## **Pascal's Triangle**

#### 1. Construction

Start with a triangle of 1's in the left and the right columns

Form each row by adding the two numbers above it



#### 2. Patterns

#### 3. Combinations

*n*-th row, *r*-th column = 
$${}_{n}C_{r} = {\binom{n}{r}} = \frac{n!}{r!(n-r)!}$$

#### 4. Binomial Theorem

$$(a+b)^{n} = \sum_{r=0}^{n} {n \choose r} a^{n-r} b^{r} = a^{n} + n \cdot a^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^{2} + \dots + b^{n}$$

Example:  $(x + 1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$ 

# **Pascal's Triangle – Patterns**



Figure 1: Pascal's Triangle values

Rows o to 8 of Pascal's Triangle Options: (1) Select an element in the triangle below to toggle it between it's value and its combinatoric representation. (2) Select one of the two buttons below to show all of the elements as values or as combinations. (3) Select one of the seven buttons below left to see some cool characteristics of Pascal's Triangle.				
Show all as Values	0C0 1C0 1	Row 0 C1 Row 1	Shov Comb	v all as inations
	2C0 2C1	2C2 Ro	w 2	
	3C0 3C1 3	C <sub>2 3</sub> C <sub>3</sub>	Row 3	
4	4C0 4C1 4C2	4C3 4C4	Row 4	
5C0	5C1 5C2 5	C3 5C4 5C	Cs Row 5	
6C0 6	6C1 6C2 6C3	6C4 6C5	6C6 Row 6	
7C0 7C1	7 <b>C</b> 2 7 <b>C</b> 3 7	C4 7C5 7C	C6 7C7 R0	ow 7
8C0 8C1 8	3C2 8C3 8C4	8C5 8C6	8C7 8C8	Row 8
(additional rows continue without end)				

Figure 2: Pascal's Triangle combinations





Rows 0 to 8 of Pascal's Triangle

Figure 3: Sums of rows

Figure 4: Prime rows



### Rows 0 to 8 of Pascal's Triangle

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The Fibonacci Sequence is a sequence of numbers beginning with a pair of 1's, and followed by successive terms where each is the sum of the two previous terms. That is, the sequence of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, ... Notice the following from Pascal's Triangle:

 $\begin{array}{cccc} 1=0+1 & 8=3+4+1 \\ 1=0+1 & 13=1+6+5+1 \\ 2=1+1 & 21=4+10+6+1 \\ 3=2+1 & 34=1+10+15+7+1 \\ 5=1+3+1 \end{array}$ 

Figure 5: Triangle numbers

Figure 6: Fibonacci numbers



Perfect squares (1, 4, 9, 16, 25, etc.) are the sums of adjacent numbers along the 3rd diagonal. Notice the following, regarding the formation of odd squares, from Pascal's Triangle:





#### Rows o to 8 of Pascal's Triangle

Options:

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Perfect squares (1, 4, 9, 16, 25, etc.) are the sums of adjacent numbers along the 3rd diagonal. Notice the following, regarding the formation of even squares, from Pascal's Triangle:

 $2^{2} = 4 = 1 + 3$   $4^{2} = 16 = 6 + 10$  $6^{2} = 36 = 15 + 21$ 

Figure 8: Even Squares