Decide if the system of equations in two variables is linear or nonlinear.

1) $\begin{cases} y = x^2 - 6 \\ x^2 + y^2 = 9 \end{cases}$

A) Linear B) Nonlinear

Note the exponents of 2 in the equations. Any exponent greater than 1 will identify a system as non-linear. This system is non-linear. **Answer B.**

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the system of equations.

2) $x - y + z = -1$
$x + y + z = -9$
$x + y - z = -3$

Note: You can download a Microsoft Excel file from the following link that will allow you to explore systems of 3 equations. Give it a try.
http://www.mathguy.us/MathTidbits.php

Typically, you would begin solving a set of three equations by selecting pairs of equations and eliminating the same variable in each pair. I'll start this one by eliminating the variable z.

Add 1st and 3rd equations

$$\begin{aligned} x - y + z &= -1 \\ x + y - z &= -3 \\ \hline 2x \qquad &= -4 \end{aligned}$$

Add 2nd and 3rd equations

$$\begin{aligned} x + y + z &= -9 \\ x + y - z &= -3 \\ \hline 2x + 2y \qquad &= -12 \end{aligned}$$

We got lucky here because we were able to eliminate two variables at the same time by adding the first and third equations. Normally, this would not happen, and you would have to solve the set of two simultaneous equations which result. Let's continue.

Solve for x:

$2x = -4$

$x = -2$

Then, solve for y:

$2x + 2y = -12$

$2 \cdot (-2) + 2y = -12$

$-4 + 2y = -12$

$2y = -8$

$y = -4$

Then, solve for z:

$x - y + z = -1$

$(-2) - (-4) + z = -1$

$2 + z = -1$

$z = -3$

Finally, test your results in one of the original equations, but not the one used to solve for z.

Second equation: $(-2) + (-4) + (-3) = -9$ ✓

Solution: $(\mathbf{-2, -4, -3})$

For problems 3 to 5, we need only write the form of the decomposition. We do not need to solve the decomposition for the constants $\boldsymbol{A}, \boldsymbol{B}$ and $\boldsymbol{C}$.

Write the form of the partial fraction decomposition of the rational expression. It is not necessary to solve for the constants.

3) $\frac{2x + 3}{(x - 6)(x + 6)}$

This rational function has no repeated linear factors in the denominator, so the decomposition is straightforward:

$$\frac{2x+3}{(x-6)(x+6)} = \frac{\boldsymbol{A}}{\boldsymbol{x-6}} + \frac{\boldsymbol{B}}{\boldsymbol{x+6}}$$

4) $\frac{3x - 1}{(x + 5)(x + 7)^2}$

This rational function has a repeated linear factor $(x+7)$, so the decomposition must include each integral exponent of $(x+7)$, up to the exponent of the term in the rational function (2):

$$\frac{3x-1}{(x+5)(x+7)^2} = \frac{\boldsymbol{A}}{\boldsymbol{x+5}} + \frac{\boldsymbol{B}}{\boldsymbol{x+7}} + \frac{\boldsymbol{C}}{\boldsymbol{(x+7)^2}}$$

5) $\frac{2x - 5}{(x + 2)(x^2 + x - 4)}$

This rational function has a quadratic function in the denominator so the decomposition must take this into account:

$$\frac{2x-5}{(x+2)(x^2+x-4)} = \frac{\boldsymbol{A}}{\boldsymbol{x+2}} + \frac{\boldsymbol{Bx+C}}{\boldsymbol{x^2+x-4}}$$

Write the partial fraction decomposition of the rational expression.

6) $\dfrac{15x - 39}{(x - 1)(x - 5)}$

Write the form of the decomposition: $\dfrac{15x-39}{(x-1)(x-5)} = \dfrac{A}{x-1} + \dfrac{B}{x-5}$

Multiply both sides by $(x-1)(x-5)$: $15x - 39 = A(x-5) + B(x-1)$

Simplify: $15x - 39 = (A+B)x + (-5A - B)$

Write the simultaneous equations and solve them:

$A + B = 15$ $-5A - B = -39$

Solve for A:

$$A + B = 15$$
$$-5A - B = -39$$
$$-4A = -24$$
$$\boldsymbol{A = 6}$$

Then, solve for B:

$$A + B = 15$$
$$6 + B = 15$$
$$\boldsymbol{B = 9}$$

So, the partial fraction decomposition is:

$$\frac{15x - 39}{(x-1)(x-5)} = \frac{\mathbf{6}}{\boldsymbol{x - 1}} + \frac{\mathbf{9}}{\boldsymbol{x - 5}}$$

