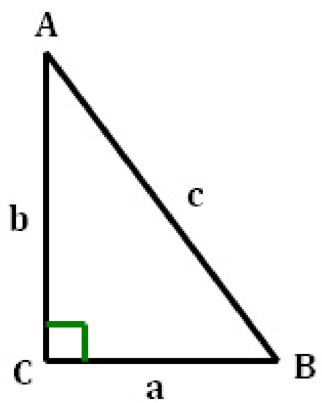
Geometry Name________________________________Period:___

Practice Test – Unit 8

Note: this page will not be available to you for the test. Memorize it!

Trigonometric Functions (p. 53 of the Geometry Handbook, version 2.1)



SOH-CAH-TOA

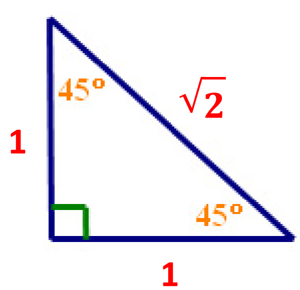
$\sin = \frac{opposite}{hypoteneuse}$ $\sin A = \frac{a}{c}$ $\sin B = \frac{b}{c}$

$\cos = \frac{adjacent}{hypoteneuse}$ $\cos A = \frac{b}{c}$ $\cos B = \frac{a}{c}$

$\tan = \frac{opposite}{adjacent}$ $\tan A = \frac{a}{b}$ $\tan B = \frac{b}{a}$

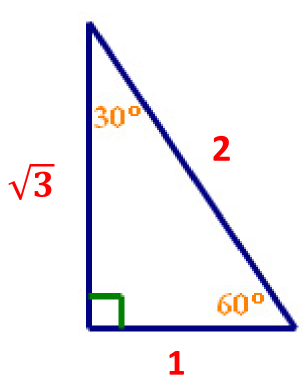
Special Triangles (p. 52 of the Geometry Handbook, version 2.1)

45^o-45^o-90^o Triangle



In a **45^o-45^o-90^o triangle**, the congruence of two angles guarantees the congruence of the two legs of the triangle. The proportions of the three sides are: $\mathbf{1 : 1 : \sqrt{2}}$. That is, the two legs have the same length and the hypotenuse is $\sqrt{2}$ times as long as either leg.

30^o-60^o-90^o Triangle

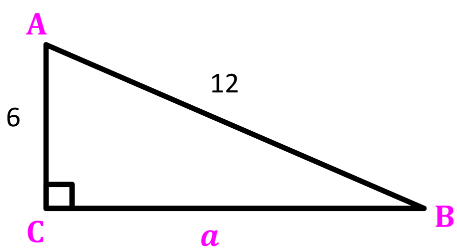


In a **30^o-60^o-90^o triangle**, the proportions of the three sides are: $\mathbf{1 : \sqrt{3} : 2}$. That is, the long leg is $\sqrt{3}$ times as long as the short leg, and the hypotenuse is **2** times as long as the short leg.

Solve each triangle below. Remember, each triangle has three answers.

For these problems, we have added names for the angles and the missing sides. We suggest you do the same.

1)



The first thing to notice about this right triangle is that the short leg is half the length of the hypotenuse. That makes this a $30° - 60° - 90°$ triangle, which has side proportions: $1 : \sqrt{3} : 2$.

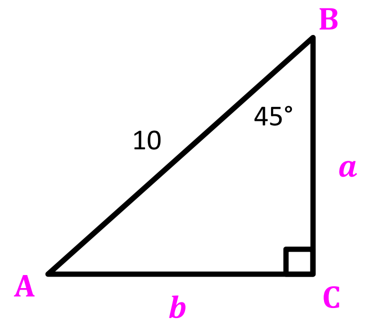
So, we have:

$$m\angle A = 60°$$

$$m\angle B = 30°$$

$$a = 6 \cdot \sqrt{3} = 6\sqrt{3}$$

2)



The first thing to notice about this right triangle is that it has a $45°$ angle. That makes this a $45° - 45° - 90°$ triangle, which has side proportions: $1 : 1 : \sqrt{2}$.

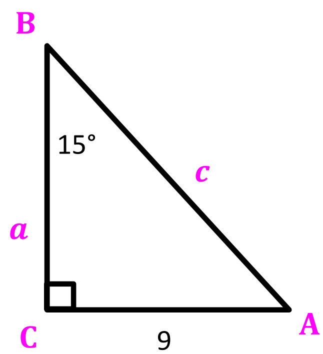
So, we have:

$$m\angle A = 45°$$

$$a = \frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

$$b = a = 5\sqrt{2}$$

3)



This is not a special triangle, so we must use Trig functions to solve it.

First, we have:

$$m\angle A = 90° - 15° = 75°$$

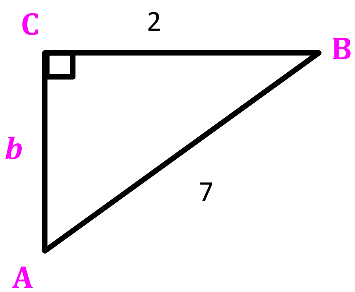
Then,

$$\tan 15° = \frac{9}{a} \quad \Rightarrow \quad a = \frac{9}{\tan 15°} = 33.59$$

$$\sin 15° = \frac{9}{c} \quad \Rightarrow \quad c = \frac{9}{\sin 15°} = 34.77$$

Tip: When calculating angles in problems where two side lengths are given, base your trig functions on the given lengths, even if you have already calculated the length of the remaining side. This will produce more accurate answers.

4)



This is not a special triangle, so we must use Trig functions to solve it.

