{1}{\cos^2 x}$

$\mathbf{\sec^2 x \quad = \quad \sec^2 x}$

Find the exact value

16) $\cos(175^\circ)\cos(55^\circ) + \sin(175^\circ)\sin(55^\circ)$

$= \cos(175^\circ - 55^\circ)$

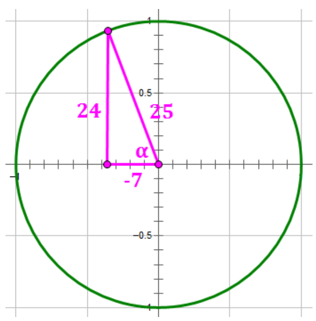
$= \cos(120^\circ)$

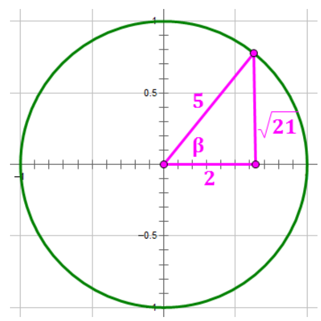
$= -\cos(60^\circ)$ Converting to an angle in Q1

$= -\frac{1}{2}$

17) $\sin\alpha = \frac{24}{25}$, α lies in quadrant II, and $\cos\beta = \frac{2}{5}$, β lies in quadrant I Find $\cos(\alpha - \beta)$.

Construct triangles for the two angles, being careful to consider the signs of the values in each quadrant:





Then,

$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

$$= \left(\frac{-7}{25}\cdot\frac{2}{5}\right) + \left(\frac{24}{25}\cdot\frac{\sqrt{21}}{5}\right)$$

$$= \frac{-14 + 24\sqrt{21}}{125}$$

18) $\sin(105^\circ)$ Note: $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$= \sin(60^\circ + 45^\circ)$ Note: both angles are in Q1, which makes things easier.

$= (\sin 60^\circ \cdot \cos 45^\circ) + (\sin 45^\circ \cdot \cos 60^\circ)$

$$= \left(\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$$

$$= \frac{\sqrt{2}\cdot(\sqrt{3}+1)}{4} \quad \text{or} \quad \frac{\sqrt{6}+\sqrt{2}}{4}$$

19) $\cos(285°)$ note: $\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$

$= \cos(240° + 45°)$ Angles in Q3 and Q1

$= (\cos 240° \cdot \cos 45°) - (\sin 240° \cdot \sin 45°)$

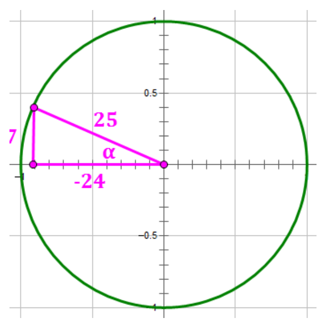
$= (-\cos 60° \cdot \cos 45°) - (-\sin 60° \cdot \sin 45°)$ Converting to Q1 angles

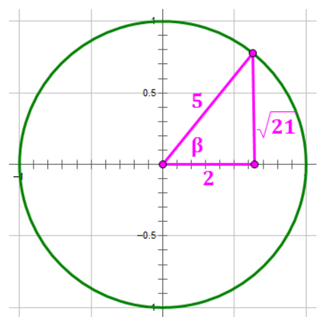
$= \left(-\frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}\right)$ Convert two negatives to a positive

$= \frac{\sqrt{2}\cdot(\sqrt{3}-1)}{4}$ or $\frac{\sqrt{6}-\sqrt{2}}{4}$

20) $\sin\alpha = \frac{7}{25}$, α lies in quadrant II, and $\cos\beta = \frac{2}{5}$, β lies in quadrant I Find $\cos(\alpha - \beta)$.