7) $\frac{36 - 7x}{x(x - 3)^2}$

Write the form of the decomposition: $\frac{-7x+36}{x(x-3)^2} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

Multiply both sides by $x(x-3)^2$: $-7x + 36 = A(x-3)^2 + Bx(x-3) + Cx$

Expand and simplify: $-7x + 36 = A(x^2 - 6x + 9) + B(x^2 - 3x) + Cx$

$$-7x + 36 = (A + B)x^2 + (-6A - 3B + C)x + 9A$$

Write the simultaneous equations and solve them:

$A + B = 0$ $-6A - 3B + C = -7$ $9A = 36$

Solve for A:	Then, solve for B:	Then, solve for C:
$9A = 36$	$A + B = 0$	$-6A - 3B + C = -7$
$\boldsymbol{A = 4}$	$4 + B = 0$	$-6(4) - 3(-4) + C = -7$
	$\boldsymbol{B = -4}$	$-24 + 12 + C = -7$
		$\boldsymbol{C = 5}$

So, the partial fraction decomposition is:

$$\frac{-7x + 36}{x(x-3)^2} = \frac{4}{x} + \frac{-4}{x-3} + \frac{5}{(x-3)^2}$$

8) $\frac{10x + 2}{(x - 1)(x^2 + x + 1)}$

Write the form of the decomposition: $\frac{10x+2}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$

Multiply both sides by $(x-1)(x^2+x+1)$: $10x + 2 = A(x^2 + x + 1) + (Bx + C)(x - 1)$

Expand and simplify: $10x + 2 = (Ax^2 + Ax + A) + (Bx^2 - Bx + Cx - C)$

$$10x + 2 = (A + B)x^2 + (A - B + C)x + (A - C)$$

Write the simultaneous equations and solve them:

$A + B = 0$ $\quad A - B + C = 10$ $\quad A - C = 2$

Now, let's eliminate C form the 2nd equation with some help from the 3rd equation.

$$\begin{array}{r} A - B + C = 10 \\ A \quad\quad - C = 2 \\ \hline 2A - B \quad\quad = 12 \end{array}$$

Then,

Solve for A:	Then, solve for B:	Then, solve for C:
$2A - B = 12$	$A + B = 0$	$A - C = 2$
$A + B = 0$	$4 + B = 0$	$4 - C = 2$
$3A = 12$	$\boldsymbol{B = -4}$	$\boldsymbol{C = 2}$
$\boldsymbol{A = 4}$		

So, the partial fraction decomposition is:

$$\frac{10x + 2}{(x - 1)(x^2 + x + 1)} = \frac{\mathbf{4}}{\boldsymbol{x - 1}} + \frac{\boldsymbol{-4x + 2}}{\boldsymbol{x^2 + x + 1}}$$

9) $\frac{x^2+3x+1}{(x^2+4)^2}$

Write the form of the decomposition: $\frac{x^2+3x+1}{(x^2+4)^2} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2}$

Multiply both sides by $(x^2+4)^2$: $x^2+3x+1 = (Ax+B)(x^2+4) + Cx+D$

Expand and simplify: $x^2+3x+1 = Ax^3+4Ax+Bx^2+4B + Cx+D$

$x^2+3x+1 = Ax^3+Bx^2+(4A+C)x+(4B+D)$

Write the simultaneous equations and solve them:

$A=0$ $B=1$ $4A+C=3$ $4B+D=1$

Solve for C:

$4A+C=3$

$4(0)+C=3$

$\boldsymbol{C=3}$

Then, solve for D:

$4B+D=1$

$4(1)+D=1$

$4+D=1$

$\boldsymbol{D=-3}$

So, the partial fraction decomposition is:

$$\frac{x^2+3x+1}{(x^2+4)^2} = \frac{\mathbf{1}}{\boldsymbol{x^2+4}} + \frac{\boldsymbol{3x-3}}{\boldsymbol{(x^2+4)^2}}$$

Solve the system by the substitution method.

