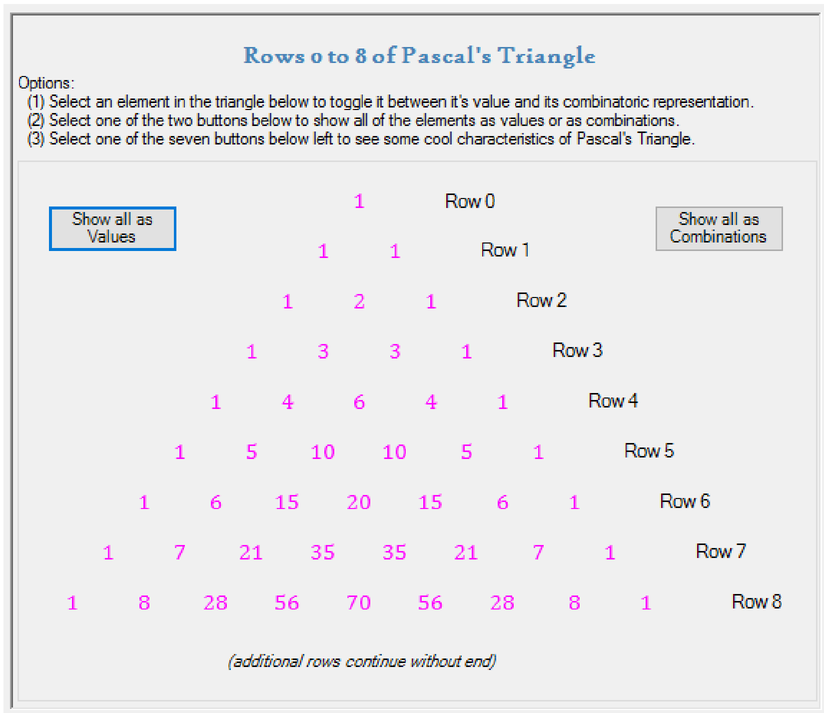
Pascal's Triangle

1. Construction

   Start with a triangle of 1's in the left and the right columns

   Form each row by adding the two numbers above it



2. Patterns

3. Combinations

   n-th row, r-th column $= {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

4. Binomial Theorem

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r}\, b^r = a^n + n \cdot a^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \cdots + b^n$$

