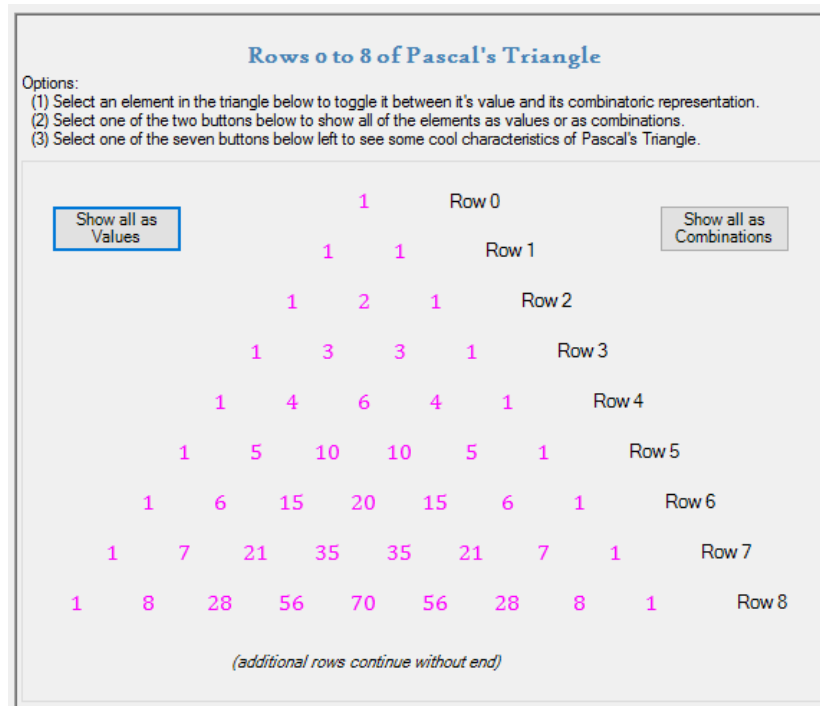


Pascal's Triangle

1. Construction

Start with a triangle of 1's in the left and the right columns

Form each row by adding the two numbers above it



2. Patterns

3. Combinations

$$n\text{-th row, } r\text{-th column} = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

4. Binomial Theorem

$$(a + b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r = a^n + n \cdot a^{n-1} b + \frac{n(n-1)}{2} a^{n-2} b^2 + \dots + b^n$$