10) $x + y = 6$
$y = x^2 - 8x + 16$

$x + y = 6 \qquad y = x^2 - 8x + 16$

$x + (x^2 - 8x + 16) = 6$

$x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$

$x = \{2, 5\}$

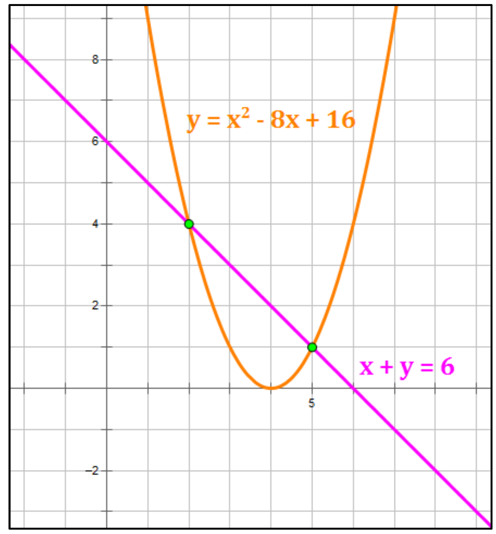
When $x = 2$, we get:

$2 + y = 6$, so $y = 4 \Rightarrow (2, 4)$ is a solution

When $x = 5$, we get:

$5 + y = 6$, so $y = 1 \Rightarrow (5, 1)$ is a solution

So, our solutions are: $\{(\mathbf{2}, \mathbf{4}), (\mathbf{5}, \mathbf{1})\}$



11) $xy = 56$
$x + y = -15$

$x + y = -15 \qquad xy = 56$

$y = -15 - x \qquad x(-15 - x) = 56$

$-x^2 - 15x = 56$

$x^2 + 15x + 56 = 0$

$(x + 7)(x + 8) = 0$

$x = \{-7, -8\}$

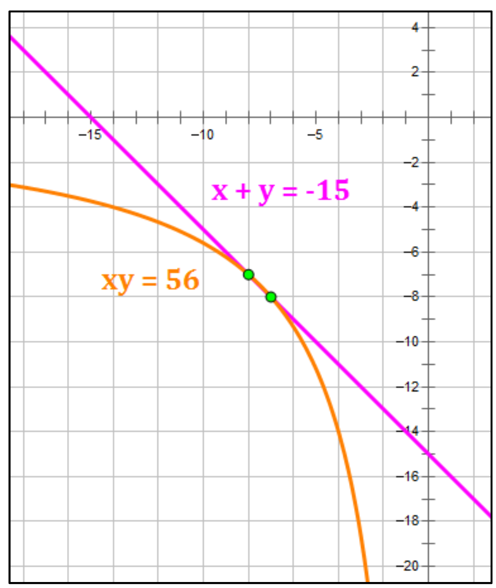
When $x = -7$, we get:

$y = -15 - (-7) = -8 \Rightarrow (-7, -8)$ is a solution

When $x = -8$, we get:

$y = -15 - (-8) = -7 \Rightarrow (-8, -7)$ is a solution

So, our solutions are: $\{(\mathbf{-7}, \mathbf{-8}), (\mathbf{-8}, \mathbf{-7})\}$



Solve the system by the addition method.

12) $2x^2 + y^2 = 66$
$x^2 + y^2 = 41$

$2x^2 + y^2 = 66 \qquad x^2 + y^2 = 41$

Let's use the Addition (i.e., Elimination) Method

$2x^2 + y^2 = 66$ multiply by (1) $2x^2 + y^2 = 66$

$x^2 + y^2 = 41$ multiply by (-1) $-x^2 - y^2 = -41$

$x^2 = 25$

$x = \pm 5$

When $x = 5$, we get:

$5^2 + y^2 = 41$

$25 + y^2 = 41$

$y^2 = 16 \quad \Rightarrow \quad y = \pm 4$

$(5, 4)$ and $(5, -4)$ are solutions

When $x = -5$, we get:

$(-5)^2 + y^2 = 41$

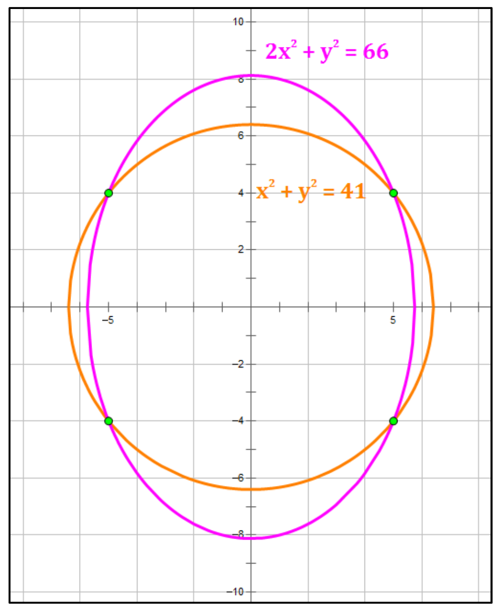
$25 + y^2 = 41$

$y^2 = 16 \quad \Rightarrow \quad y = \pm 4$

$(-5, 4)$ and $(-5, -4)$ are solutions

So, the entire solution set is:

$\{(\mathbf{5}, \mathbf{4}), (\mathbf{5}, -\mathbf{4}), (-\mathbf{5}, \mathbf{4}), (-\mathbf{5}, -\mathbf{4})\}$



Note: you can graph multiple conic sections (and lines) using the Algebra App, available at: http://www.mathguy.us/Apps/AboutAlgebraMainApp.php.