   Example: $(x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

Pascal’s Triangle – Patterns

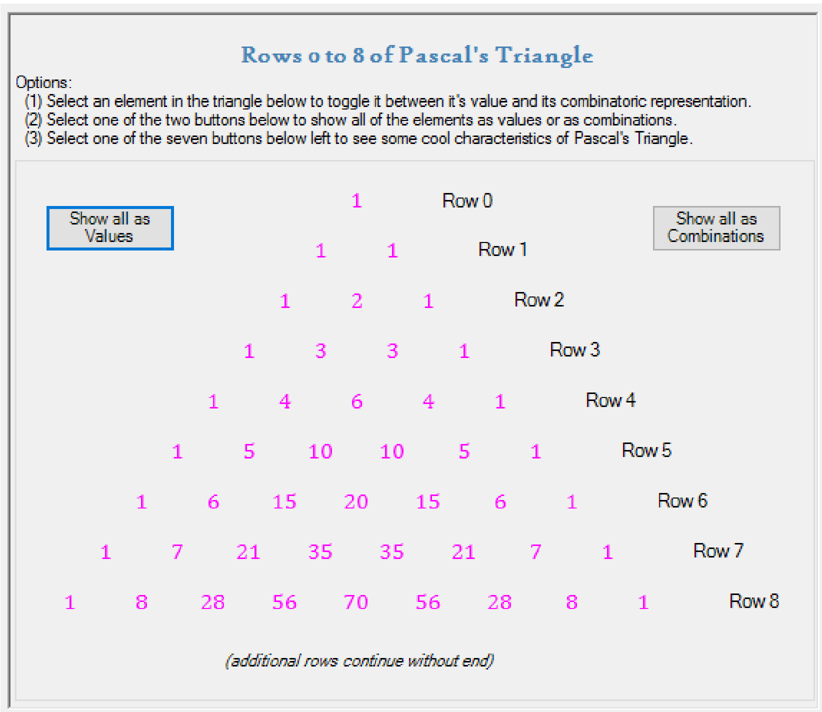


Figure 1: Pascal's Triangle values

