2009 Sample Math Proficiency Exam
Worked Out Problems

The following solutions to the practice exam provide the methods that I would use to get a solution. These are not the only ways to get the correct answer, so if you have another method that works, go ahead and use it. I hope this packet helps you prepare for the Exam. Earl

#1) Use Method 1:

$$\sqrt{3^2 + 3^2} = \sqrt{2 \cdot 3^2} = \sqrt{2} \cdot \sqrt{3^2} = \sqrt{2} \cdot 3 = 3\sqrt{2}$$ **B**

Or Method 2:

$\sqrt{3^2 + 3^2} = \sqrt{9 + 9} = \sqrt{18}$ Now, notice: $18 = 2 \cdot 3 \cdot 3$

So, $\sqrt{18} = \sqrt{2 \cdot 3 \cdot 3}$

Pull the pair of 3's out front to get: $3\sqrt{2}$

#2) $(4 + x) + 3x = 3x + (4 + x)$ is the **commutative property** of addition. **A**

$3x(4 + x) + 0 = 3x(4 + x)$ is the **identity property** of addition.

$(4 + x) + 3x = 4 + (x + 3x)$ is the **associative property** of addition.

$3x(4 + x) = 3x(4) + 3x(x)$ is the **distributive property** of multiplication over addition.

#3) Since information for Al is in the first column of the table, look for the values 3 and 2 in the first column. That narrows the answers down to B and D.

The difference between answers B and D is in Lisa's information. Lisa tutored 4 individuals, which means the 4 should be in the first row, under "Lisa." **D**

#4) This problem is an example of the counting rule. Just organize the information and multiply the numbers.

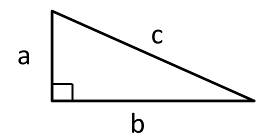
- 3 shirts
- 2 pairs of pants
- 2 pairs of socks

Since each item can be worn with each other item, the answer is:

$3 \cdot 2 \cdot 2 = 12 \; possible\ outfits$ **D**

#5) This problem requires you to remember the Pythagorean Theorem, which is:

$$a^2 + b^2 = c^2$$



where a and b are the legs, and c is the hypotenuse.

Note that either leg can be a or b as long as the side opposite the right angle, the hypotenuse, is c.

Then, from the numbers in the problem, we have:

$$3^2 + b^2 = 8^2$$

$$9 + b^2 = 64$$

$$\underline{-9 \qquad\quad -9}$$

$$b^2 = 55$$

$$b = \sqrt{55} \qquad \textbf{C}$$

Note that it would be very helpful for you to memorize the following squares:

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$

#6) This is a hard problem, so it is one that should be left for after you have completed all of the easier problems. Here are the steps:

$$\frac{4x^2-1}{2y} - \frac{9y-5}{3} = z - 3y$$

Multiply both sides by the product of the denominators, $\mathbf{2y} \cdot \mathbf{3}$ to get:

$$\frac{\mathbf{2y} \cdot \mathbf{3} \cdot (4x^2-1)}{2y} - \frac{\mathbf{2y} \cdot \mathbf{3} \cdot (9y-5)}{3} = \mathbf{2y} \cdot \mathbf{3} \cdot (z - 3y)$$

Cancel like factors from the numerators and denominators:

$$3 \cdot (4x^2 - 1) - 2y \cdot (9y - 5) = \mathbf{6y} \cdot (z - 3y)$$

Multiply terms where indicated: $12x^2 - 3 - 18y^2 + 10y = 6yz - 18y^2$

Add $18y^2$ to both sides: $\qquad + 18y^2 \qquad\qquad + 18y^2$

$$12x^2 - 3 + 10y = 6yz$$

Collect the y-terms on one side: $\qquad -10y \quad -10y$

$$12x^2 - 3 = 6yz - 10y$$

Factor the terms:

Divide by: $2\,(3z - 5)$

$$\frac{3\,(4x^2 - 1)}{2\,(3z - 5)} = \frac{2\,(3z - 5)y}{2\,(3z - 5)}$$

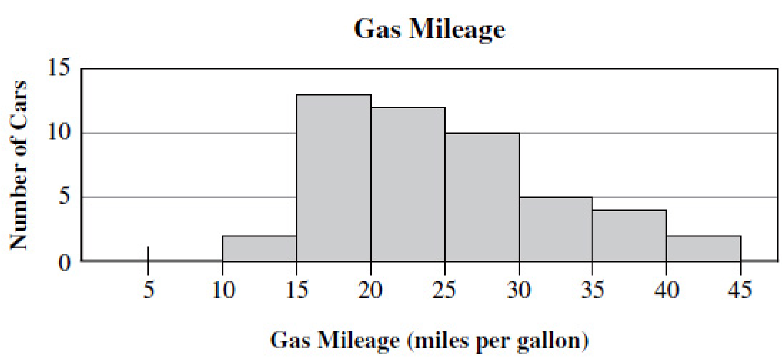
$$\frac{3\left(4x^2 - 1\right)}{2\,(3z-5)} = y \qquad \textbf{A}$$

#7) Here are the definitions of the terms in this question:

- **Test:** a methodology used to evaluate a hypothesis about a population. Example: you might speculate that 40% of all major league baseball players weigh over 200 pounds. Then you devise a test to see if you are right.
- **Sample:** a ***subset*** of individuals is surveyed to estimate characteristics of the entire population. Example: one out of five students going on a field trip is asked what they want for lunch, in order to estimate what is needed for the entire group of students going on the field trip.
- **Census:** an ***entire*** population is surveyed to determine its characteristics. Example: the United States government counts all of its citizens every 10 years by attempting to contact all 300 million of us!
- **Statistic:** a value that measures something about a population. Examples: mean, median, mode, variance, etc.

In question 7, the group of people surveyed is a subset of the whole, so the answer is **B**, a sample.

#8) Identify the numbers of cars in each category, then add them up across the page:



Number in category:	2	13	12	10	5	4	2
Adding across:	2	15	27	37	42	46	48

Note: The values in these two rows relate to the bars directly above them in the chart.

The median, then, is at the half-way point, or the point between the 24^{th} and 25^{th} car. This occurs between 15 and 27 in the second line above, which is in the 20-25 miles per gallon category. **B**

#9) Rules for determining how many solutions a pair of lines has:

- If two lines **intersect**: exactly 1 solution
- If two lines are **parallel** and are **NOT the same line**: 0 solutions
- If two lines are **the same line**: infinitely many solutions

Based on this, the answer is **A** because the lines are parallel and not the same line.

#10) Venn diagrams show the intersections of sets. In this problem, Bart has 3 exercises he may do in a day. Each of the four answers has to be carefully read and checked against the diagram. Though this sounds like a lot of effort, it is not all that difficult once you get used to reading Venn diagrams. And, getting one or two more problems correct on the exam is well worth the effort.

Comments on each of the four answers:

A. Bart ran 2 + 2 + 5 + 9 = **18** days, and on 9 + 5 = **14** of those he also biked. So, A is false.
B. Bart ran 2 + 2 + 5 + 9 = **18** days. He swam for 11 + 0 = **11** days that he did not run. This totals **29** days, but July has **31** days. He took 2 days off. So, B is false.
C. Bart swam for 2 + 5 + 11 = **18** days. He ran 2 + 2 + 5 + 9 = **18** days. Since both numbers are the same, C is false.
D. Bart biked for 9 + 5 = **14** days. Notice in the Venn diagram that both the 9 and the 5 are contained in the intersection of the biking and running circles. So, **D** is true and is the correct answer.

#11) First, find which integers $\sqrt{180}$ is between. From the chart in #5 above, that you have undoubtedly memorized by now, we know that:

$\sqrt{169} = 13$

$\sqrt{196} = 14$

So, $\sqrt{180}$ is between 13 and 14.

Then, $3\sqrt{180}$ is between $3 \cdot 13 = \mathbf{39}$ and $3 \cdot 14 = \mathbf{42}$.