- On the opening page, click on the "Conic Sections" button in the "More Algebra" column.
- On the left hand side of the Conic Sections page, click on the "Graph Multiple Equations" button
- You may enter up to 4 equations, using either General Form or Standard Form for each.

Solve by the method of your choice.

13) $x^2 + y^2 = 29$
$4x + y^2 = 17$

$x^2 + y^2 = 29$ $\quad$ $4x + y^2 = 17$

Let's use the Addition (i.e., Elimination) Method

$x^2 + y^2 = 29$ multiply by (1)

$4x + y^2 = 17$ multiply by (-1)

$$\begin{aligned} x^2 \quad + y^2 &= \quad 29 \\ -4x - y^2 &= -17 \\ \hline x^2 - 4x &= \quad 12 \end{aligned}$$

$$x^2 - 4x - 12 = 0$$

$$(x+2)(x-6) = 0$$

$$x = \{-2, 6\}$$

When $x = -2$, we get:

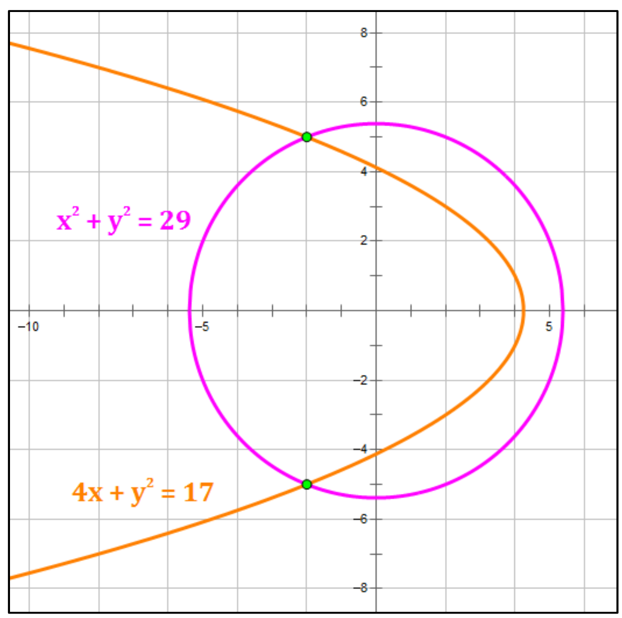
$4x + y^2 = 17$

$4(-2) + y^2 = 17$

$-8 + y^2 = 17$

$y^2 = 25 \quad \Rightarrow \quad y = \pm 5$

$(-2, 5)$ and $(-2, -5)$ are solutions



When $x = 6$, we get:

$4x + y^2 = 17$

$4(6) + y^2 = 17$

$24 + y^2 = 17$

$y^2 = -7$

Result: no real solutions when $x = 6$

So, the entire solution set is:

$\{(\mathbf{-2, 5}), (\mathbf{-2, -5})\}$

Graph the inequality.