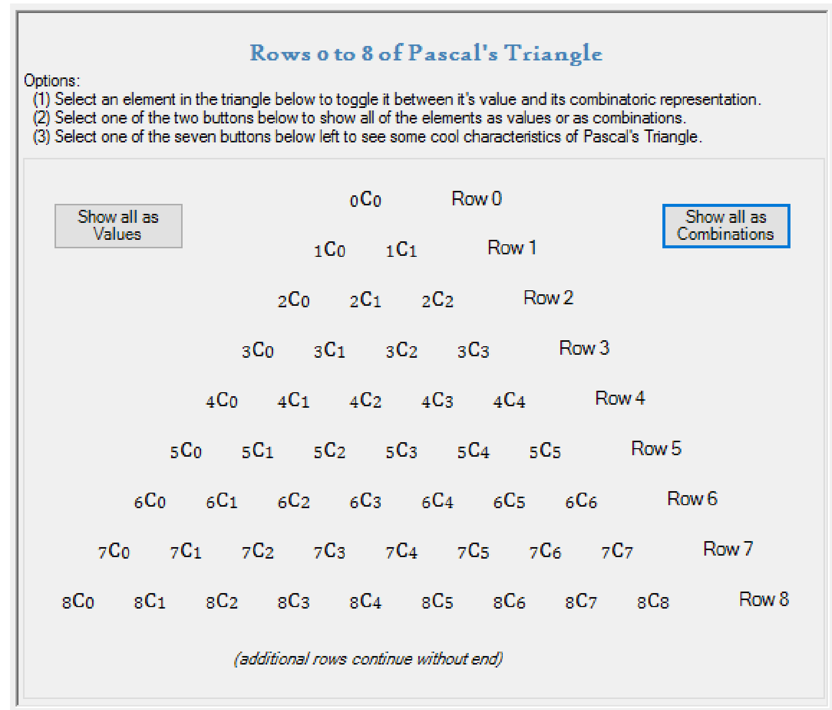


Figure 2: Pascal's Triangle combinations

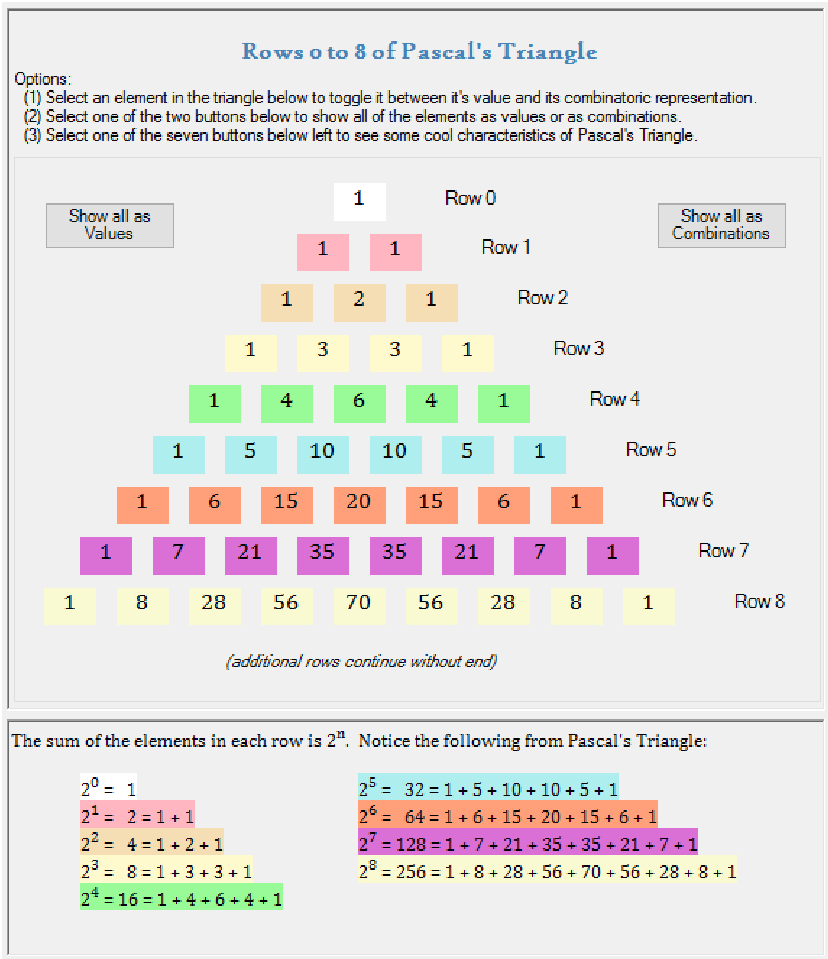


Figure 3: Sums of rows