The answer is: Between 39 and 42. **C**

#12) Recall that the diameter is two times the radius ($d = 2r$). So our approach should be to find the radius, then double it.

We are given the following: $V = \frac{4\pi}{3} r^3$ and, also: $V = 36\pi$

So, $\frac{4\pi}{3} r^3 = 36\pi$

Next, multiply both sides by $\frac{3}{4\pi}$: $\frac{3}{4\pi} \cdot \frac{4\pi}{3} r^3 = \frac{3 \cdot 36\pi}{4\pi}$

Simplify: $r^3 = 27$

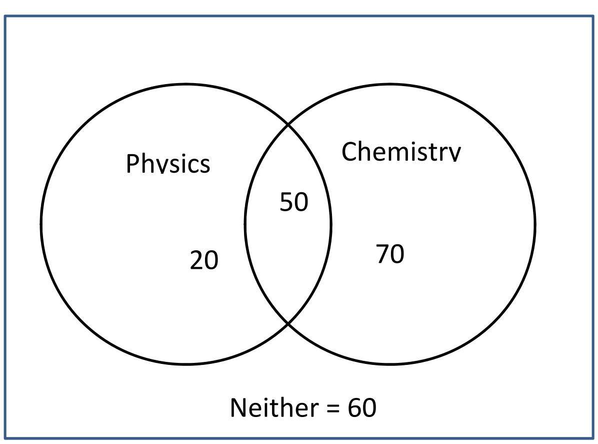
From this, we gather that: $r = 3$ Then, $d = 2r = 2 \cdot 3 = 6\ cm$ **B**

#13) Some points about absolute value inequalities:

- $\leq$ or $\geq$ require a closed circle at the endpoints.
- $<$ or $>$ require an open circle at the endpoints.
- $\geq$ or $>$ can be thought of as containing "great**or**" signs, meaning the inequality will be an "OR" inequality. "OR" inequalities are typically graphed on the outsides of a number line, Kind of like the *oars* of a boat.
- $\leq$ or $<$ can be thought of as containing "less th**and**" signs, meaning the inequality will be an "AND" inequality. "AND" inequalities are typically graphed in the middle of a number line, with nothing on the outside of the boat (no oars).

So, OR's are on the outside and AND's are in the middle. This problem has a "less th**and**" sign, so the graph must be in the middle. The only answer like this is **D**.

#14) If you like Venn diagrams, you can draw one. See if you think this helps.



50 students have both subjects.

70 take physics, so 70 – 50 = 20 must be taking only physics.

120 take chemistry, so 120 – 50 = 70 must be taking only chemistry.

That leaves 200 – 20 – 50 – 70 = 60 taking neither subject.

$P = 60 \div 200 = .30$ **C**

#15) Draw a mural: $Length = 3x$

$Width = x$

← mural

The area of the mural is: $(length \cdot width)$, which is: $(3x \cdot x) = 3x^2$

So, we are looking for the graph of: $y = 3x^2$

This must be A or C since a quadratic equation graphs as a parabola (B and D are lines). Testing the value of $x = 0$ gives us: $y = 3x^2 = 3 \cdot 0^2 = 0$. So, the ordered pair $(0, 0)$ must be on the graph, and this tells us that A is the answer. **A**

#16) Factor: $x^2 - 10x - 24$

Notice the following:

- The answer must take the form: $(x \pm \_) \cdot (x \pm \_)$ because the lead term is x^2.
- The two values we are looking for multiply to get -24, so one must be negative and one must be positive.
- The two values we are looking for must add to -10.

Factors of 24, in pairs, are 1 and 24, 2 and 12, 3 and 8, 4 and 6. Testing these values based on the above constraints leads us to the factors -12 and 2. So,

$$x^2 - 10x - 24 = (x - 12)(x + 2)$$ **D**

#17) Create a table of differences for the denominators, as follows:

Values	1st **D**	2nd **D**
2		
	6	
8		4
	10	
18		4
	14	
32		*4*
	18	
50		*4*
	22	
72		*4*
	26	
98		

Keep making difference columns until the same number shows up repeatedly in a column. Then, reverse the difference process, adding successive values until you get the number you need.