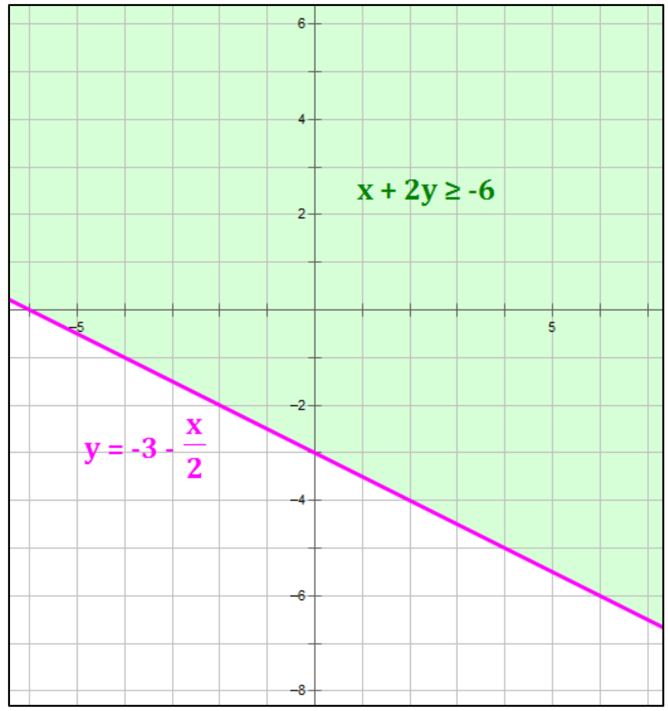
14) x + 2y ≥ -6

Let's start by solving this for y.

$$\boldsymbol{x + 2y \geq -6}$$

$$2y \geq -6 - x$$

$$y \geq -3 - \frac{x}{2}$$

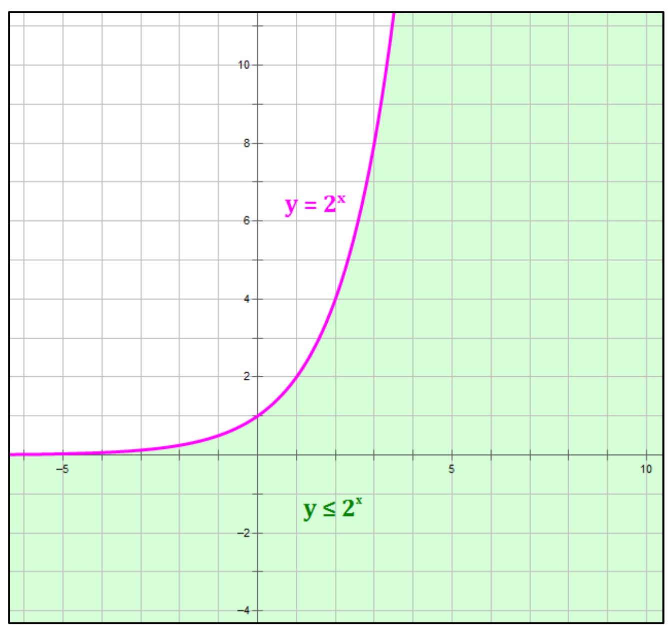


To graph this, do the following:

- Graph the line: $y = -3 - \frac{x}{2}$.
- The line will be solid because there is an "equal sign" included in the inequality.
- Fill in the portion of the graph above the line because of the "greater than" portion of the inequality.

15) y ≤ 2ˣ

$$\boldsymbol{y \leq 2^x}$$



To graph this inequality, do the following:

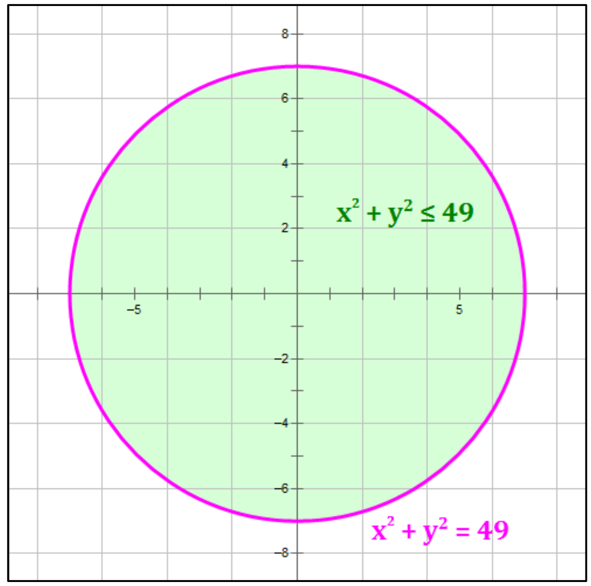
- Graph the curve: $y = 2^x$.
- Some points on the curve: $(-2, 0.25), (0, 1), (2, 4)$
- The curve will be solid because there is an "equal sign" included in the inequality.
- Fill in the portion of the graph below the curve because of the "less than" portion of the inequality.

16) $x^2 + y^2 \leq 49$

$\boldsymbol{x^2 + y^2 \leq 49}$ is the interior of a circle with center $(\mathbf{0}, \mathbf{0})$ and radius $\boldsymbol{r} = \sqrt{\mathbf{49}} = \mathbf{7}$.