Construct triangles for the two angles, being careful to consider the signs of the values in each quadrant:





Then,

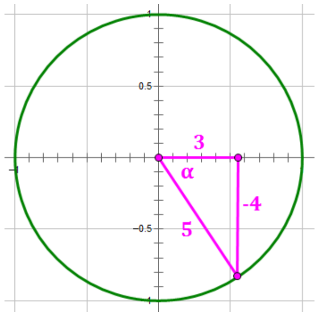
$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

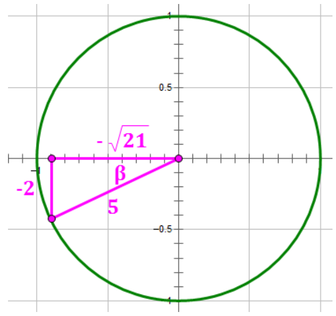
$= \left(\frac{-24}{25}\cdot\frac{2}{5}\right) + \left(\frac{7}{25}\cdot\frac{\sqrt{21}}{5}\right)$

$= \frac{-48+7\sqrt{21}}{125}$

21) $\sin\alpha = -\frac{4}{5}$, α lies in quadrant IV, and $\cos\beta = -\frac{\sqrt{21}}{5}$, β lies in quadrant III Find $\sin(\alpha - \beta)$.

Construct triangles for the two angles, being careful to consider the signs of the values in each quadrant:





Then,

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \sin(\beta)\cos(\alpha)$$

$$= \left(\frac{-4}{5} \cdot \frac{-\sqrt{21}}{5}\right) - \left(-\frac{2}{5} \cdot \frac{3}{5}\right)$$

$$= \frac{6 + 4\sqrt{21}}{25}$$

22) $\tan 255° = \tan(315° - 60°)$ note: $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta}$

$$= \frac{\tan 315° - \tan 60°}{1 + \tan 315° \cdot \tan 60°}$$ Angles in Q4 and Q1

$$= \frac{-\tan 45° - \tan 60°}{1 + (-\tan 45°) \cdot \tan 60°}$$ Converting to Q1 angles

$$= \frac{-1 - \sqrt{3}}{1 + (-1) \cdot \sqrt{3}}$$

$$= \frac{-(1 + \sqrt{3})}{1 - \sqrt{3}}$$

$$= \frac{-(1 + \sqrt{3})}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \frac{-(4 + 2\sqrt{3})}{-2} = 2 + \sqrt{3}$$

23) $\sin(255^\circ)\cos(15^\circ) - \cos(255^\circ)\sin(15^\circ)$

$= \sin(255^\circ - 15^\circ)$

$= \sin(240^\circ)$

$= -\sin(60^\circ)$ Converting to an angle in Q1

$= -\frac{\sqrt{3}}{2}$

24) $\cos(15^\circ)\cos(45^\circ) - \sin(15^\circ)\sin(45^\circ)$

$= \cos(15^\circ + 45^\circ)$

$= \cos(60^\circ)$

$= \frac{1}{2}$

25) $\dfrac{\tan 70^\circ + \tan 80^\circ}{1 - \tan 70^\circ \cdot \tan 80^\circ}$ use: $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$

$= \tan(70^\circ + 80^\circ)$

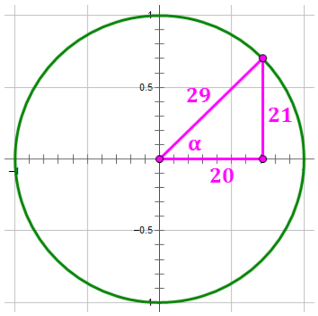
$= \tan(150^\circ)$

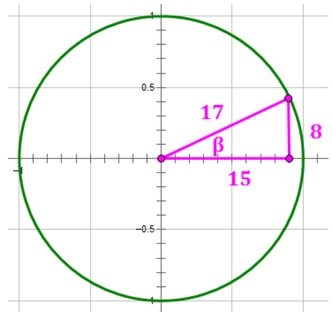
$= -\tan(30^\circ)$ Converting to an angle in Q1

$= -\frac{\sqrt{3}}{3}$

26) $\sin\alpha = \frac{21}{29},\ 0<\alpha<\frac{\pi}{2};\quad \cos\beta = \frac{15}{17},\ 0<\beta<\frac{\pi}{2}$ Find $\tan(\alpha+\beta)$.