The solution, then, is: 1/98 **B**

#18) First, notice that $\angle DGF$ is a right angle, so the measure of minor arc $\widehat{DF}$ is 90˚. This represents $\frac{90}{360} = \frac{1}{4}$ of the circumference of the circle.

The circumference is: $C = 2\pi r$ or, in this case, $C = 2\pi x$.

The length of the arc is ¼ of this, or $\frac{2\pi x}{4} = \frac{\pi x}{2}$ **A**

#19) Circle the values in the *number chart* based on what is shown in the *bar graph*.

School Lunches

	Mon.	Tue.	Wed.	Thur.	Fri.
Week 1	425	400	476	(500)	478
Week 2	(501)	478	404	456	(493)
Week 3	417	(485)	(515)	468	475

What do the circled numbers have in common? They are the largest number in each column. **D**

#20) To multiply a matrix by a scalar (i.e., a number), multiply each value in the matrix by that scalar.

$$-3\begin{bmatrix} -1 & 3 \\ 4 & 7 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -9 \\ -12 & -21 \\ -6 & 6 \end{bmatrix}$$ **A**

#21) In a triangle that contains a line parallel to one of the sides, the corresponding parts of the sides are proportional. There are a number of ways to approach this problem, but the solution below is probably the easiest.

$$\frac{whole\ side}{segment\ closest\ to\ R} = \frac{RW}{RT} = \frac{RV}{RS} \Longrightarrow \frac{RW}{7} = \frac{7}{4}$$

Multiply both sides by 7: $$\frac{7 \cdot RW}{7} = \frac{7 \cdot 7}{4}$$

$$RW = \frac{49}{4} = 12.25\ cm$$

C

#22) Some definitions from statistics:

- **Mean** is the average; add all the numbers and divide by how many there are.
- **Median** is the middle number when you line them up from low to high; if there are two values in the middle, the median is their average.
- **Mode** is the value that shows up most often. A setoff data may have more than one mode.
- **Range** is the difference between the highest value and the lowest value.

In this problem, the highest value is \$1.70 (in month 9) and the lowest is \$1.10 (in month 5). Then, $range = \$1.70 - \$1.10 = \$0.60.$ **B**

#23) A solution to this problem is given here. But, if you forget how to do this, substitute the values from each answer (A, B, C, D) into the two equations. The correct solution will make both equations true. (The box below shows how to do this.)

Solution: Start with the two equations given, and multiply each by whatever values will make one of the variables disappear when the equations are added.

$2x - 4y = -5$	$\cdot 5$	$10x - 20y = -25$	*A note on how to select the multipliers is provided in the solution to problem #32 below.*
$3x + 5y = 9$	$\cdot 4$	$12x + 20y = 36$	
Next, add the equations:		$22x = 11$	
Divide by 22:		$\frac{22x}{22} = \frac{11}{22}$	
		$x = 0.5$	**B**

If you also needed to solve for y, substitute $x = \frac{1}{2}$ into one of the original equations and solve for y, like this:

$$2x - 4y = -5$$
$$2(\tfrac{1}{2}) - 4y = -5$$
$$1 - 4y = -5$$
$$-1 \qquad -1$$
$$\frac{-4y}{-4} = \frac{-6}{-4}$$

So, $y = 1.5$

To check your answer, substitute $x = 0.5$ and $y = 1.5$ into the original equations:

$2x - 4y = -5$	$3x + 5y = 9$
$2(0.5) - 4(1.5) = -5$	$3(0.5) + 5(1.5) = 9$
$1 - 6 = -5$	$1.5 + 7.5 = 9$
True	True

If both results are true, this is the correct solution.

#24) We will let Kevin pick his cubes one at a time:

- There are 5 possible cubes Kevin can pick first. After that …
- There are 4 possible cubes Kevin can pick next. After that …
- There are 3 possible cubes Kevin can pick next. After that …
- There are 2 possible cubes Kevin can pick next. After that …
- There is only one cube left and Kevin must pick it.