To graph this inequality, do the following:

- Graph the circle: $x^2 + y^2 = 49$.
- Some points on the curve: $(0, 7), (0, -7), (7, 0), (-7, 0)$
- The curve will be solid because there is an "equal sign" included in the inequality.
- Fill in the interior of the circle because of the "less than" portion of the inequality.



Graph the solution set of the system of inequalities or indicate that the system has no solution.

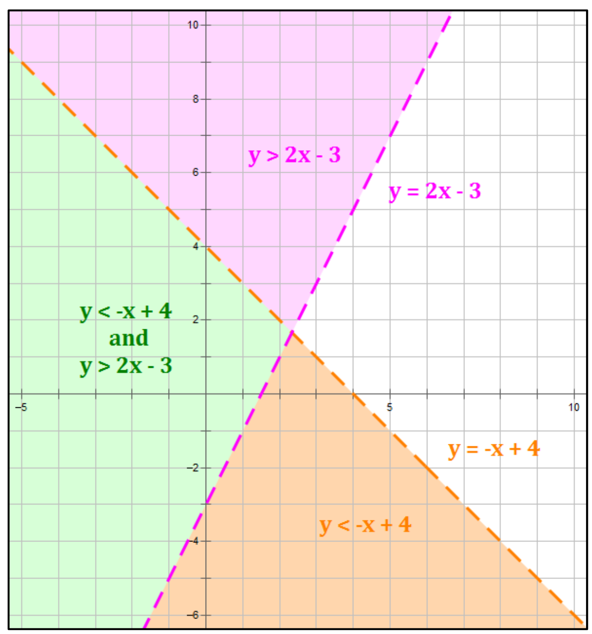
17) $y < -x + 4$
$y > 2x - 3$

$\boldsymbol{y < -x + 4}$ (orange and green areas)

- Graph the line: $y = -x + 4$.
- The line will be dashed because there is no "equal sign" included in the inequality.
- Fill in the portion of the graph below the line because of the "less than" portion of the inequality.

$\boldsymbol{y > 2x - 3}$ (violet and green areas)

- Graph the line: $y = 2x - 3$.
- The line will be dashed because there is no "equal sign" included in the inequality.
- Fill in the portion of the graph above the line because of the "greater than" portion of the inequality.

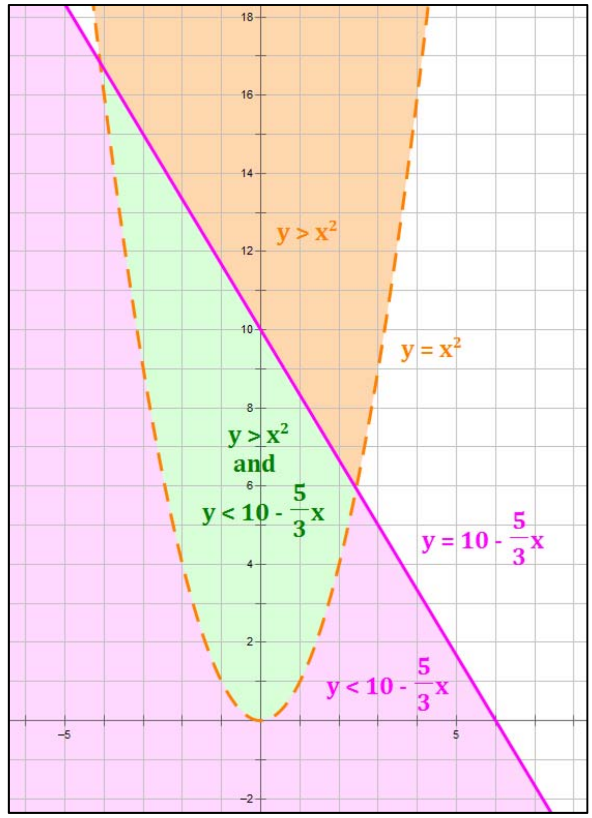


The green area is the area of intersection of the two linear inequalities.

18) $y > x^2$
$10x + 6y \leq 60$

$y > x^2$ (orange and green areas)

- Graph the parabola: $y = x^2$.
- Some points on the curve: $(0, 0), (2, 4), (-2, 4)$
- The curve will be dashed because there is no "equal sign" included in the inequality.
- Fill in the portion of the graph above the curve because of the "greater than" portion of the inequality.