Example: $(x + 1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

Pascal's Triangle - Patterns

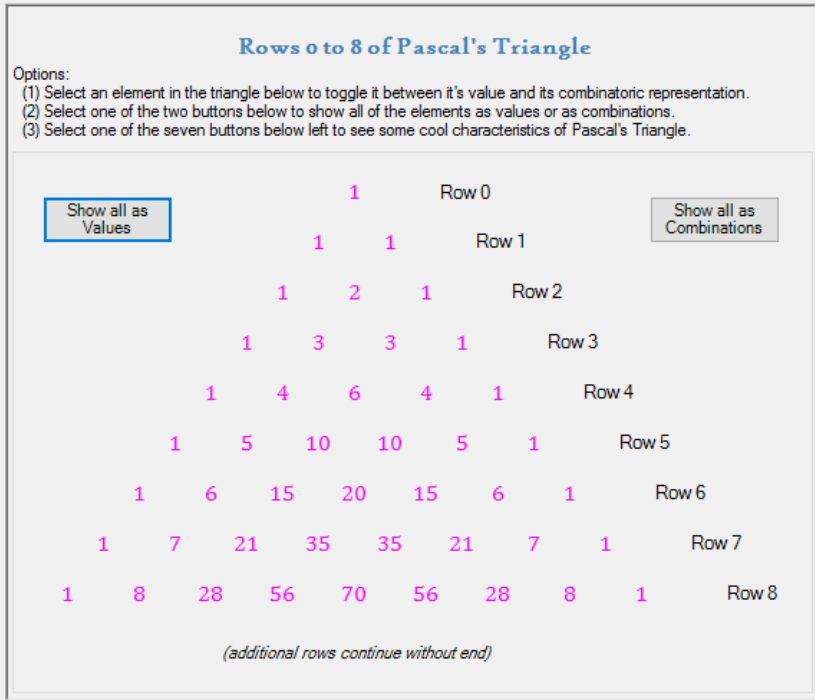


Figure 1: Pascal's Triangle values

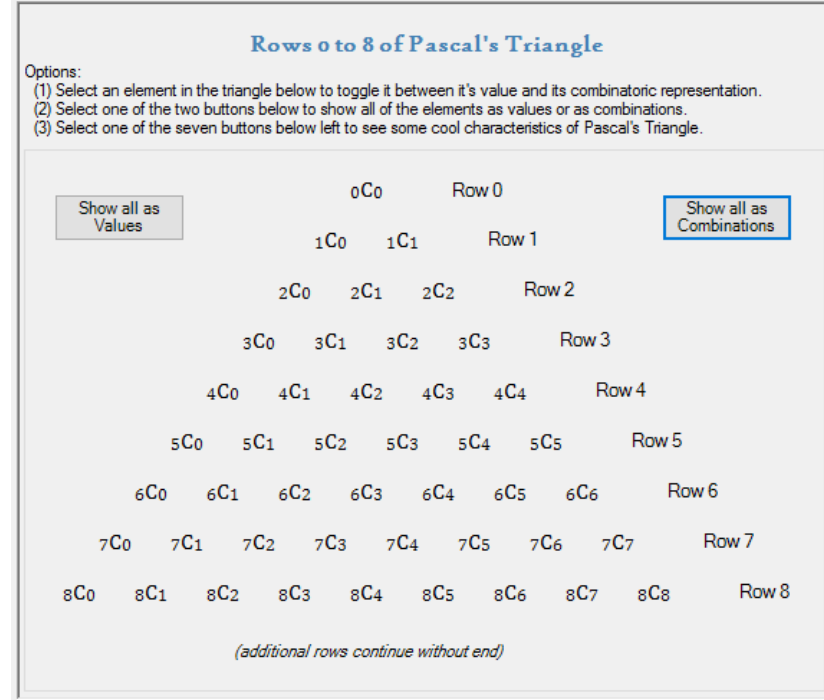