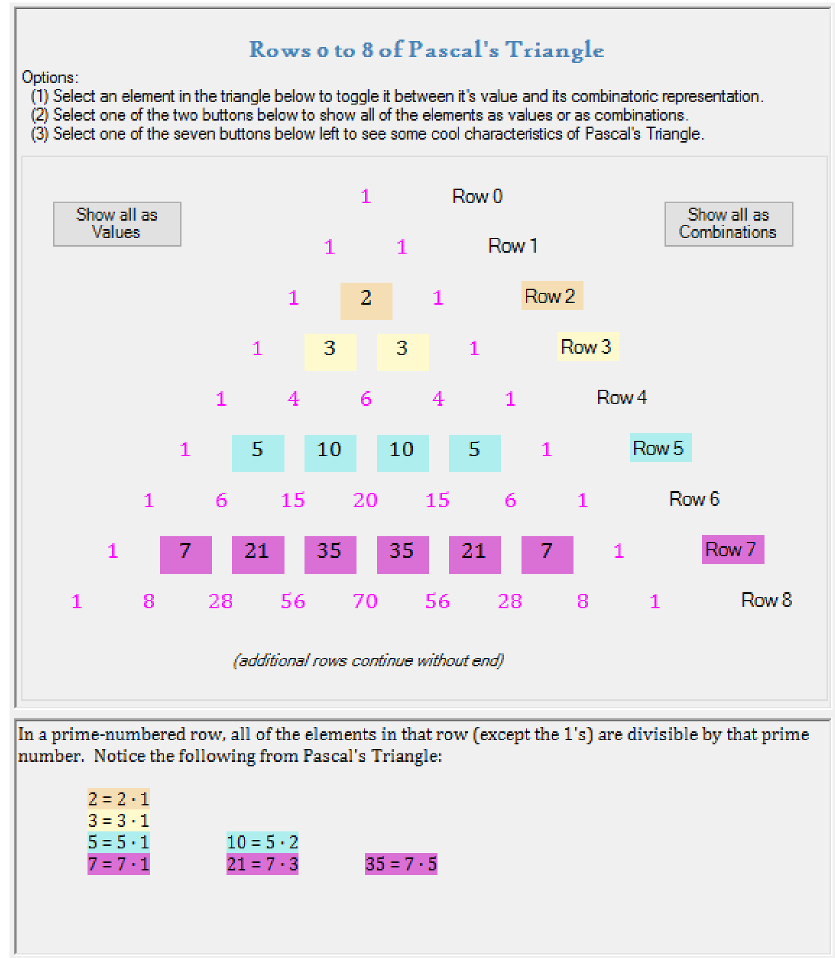


Figure 4: Prime rows

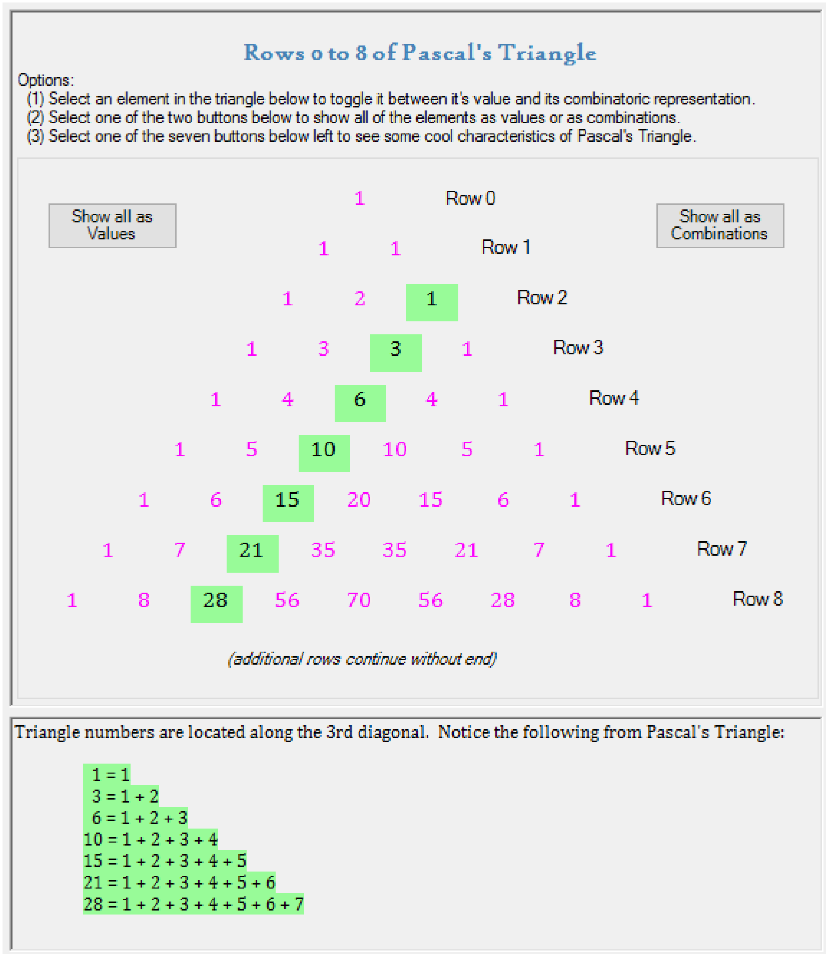


Figure 5: Triangle numbers

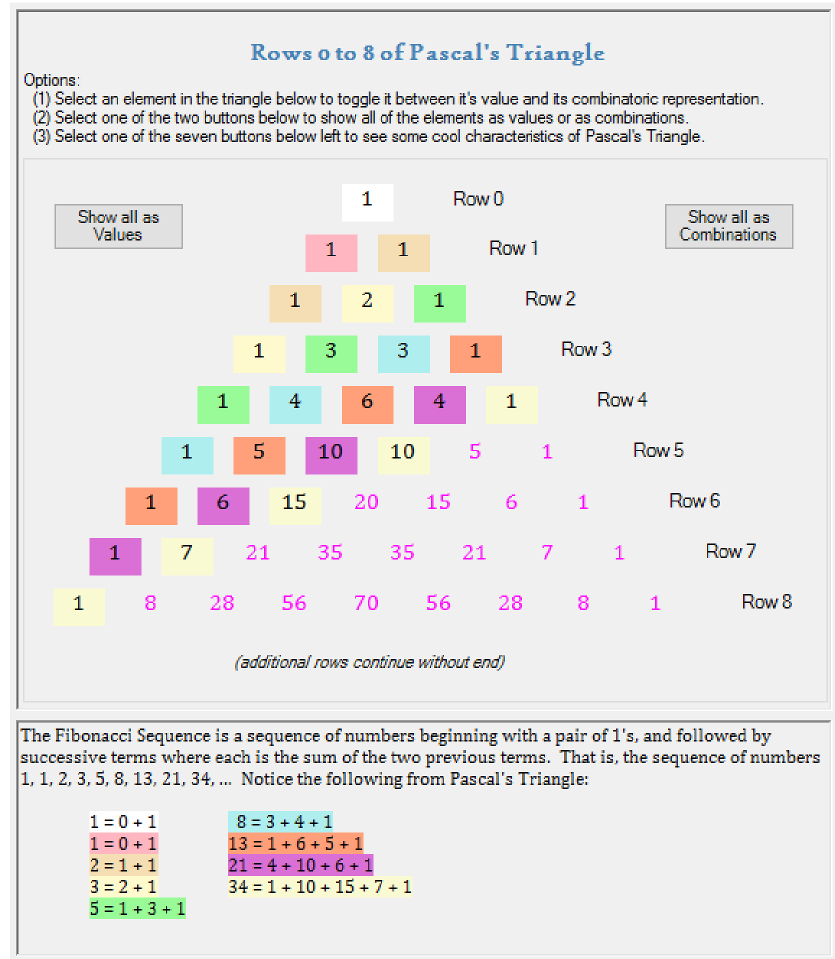