The number of possibilities is: $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$ **D**

#25) Housing may use up to 30% of Mario's salary. In equation form, this is:

$$30\% \cdot salary = Housing\ Expense$$

For a \$750 apartment, $30\% \cdot salary = 750$

Convert the % to a decimal: $.30 \cdot salary = 750$

Divide by .30: $\frac{.30 \cdot salary}{.30} = \frac{750}{.30}$

Result: $salary = \$2{,}500$ per month

Multiply by 12 months: $salary = \$30{,}000$ per year **C**

#26) This is an odd looking problem until you see the solution is *the distance between the two roots* of the equation. So, let's solve the equation:

$$0 = \frac{1}{6}(x^2 - 18x + 45)$$

The $\frac{1}{6}$ factor does not affect the roots, so $0 = (x^2 - 18x + 45)$

This time, when factoring, note that the sign on the constant term (45) is "+".

- This requires both values in the factoring process to have the same sign.
- The middle coefficient is negative, so the signs on our values must both be "−".

The factors of 45, in pairs, are 1 and 45, 3 and 15, 5 and 9. Using this information, we factor to get:

$$(x^2 - 18x + 45) = (x - 3)(x - 15) = 0$$

This requires that either: $x - 3 = 0$ or $x - 15 = 0$

Solving for x: $+3 \quad +3$ $\qquad$ $+15 \quad +15$

So, the two solutions are: $x = 3$ or $x = 15$

The difference is: $15 - 3 = 12$ **A**

#27) $\sqrt{5^3} = \sqrt{125}$

From the table above (problem 5), we know that:

$\sqrt{121} = 11$ and $\sqrt{144} = 12$

See why you want to memorize that table? 

So, $\sqrt{5^3}$ is closest to 11. **C**

#28) The center of the tolerance range is the mean (average) of the two values shown:

$$\frac{30.010 + 30.020}{2} = 30.015$$

Shortcut: In calculating the average of the two scary numbers, notice that the numbers are the same except for the last two digits. To get the average, calculate the average of the last two digits, 10 and 20, which is 15, and leave the rest of the number alone.

The "$\pm$" piece is the difference between this center value and either of the values shown in the inequality. So,

"$\pm$" piece $= 30.015 - 30.010 = .005$ (Shortcut: $15 - 10 = 5$)

So, the final answer is: $30.015 \pm .005$ **A**

#29) The probability of each coin landing with heads facing up is $\frac{1}{2}$.

The probability of three coins landing with heads facing up, then, is:

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$ **B**

#30) $\widehat{KPM}$ is a **major arc** (i.e., the long way around the circle), and includes all of the circle's 360˚ except those in **minor arc** $\widehat{KM}$.

Minor arc $\widehat{KM}$ has the same measure as **central angle** $\angle KLM$, which is 120˚. (Note that an arc and its associated central angle always have the same measure.)

So, the measure of major arc $\widehat{KPM} = 360^\circ - 120^\circ = 240^\circ$ **D**

#31) The numbers in this sequence are rising too rapidly to use the difference table approach we used in problem #17. Large increases usually means multiplication, so let's look at it with a multiplication state of mind.

Toward the end of the sequence, you may notice that the ratio of successive values is close to 3. So let's see what happens if we multiply each value by 3.

Value	3 · Value	Next Value
4	12	10
10	30	28
28	84	82
82	246	244

It looks like we have a pattern. After multiplying by 3, if we subtract 2, we get the next value in the sequence. To get the 6^{th} value, we need to apply this rule to the 5^{th} value, 244.

$$3 \cdot 244 - 2 = 732 - 2 = 730$$ **C**

#32) Another system of equations. These are popular. Let's use the same method we did in problem #23. Start with the two equations given, and multiply them by whatever will make one of the variables disappear when the equations are added.