Figure 2: Pascal's Triangle combinations

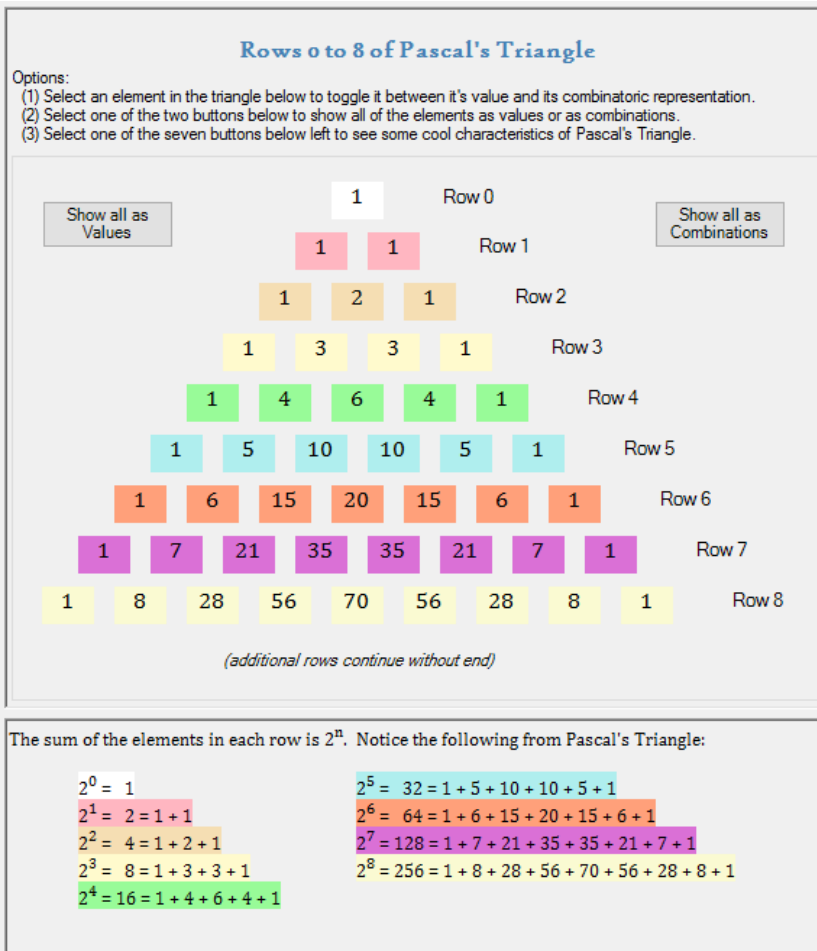


Figure 3: Sums of rows

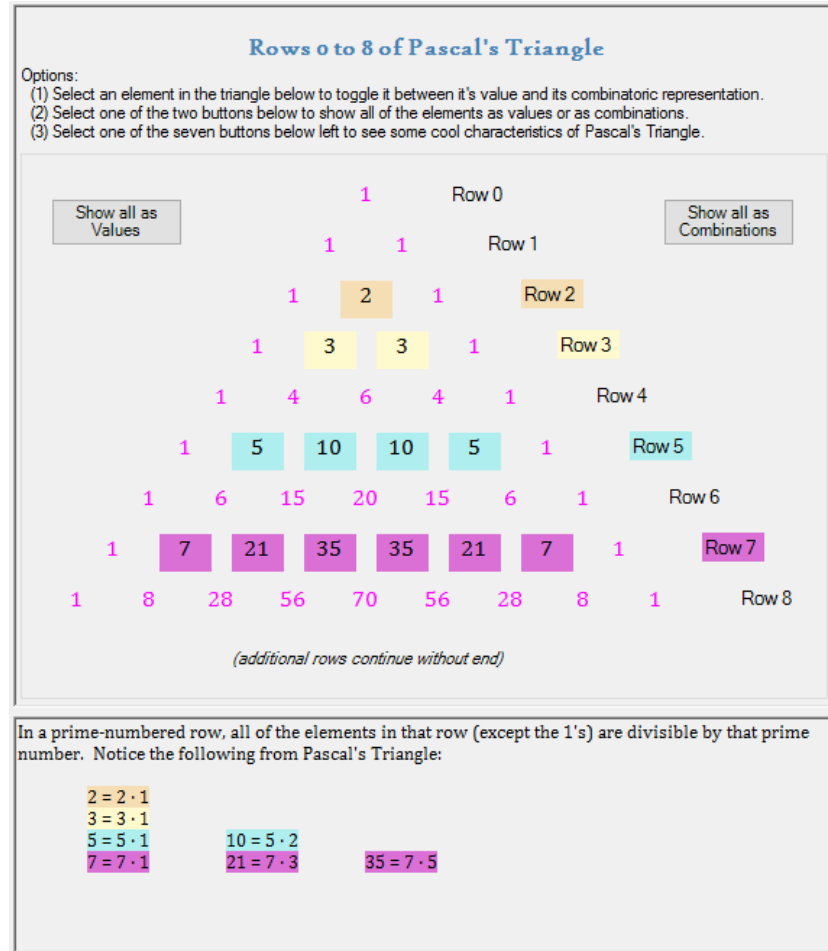
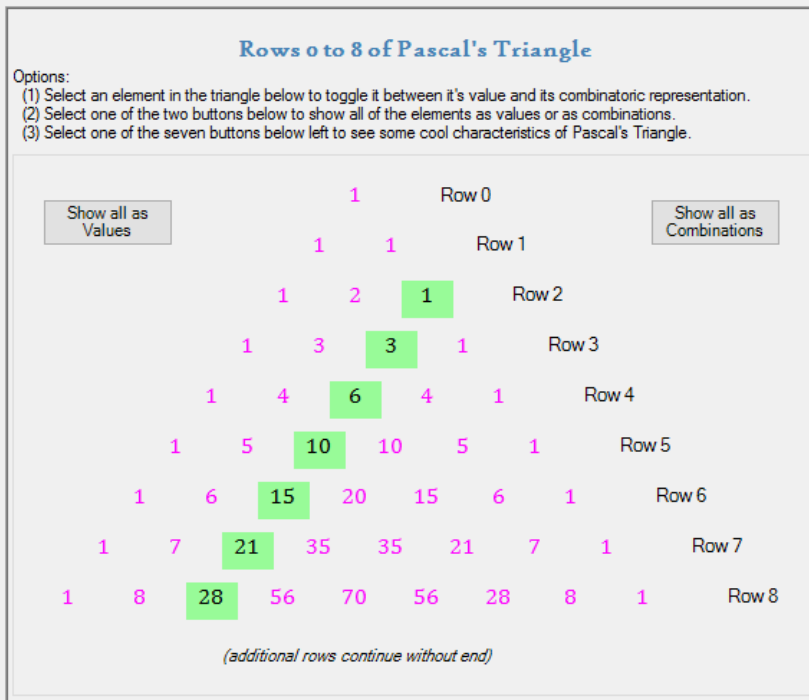