Figure 6: Fibonacci numbers

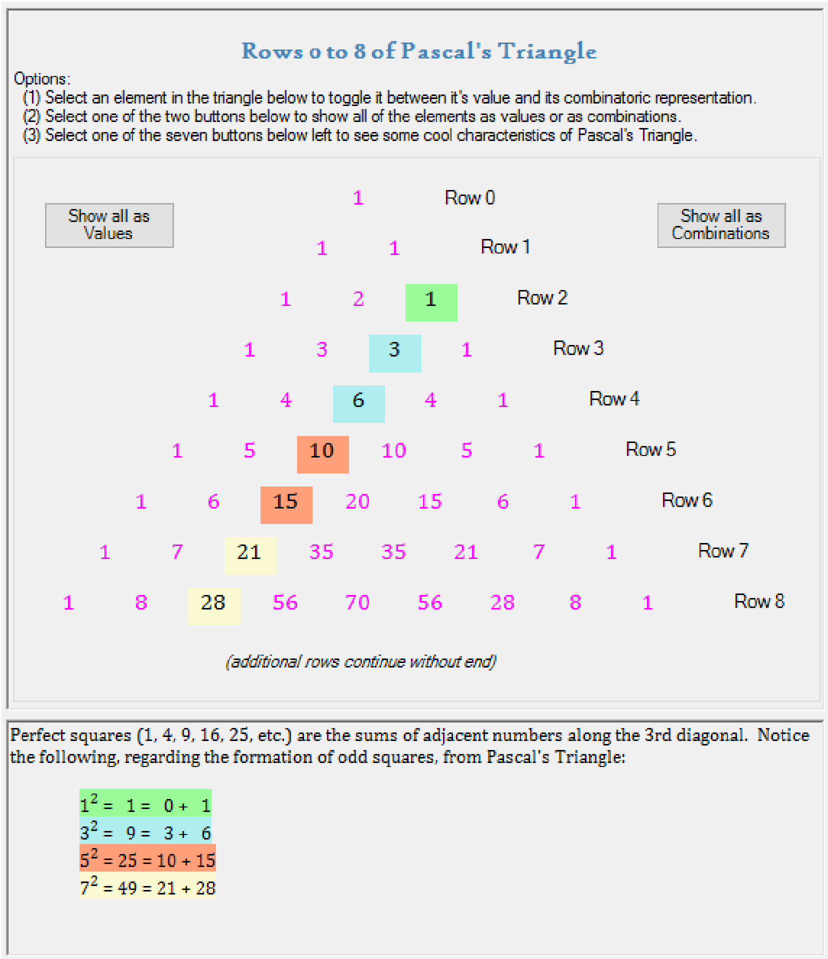


Figure 7: Odd squares

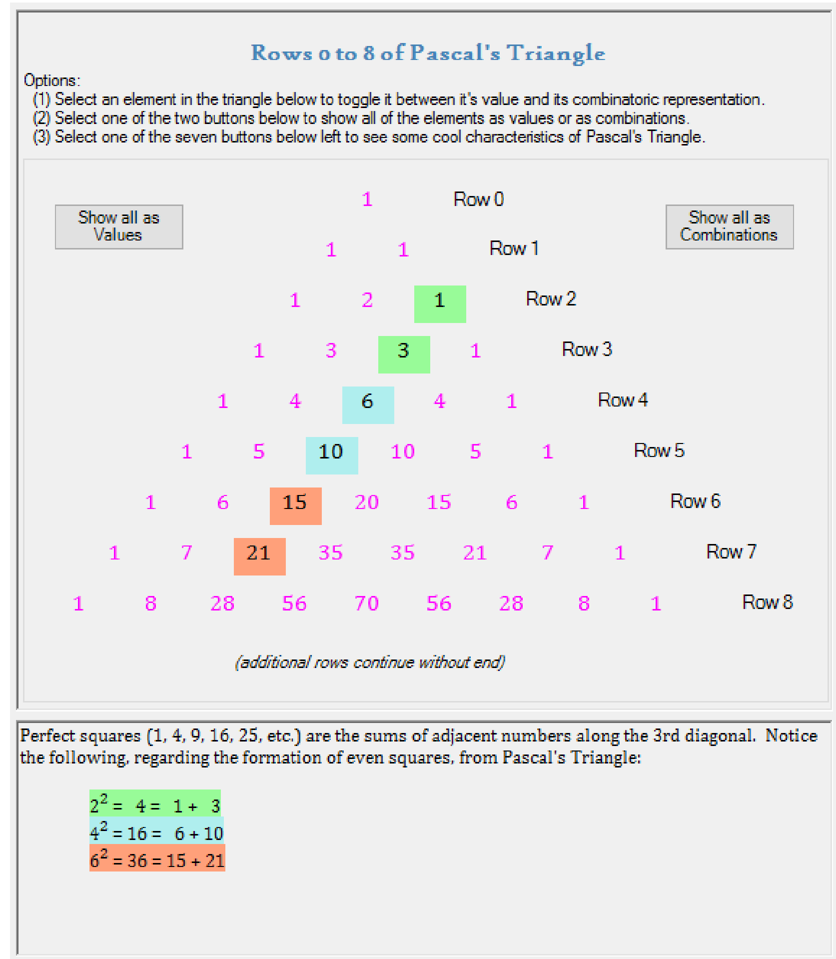


Figure 8: Even Squares