$3x - 2y = 8$ $\quad \cdot 3 \quad$ $9x - 6y = 24$

$-x + 3y = -5$ $\quad \cdot 2 \quad$ $-2x + 6y = -10$

Next, add the equations: $7x = 14$

Divide by 7: $\frac{7x}{7} = \frac{14}{7}$

$$x = 2$$ **B**

A note about selecting the multipliers: Notice the coefficients of y in the equations are -2 and 3. All I did to select multipliers is reverse them and change the sign on one of them. This method ALWAYS works. Sometimes it does not give you the numbers in simplest form, but you can reduce your result to lowest terms at the end.

#33) Solving an ugly equation for one of its variables may seem daunting at first, but by following a couple of rules, you can become a master equation solver.

Let's start with the equation given, circle the variable we want to solve for, and see what we have before us.

$$T = 2\pi\sqrt{\frac{m}{K}}$$

We want to solve for m.

So, we have a 2π in front, and a square root to deal with. You may remember the order of operations and its mnemonic PEMDAS. The order is:

- **P**arentheses first
- **E**xponents second (including roots, which are basically fractional exponents)
- **M**ultiplication and **D**ivision next (performed left-to-right when mixed)
- **A**ddition and **S**ubtraction last (performed left-to-right when mixed)

When solving an equation, you do PEMDAS in reverse (SADMEP). In our equation, we have no addition or subtraction, so we will divide both sides by 2π to begin.

$$\frac{T}{2\pi} = \frac{2\pi}{2\pi}\sqrt{\frac{m}{K}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{m}{K}}$$

Why have I circled m all the way down the page? Remember that m is the variable we are solving for, and we do not want to lose it in the alphabet soup of this problem.

Now let's square both sides to get rid of the square root.

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{m}{K}}\right)^2$$

$$\frac{T^2}{4\pi^2} = \frac{m}{K}$$

If we now multiply by K, we have a solution for m.

$$\frac{K \cdot T^2}{4\pi^2} = \frac{K \cdot m}{K}$$

which gives:

$$\frac{K \cdot T^2}{4\pi^2} = m$$

A

#34) This is another hard problem that can be left to the end. Here's how to solve it:

We need to determine Jim's net gain under two scenarios; 1) he sells the car as is, and 2) he repairs the car and sells it for more because it is fixed up. Let's do this in a series of steps.

Scenario 1:

- $Current\ value\ = \$30{,}000 \cdot 27\% = \$8{,}100$

Scenario 2:

- $Repair\ cost = \ \$8{,}100 \cdot 8\% = \$648$
- $Price\ of\ car\ after\ repairs = \$30{,}000 \cdot 29\% = \$8{,}700$
- $Net\ Revenue\ after\ repairs = \$8{,}700 - \$648 = \$8{,}052$

By selling the car in its current condition Jim is better off by $48, calculated as:

- $\$8{,}100 - \$8{,}052 = \$48$ **A**

Problems like this require you to logically proceed step-by-step to get the eventual answer. It is close to a real life situation, though it is unlikely the values indicated in the problem could be known with certainty. Still, for students who ask "when am I ever going to use this stuff in real life?", this is an example of "when."

#35) This problem can be solved in a straightforward manner by using reverse PEMDAS:

Divide by 5: $$\frac{5x^2}{5} = \frac{125}{5}$$

$$x^2 = 25$$

Don't forget the "±" sign: $x = \pm 5$ **B**

#36) **Bias** is introduced when a sample is **not random**, but rather includes characteristics that affect what is being measured. The classic example would be estimating the average height of people in a school by surveying the height of its basketball players. The bias introduced in this case is that basketball players tend to be tall, so they are not representative of the average student at the school.

Since problem #36 involves average time on a bus of ALL students in the school, any sample that limits the survey to students who use a particular mode of transportation are biased. You need a sample that is not affected by the surveyed students' modes of transportation. **D**

#37) Going back to the definition of properties in problem #2 above, we see that this is the **distributive property** of multiplication over addition. **C**

#38) Illustrations of parallel lines cut by a transversal (a line that crosses and intersects the two parallel lines) are common and have a few basic properties:

- The angles in the illustration have **only two measures**. There is a **large angle** (usually > 90˚, but could = 90˚) and a **small angle** (usually < 90˚, but could = 90˚).
- All of the small angles have the same measure.
- All of the large angles have the same measure.
- The measures of a small angle and a large angle **add to 180˚**.