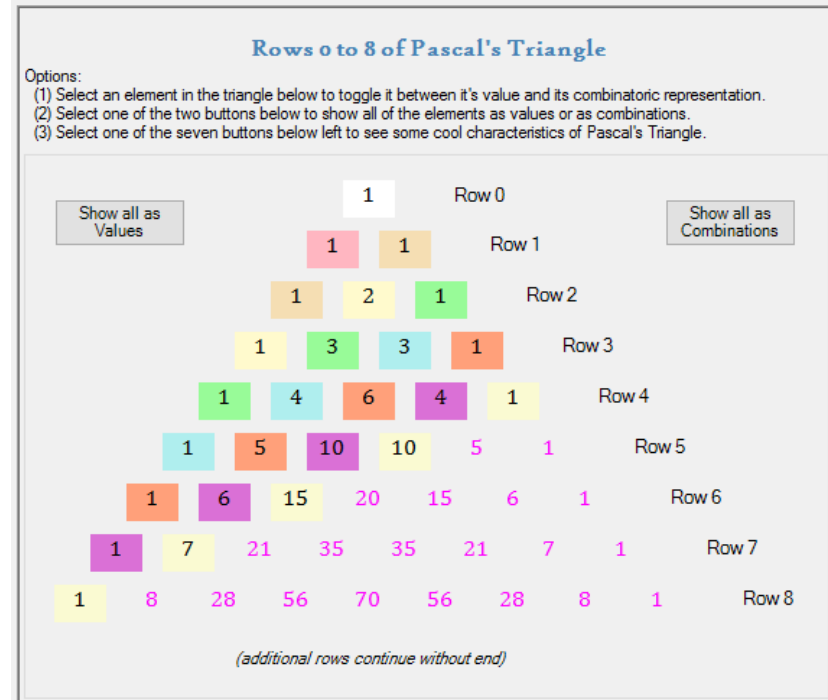
Figure 4: Prime rows



Triangle numbers are located along the 3rd diagonal. Notice the following from Pascal's Triangle:

$$\begin{aligned}
 1 &= 1 \\
 3 &= 1 + 2 \\
 6 &= 1 + 2 + 3 \\
 10 &= 1 + 2 + 3 + 4 \\
 15 &= 1 + 2 + 3 + 4 + 5 \\
 21 &= 1 + 2 + 3 + 4 + 5 + 6 \\
 28 &= 1 + 2 + 3 + 4 + 5 + 6 + 7
 \end{aligned}$$

Figure 5: Triangle numbers



The Fibonacci Sequence is a sequence of numbers beginning with a pair of 1's, and followed by successive terms where each is the sum of the two previous terms. That is, the sequence of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, ... Notice the following from Pascal's Triangle:

$$\begin{aligned}
 1 &= 0 + 1 & 8 &= 3 + 4 + 1 \\
 1 &= 0 + 1 & 13 &= 1 + 6 + 5 + 1 \\
 2 &= 1 + 1 & 21 &= 4 + 10 + 6 + 1 \\
 3 &= 2 + 1 & 34 &= 1 + 10 + 15 + 7 + 1 \\
 5 &= 1 + 3 + 1 & &
 \end{aligned}$$