$10x + 6y \leq 60$ (violet and green areas)

- Put this in slope intercept form

$$10x + 6y \leq 60$$
$$6y \leq 60 - 10x$$
$$y \leq 10 - \frac{5}{3}x$$

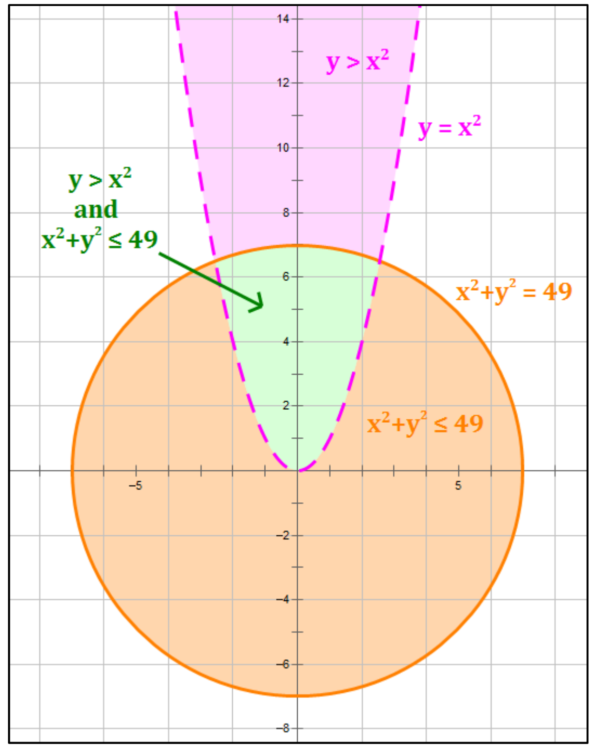
- Graph the line: $y = 10 - \frac{5}{3}x$.
- The line will be solid because there is an "equal sign" included in the inequality.
- Fill in the portion of the graph below the line because of the "less than" portion of the inequality.

The green area (and contiguous magenta line) is the area of intersection of the two linear inequalities.

19) $x^2 + y^2 \le 49$
$y - x^2 > 0$

$x^2 + y^2 \le 49$ (orange and green areas)

- Graph the circle: $x^2 + y^2 = 49$.
- Some points on the curve: $(0,7), (0,-7), (7,0), (-7,0)$
- The curve will be solid because there is an "equal sign" included in the inequality.
- Fill in the interior of the circle because of the "less than" portion of the inequality.



$y - x^2 > 0$ (violet and green areas)

- Put this in "$y >$" form
  $y - x^2 > 0$
  $y > x^2$
- Graph the parabola: $y = x^2$.
- Some points on the curve: $(0,0), (2,4), (-2,4)$
- The curve will be dashed because there is no "equal sign" included in the inequality.
- Fill in the portion of the graph above the curve because of the "greater than" portion of the inequality.

The green area (and contiguous orange curve) is the area of intersection of the two linear inequalities.

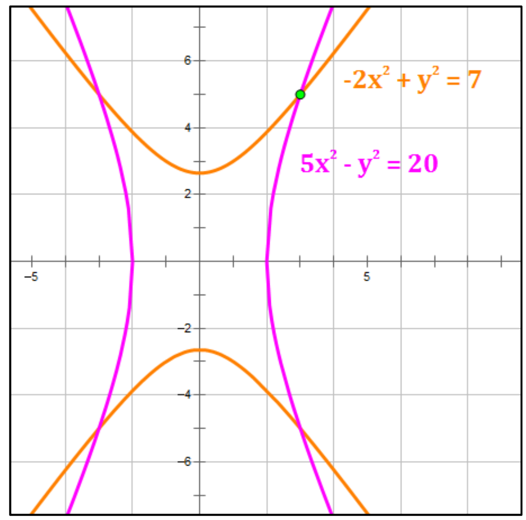
Solve the problem.

20) A system for tracking ships indicated that a ship lies on a hyperbolic path described by $5x^2 - y^2 = 20$. The process is repeated and the ship is found to lie on a hyperbolic path described by $y^2 - 2x^2 = 7$. If it is known that the ship is located in the first quadrant of the coordinate system, determine its exact location.

Let's use the Addition (i.e., Elimination) Method

$$\begin{aligned} 5x^2 - y^2 &= 20 \\ -2x^2 + y^2 &= 7 \\ \hline 3x^2 \quad &= 27 \end{aligned}$$

$$x = \{\pm 3\}$$



Since the ship is in Q1, we must have positive x and a positive y. So, $x = 3$. Then,

$$5x^2 - y^2 = 20$$

$$5(3)^2 - y^2 = 20$$

$$45 - y^2 = 20$$

$y^2 = 25 \quad \Rightarrow \quad y = 5$ in Q1 So, $(3, 5)$ **is the exact location**

Let x represent one number and let y represent the other number. Use the given conditions to write a system of nonlinear equations. Solve the system and find the numbers.