In this problem, we have a large angle and a small angle, so,

$$(4x + 10) + (x + 40) = 180$$

Combine like terms: $5x + 50 = 180$

Subtract 50 from both sides: $\quad -50 \quad -50$

$$5x = 130$$

Divide by 5: $\dfrac{5x}{5} = \dfrac{130}{5}$

$$x = 26$$

After finding x, it is a good idea to go back and re-read what the problem is asking for. This problem is asking for the measure of $\angle GJH$, one of the small angles:

$$m\ \angle GJH = x + 40 = 26 + 40 = 66^{\circ}$$ **C**

#39) The two distributions shown are different in term of how they show data, but each provides information about the distribution of plant heights. Comments on each answer:

A. The *median* heights are at the middle of both distributions, not the *mean* heights. Although the median and mean are the same in a normal (left-side) distribution, they are not necessarily the same in the box-and-whisker plot (on the right side).
B. The % of plants from 80 to 86 cm is 34% (= ½ · 68%) on the left and 25% on the right.
C. 86 cm is the mode on the left, but we do not have information about the mode in the distribution on the right. Box-and-whisker plots do not show a mode.
D. Since 86 cm is the median of both distributions, 50% of plants are greater than or equal to 86 cm in both distributions. Although the answer omits the phrase "or equal to," this is the best answer. **D**

#40) Remember that the “>” sign implies that the solution is an “OR” solution. So, is the solution A or B? Notice that “1” is a solution in A, but not in B. So, the easiest way to solve this is to see if the inequality is true for $x = 1$.

$$|3x - 4| > 5$$

$$|(3 \cdot 1) - 4| > 5$$ If this is true the answer is A; if not true, the answer is B.

$$|-1| > 5$$

$1 > 5$, which is NOT true. So, the answer is **B**.

To solve the problem directly with math rather than logic,

$$|3x - 4| > 5$$

Notice the switching of the < sign to > and the use of a minus sign in the second inequality.

Create two inequalities: $3x - 4 > 5$ OR $3x - 4 < -5$

Add 4 to all sides: $+4 \quad +4$ $\qquad$ $+4 \quad +4$

$$3x > 9 \quad \text{OR} \quad 3x < -1$$

Divide by 3: $\frac{3x}{3} > \frac{9}{3}$ $\qquad$ $\frac{3x}{3} < \frac{-1}{3}$

$$x > 3 \quad \text{OR} \quad x < -\frac{1}{3}$$

#41) Divide distance by hours to get speed:

Don’s Bicycle Rides			
Day	**Miles**	**Hours**	**Speed**
Monday	38	2.0	$38 \div 2 = 19\ mph$
Wednesday	33	1.5	$33 \div 1.5 = 22\ mph$
Friday	18	0.75	$18 \div .75 = 24\ mph$
Saturday	45	2.25	$45 \div 2.25 = 20\ mph$

The greatest average speed, then, is $24\ mph$ on Friday. **C**

#42) This problem involves the Pythagorean Theorem and the squares you memorized from problem 5. You did memorize them, didn't you? 

Route 1: $5 + 12 = 17\ miles$

Route 2: Use the Pythagorean Theorem. Let c be the distance along the hypotenuse of the triangle shown in the problem. Then,

$$5^2 + 12^2 = c^2$$

$$25 + 144 = c^2$$

$$169 = c^2$$

$$13 = c$$

The difference between routes 1 and 2 is:

$$17 - 13 = 4\ miles$$ **A**

#43) Section F is $\frac{1}{3}$ of $\frac{1}{2}$ of the spinner. $\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$ **A**

#44) Each day, Maya swims: $20 \cdot 75 = 1{,}500\ ft.$

To swim 1 mile takes: $5{,}280 \div 1{,}500 = 3.52\ days$

So, Maya achieves the 1-mile mark on day 4. **B**

#45) The easiest way to solve this problem is to substitute the values (-4 and 2) into the equations in the answers and see which solution produces true equations. **C**

#46) The properties illustrated by each answer are as follows:

A. Inverse property of multiplication
B. Identity property of multiplication
C. Zero property of multiplication
D. Commutative property of multiplication **A**

#47) **Outliers** are unusual values, generally significantly different from most of the other values in a sample. The only group identified for which 7-ft tall people would not be outliers is a men's professional basketball team. **B**

#48) This is a difficult problem because it requires the student to calculate "divided differences." Here's how it works: first subtract successive values to get differences, then divide the "dollar differences" by the "hour differences" to get something useful.