Construct triangles for the two angles, being careful to consider the signs of the values in each quadrant:





Then,

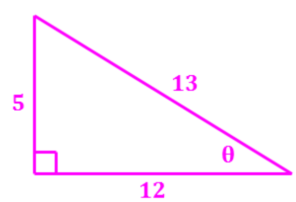
$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta}$$

$$= \frac{\frac{21}{20}+\frac{8}{15}}{1-\frac{21}{20}\cdot\frac{8}{15}} = \frac{\frac{95}{60}}{1-\frac{168}{300}}$$

$$= \frac{\frac{19}{12}}{1-\frac{14}{25}} = \frac{\frac{19}{12}}{\frac{11}{25}} = \frac{19}{12}\cdot\frac{25}{11}$$

$$= \mathbf{\frac{475}{132}}$$

27) Find $\sin 2\theta$



$$\sin 2\theta = 2\cdot\sin\theta\cdot\cos\theta$$

$$= 2\cdot\frac{5}{13}\cdot\frac{12}{13}$$

$$= \mathbf{\frac{120}{169}}$$

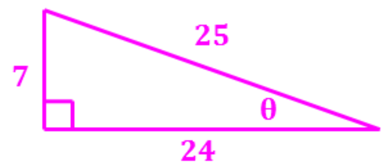
28) Find $\tan 2\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\cdot\frac{7}{24}}{1-\left(\frac{7}{24}\right)^2}$$

$$= \frac{\frac{7}{12}}{\frac{527}{576}} = \frac{7}{12}\cdot\frac{576}{527}$$

$$= \frac{336}{527}$$

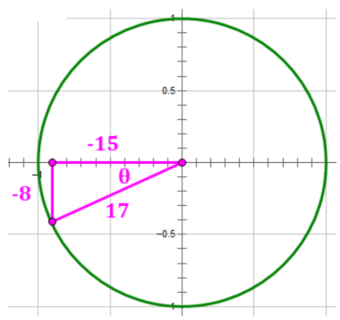


29) $\tan\theta = \frac{8}{15}$, θ lies in quadrant III Find sin 2θ.

$$\sin 2\theta = 2\cdot\sin\theta\cdot\cos\theta$$

$$= 2\cdot\frac{-8}{17}\cdot\frac{-15}{17}$$

$$= \frac{240}{289}$$



30) $2\cdot\sin 15°\cdot\cos 15°$

$$= \sin(2\cdot 15)°$$

$$= \sin(30)°$$

$$= \frac{1}{2}$$

31) $\dfrac{2\tan\left(\frac{\pi}{8}\right)}{1-\tan^2\left(\frac{\pi}{8}\right)}$

$$= \tan\left(2\cdot\frac{\pi}{8}\right)$$

$$= \tan\left(\frac{\pi}{4}\right)$$ Recall that $\frac{\pi}{4}$ radians is 45°

$$= 1$$

32) $\cos^4 x$ — Note that $\cos^4 x = (\cos^2 x)^2$

$$= \left(\frac{1+\cos 2x}{2}\right)^2$$

Using the power reducing formula: $\cos^2 x = \dfrac{1+\cos 2x}{2}$

$$= \frac{1}{4}\cdot(1 + 2\cdot\cos 2x + \cos^2 2x)$$

$$= \frac{1}{4}\cdot\left(1 + 2\cdot\cos 2x + \frac{1+\cos 4x}{2}\right)$$

Using the power reducing formula again

$$= \frac{1}{4}\cdot\left(\frac{2}{2} + \frac{4\cdot\cos 2x}{2} + \frac{1+\cos 4x}{2}\right)$$

$$= \frac{1}{4}\cdot\frac{1}{2}\cdot(2 + 4\cdot\cos 2x + 1 + \cos 4x)$$

$$= \mathbf{\frac{3+4\cdot\cos 2x+\cos 4x}{8}}$$

33) $7\sin^2 x\ \cos^2 x$

$$= 7\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$$

Using: $\sin^2 x = \dfrac{1-\cos 2x}{2}$ $\quad\cos^2 x = \dfrac{1+\cos 2x}{2}$

$$= \frac{7}{4}(1 - \cos^2 2x)$$

$$= \frac{7}{4}(\sin^2 2x)$$

$$= \frac{7}{4}\left(\frac{1-\cos 4x}{2}\right)$$

Using the power reducing formula again

$$= \mathbf{\frac{7}{8}(1 - \cos 4x)}$$

34) $\cos 15°$ — Note that $15°$ is in Q1, so the value of $\cos 15°$ is positive.

$$= \cos\left(\frac{30°}{2}\right)$$

$$= \sqrt{\frac{1+\cos 30°}{2}}$$

Using the half-angle formula: $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

$$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \mathbf{\frac{\sqrt{2+\sqrt{3}}}{2}}$$

Yes, you are right. This answer is weird!