Figure 6: Fibonacci numbers

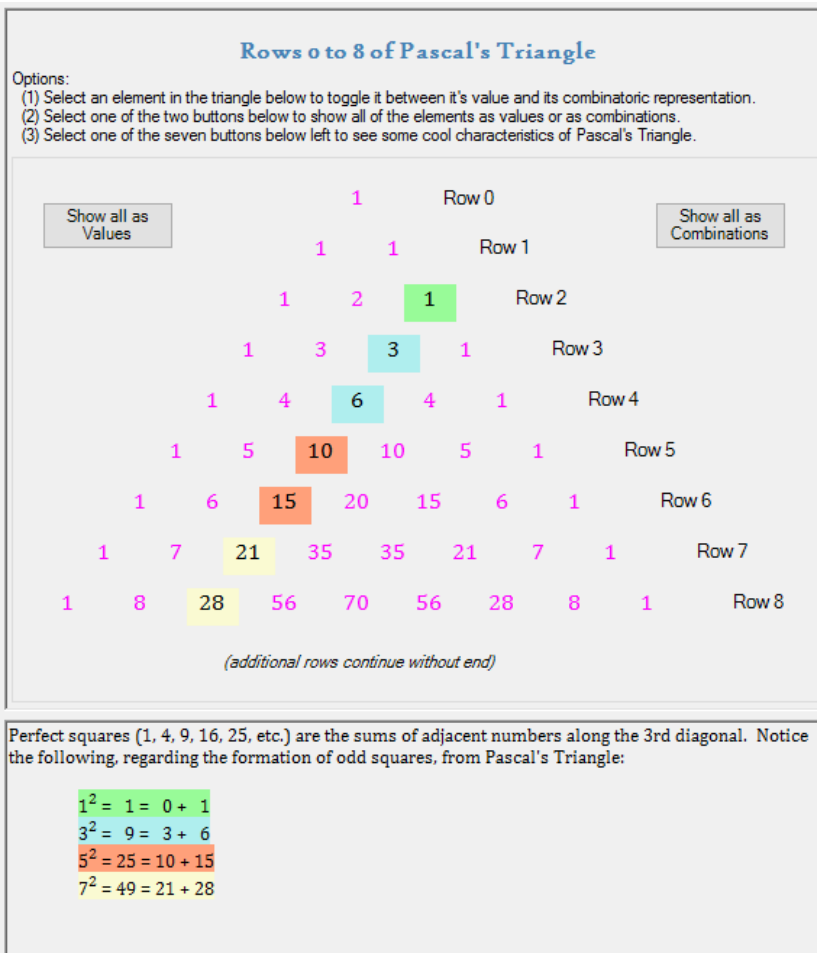


Figure 7: Odd squares

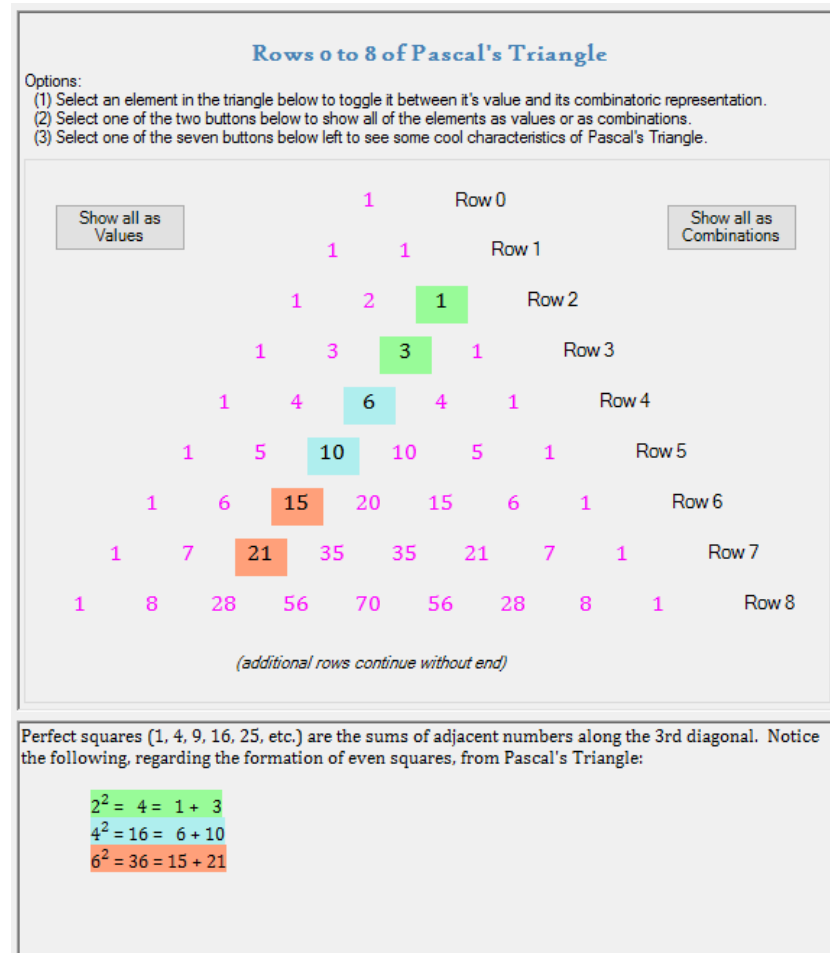


Figure 8: Even Squares