D	Hours	Dollars	**D**	Divided Diff's
	3	195		
2			80	80 / 2 = 40
	5	275		
7			280	280 / 7 = 40
	12	555		
4			160	160 / 4 = 40
	16	715		
3			120	120 / 3 = 40
	19	835		

The constant divided difference of 40 means that the cost of each additional hour worked is $40. The equation of the Repair Charges (y) based on the number of hours (x) can be developed as follows (note that we already know that the slope of the line is the constant difference value of 40).

With a slope of 40, the equation must be of the form: $y = 40x + b$. Let us calculate the value of b based on the cost of the job for 3 hours. (Note that we could have used information from any of the lines in the table for this purpose. I picked 3 hours.)

Start with: $195 = 3 \cdot 40 + b$

Multiply: $195 = 120 + b$

Subtract 120: $-120 \quad -120$

$$75 = b$$

Result: the equation is: $y = 40x + 75$

To get the number of hours that generates a job of over $5,000, set $y = 5{,}000$ and solve:

$$5{,}000 = 40x + 75$$

Subtract 75: $-75 \quad -75$

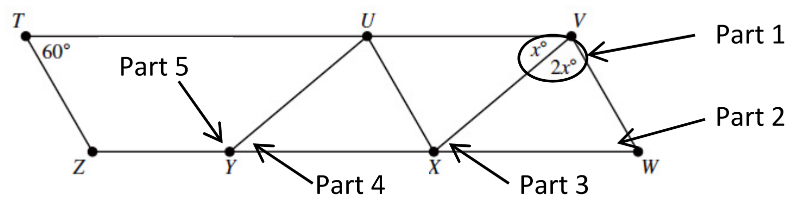
$$4{,}925 = 40x$$

Divide by 40: $\frac{4{,}925}{40} = \frac{40x}{40}$

$$123.1 = x$$

So, it would require 124 days to generate a $5,000 job. **C**

#49) This problem must be solved one angle at a time. And, yes, it is long. Here we go.



Part 1: Parallel lines cut by a transversal: $m\angle ZTU + m\angle UVW = 180^\circ$

Substitute values: $60 + 3x = 180$

Subtract 60: $-60 \quad -60$

$3x = 120$

Divide by 3: $\frac{3x}{3} = \frac{120}{3}$

$x = 40$

Measures of first two angles: $\boldsymbol{m\angle UVX} = x = 40^\circ$

$\boldsymbol{m\angle XVW} = 2x = 80^\circ$

Part 2: Opposite angles in parallelogram TVWZ: $\boldsymbol{m\angle XWV} = m\angle ZTU = 60^\circ$

Part 3: Three angles in a triangle add to 180˚:

$\boldsymbol{m\angle VXW} + m\angle XVW + m\angle XWV = 180^\circ$

$\boldsymbol{m\angle VXW} + 80^\circ + 60^\circ = 180^\circ$

$\boldsymbol{m\angle VXW} = 40^\circ$

Part 4: Parallel lines cut by a transversal: $\boldsymbol{m\angle UYX} = m\angle VXW = 40^\circ$

Part 5: Supplementary angles: $m\angle UYX + \boldsymbol{m\angle ZYU} = 180^\circ$

Substitute value: $40^\circ + \boldsymbol{m\angle ZYU} = 180^\circ$

And, finally: $\boldsymbol{m\angle ZYU} = 140^\circ$ **D**

#50) Precision is greatest when the units are smallest. If two measures have the same units, the more precise value contains more decimals. **C**