35) $\cos\frac{5\pi}{12}$

Note that $\frac{5\pi}{12}$ is in Q1, so the value of $\cos\frac{5\pi}{12}$ is positive.

$$= \cos\left(\frac{\frac{5\pi}{6}}{2}\right)$$

$$= \sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}}$$

Using the half-angle formula: $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$$= \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}}$$

Converting to an angle in Q1

$$= \sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \mathbf{\frac{\sqrt{2-\sqrt{3}}}{2}}$$

Yes, this answer is weird too!

36) $\sec\theta = 4$, θ lies in quadrant I Find $\cos\frac{\theta}{2}$.

Note that if θ is in Q1, then $\frac{\theta}{2}$ is also in Q1, so the value of $\cos\frac{\theta}{2}$ is positive.

$\cos\theta = \frac{1}{\sec\theta}$ so, $\cos\theta = \frac{1}{4}$

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$$

$$= \sqrt{\frac{1+\frac{1}{4}}{2}} = \sqrt{\frac{5}{8}}\cdot\sqrt{\frac{2}{2}} = \sqrt{\frac{10}{16}} = \mathbf{\frac{\sqrt{10}}{4}}$$

37) $\cos\theta = -\frac{3}{5}$, θ lies in quadrant III Find $\cos\frac{\theta}{2}$.

Note that if θ is in Q3, then $\frac{\theta}{2}$ is in Q2, so the value of $\cos\frac{\theta}{2}$ is negative.

$$\cos\frac{\theta}{2} = -\sqrt{\frac{1+\cos\theta}{2}}$$

$$= -\sqrt{\frac{1-\frac{3}{5}}{2}} = -\sqrt{\frac{1}{5}}\cdot\sqrt{\frac{5}{5}} = -\sqrt{\frac{5}{25}} = \mathbf{-\frac{\sqrt{5}}{5}}$$

38) $\sin 8x \cdot \cos 5x$

Use: $\boldsymbol{\sin\alpha \cdot \cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]}$

$$\sin 8x \cdot \cos 5x = \frac{1}{2}\left[\sin(8x+5x) + \sin(8x-5x)\right]$$
$$= \boldsymbol{\frac{1}{2}\left[\sin(13x) + \sin(3x)\right]}$$

39) $\cos\frac{7x}{2} \cdot \cos\frac{x}{2}$

Use: $\boldsymbol{\cos\alpha \cdot \cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]}$

$$\cos\frac{7x}{2} \cdot \cos\frac{x}{2} = \frac{1}{2}\left[\cos\left(\frac{7x}{2}-\frac{x}{2}\right) + \cos\left(\frac{7x}{2}+\frac{x}{2}\right)\right]$$
$$= \boldsymbol{\frac{1}{2}\left[\cos(3x) + \cos(4x)\right]}$$

40) $\sin 8x + \sin 2x$

Use: $\boldsymbol{\sin\alpha + \sin\beta = 2 \cdot \sin\left(\frac{\alpha+\beta}{2}\right) \cdot \cos\left(\frac{\alpha-\beta}{2}\right)}$

$$\sin 8x + \sin 2x = 2 \cdot \sin\left(\frac{8x+2x}{2}\right) \cdot \cos\left(\frac{8x-2x}{2}\right)$$
$$= \boldsymbol{2 \cdot \sin(5x) \cdot \cos(3x)}$$

41) $\cos 8x - \cos 2x$

Use: $\boldsymbol{\cos\alpha - \cos\beta = -2 \cdot \sin\left(\frac{\alpha+\beta}{2}\right) \cdot \sin\left(\frac{\alpha-\beta}{2}\right)}$

$$\cos 8x - \cos 2x = -2 \cdot \sin\left(\frac{8x+2x}{2}\right) \cdot \sin\left(\frac{8x-2x}{2}\right)$$
$$= \boldsymbol{-2 \cdot \sin(5x) \cdot \sin(3x)}$$