21) The sum of two numbers is -7 and their product is -144. Find the numbers.

$x + y = -7 \qquad xy = -144$

Let's use the Substitution Method

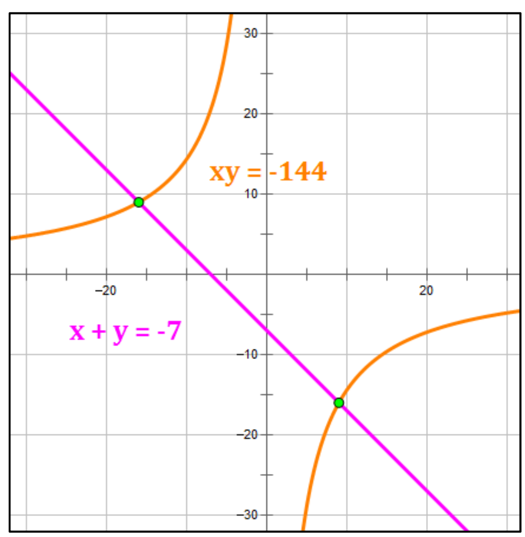
$y = -7 - x \qquad x(-7 - x) = -144$

$$-x^2 - 7x = -144$$

$$x^2 + 7x - 144 = 0$$

$$(x + 16)(x - 9) = 0$$

$$x = \{-16, 9\}$$



When $x = -16$, we get:

$y = -7 - (-16) = 9 \Rightarrow (-16, 9)$ is a solution

When $x = 9$, we get:

$y = -7 - (9) = -16 \Rightarrow (9, -16)$ is a solution

So, the two numbers are: $-\mathbf{16}$ and $\mathbf{9}$

22) The sum of the squares of two numbers is 37. The sum of the two numbers is -5. Find the two numbers.

$x^2 + y^2 = 37 \qquad x + y = -5$

Let's use the Substitution Method

$y = -x - 5 \qquad x^2 + (-x - 5)^2 = 37$

$x^2 + (x^2 + 10x + 25) = 37$

$2x^2 + 10x - 12 = 0$

$x^2 + 5x - 6 = 0$

$(x + 6)(x - 1) = 0$

$x = \{-6, 1\}$

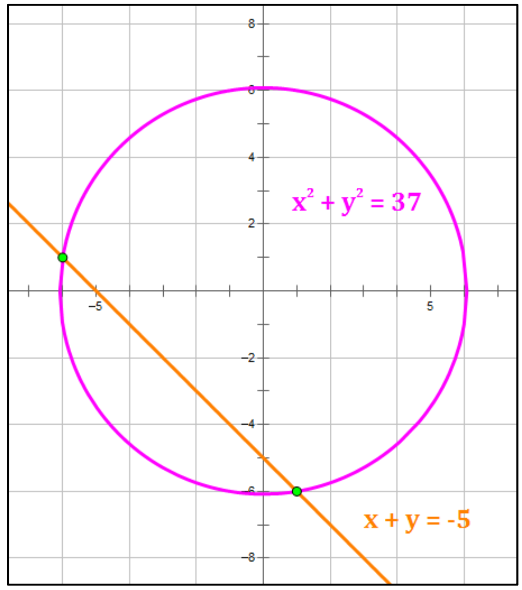
When $x = -6$, we get:

$y = -(-6) - 5 = 1 \Rightarrow (-6, 1)$ is a solution

When $x = 1$, we get:

$y = -(1) - 5 = -6 \Rightarrow (1, -6)$ is a solution

So, the two numbers are: **-6** and **1**



Solve the problem.

23) Find the dimensions of a rectangle whose perimeter is 42 feet and whose area is 90 square feet.

$2x + 2y = 42 \qquad xy = 90$ with the dimensions of the rectangle being: x by y

Let's use the Substitution Method

$y = 21 - x \qquad x(21 - x) = 90$

$-x^2 + 21x = 90$

$x^2 - 21x + 90 = 0$

$(x - 15)(x - 6) = 0$

$x = \{6, 15\}$

When $x = 6$, we get:

$y = 21 - (6) = 15 \Rightarrow (6, 15)$ is a solution

When $x = 15$, we get:

$y = 21 - (15) = 6 \Rightarrow (15, 6)$ is a solution

So, the dimensions are: **15** ft. by **6** ft.