First, we have:

$$2^2 + b^2 = 7^2 \quad \Rightarrow \quad 4 + b^2 = 49 \quad \Rightarrow \quad b^2 = 45$$

$$\boldsymbol{b = \sqrt{45} = 3\sqrt{5} \sim 6.71}$$

Then,

$$\sin A = \frac{2}{7} \quad \Rightarrow \quad \boldsymbol{m\angle A} = \sin^{-1}\frac{2}{7} = \boldsymbol{16.6^\circ}$$

$$\boldsymbol{m\angle B = 90^\circ - 16.6^\circ = 73.4^\circ}$$

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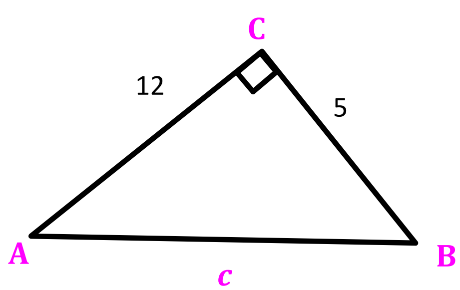
5)



Hope you like Trig functions!

First, we have:

$$5^2 + 12^2 = c^2 \quad \Rightarrow \quad 25 + 144 = c^2 \quad \Rightarrow \quad 169 = c^2$$

$$\boldsymbol{c = \sqrt{169} = 13}$$

Then,

$$\tan A = \frac{5}{12} \quad \Rightarrow \quad \boldsymbol{m\angle A} = \tan^{-1}\frac{5}{12} = \boldsymbol{22.6^\circ}$$

$$\boldsymbol{m\angle B = 90^\circ - 22.6^\circ = 67.4^\circ}$$

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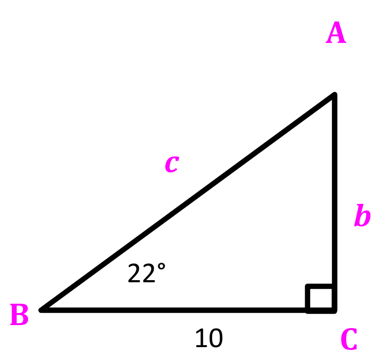
6)



Here they are again!

First, we have:

$$\boldsymbol{m\angle A = 90^\circ - 22^\circ = 68^\circ}$$

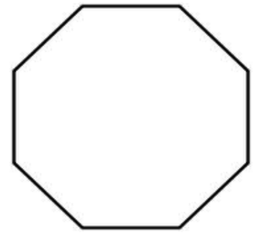
Then,

$$\tan 22^\circ = \frac{b}{10} \quad \Rightarrow \quad \boldsymbol{b} = 10 \cdot \tan 22^\circ \sim \boldsymbol{4.04}$$

$$\cos 22^\circ = \frac{10}{c} \quad \Rightarrow \quad \boldsymbol{c} = \frac{10}{\cos 22^\circ} \sim \boldsymbol{10.79}$$

7) A regular octagon has a perimeter of 80 inches and an apothem of 12.07 inches. Find the area of the regular octagon, rounded to one decimal place.

The formula for the area of a regular polygon is $A = \frac{1}{2}ap$, where a is the length of the apothem and p is the perimeter of the polygon.



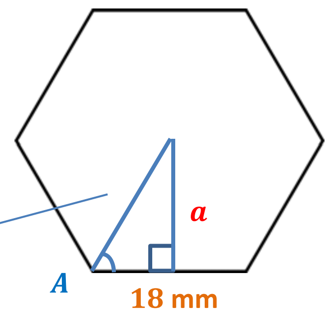
We are given: $p = 80$, $a = 12.07$

So, $Area = \frac{1}{2}ap = \frac{1}{2}(12.07)(80) = \mathbf{482.8}$ **in²**

8) Find the area of the regular hexagon shown below. Leave your answer as a radical.

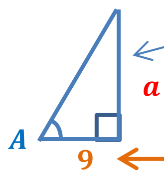
Step 1: How many sides? 6

Step 2: Find the perimeter: $P = 6 \cdot 18\text{ mm} = 108\text{ mm}$



Step 3: Find the apothem:

Create the "little guy" triangle:



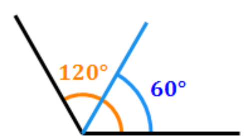
Our goal is to find $\boldsymbol{a}$.

The length of the base of the "little guy" triangle is: $\mathbf{18} \div 2 = \mathbf{9\text{ mm}}$

The sum of the angles in the figure (upper right) is: $(6-2)\cdot 180^\circ = 720^\circ$

Each angle of the figure measures: $720^\circ \div 6 = 120^\circ$

$m\angle A = 120^\circ \div 2 = 60^\circ$



Then, the "little guy" triangle is a 30°- 60°- 90° triangle. So, $\boldsymbol{a} = \mathbf{9\sqrt{3}}$.

Step 4: Calculate the area:

$$Area = \frac{1}{2}\,a\,P = \frac{1}{2}\cdot \mathbf{9\sqrt{3}} \cdot \mathbf{108} = \mathbf{486\sqrt{3}\ mm^2}$$

Step 5: (Optional) Compare result to the area of a square with side $2a$.

The area in Step 4 is $486 \cdot \sqrt{3} \sim \mathbf{841.8\ mm^2}$

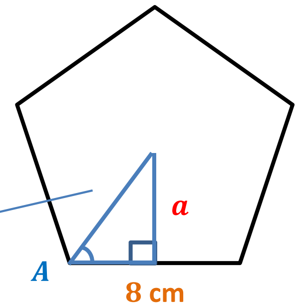
The area of a square with side $2a = 18\sqrt{3}$ should be a little more than this.

Square area is: $Area = 18\sqrt{3} \cdot 18\sqrt{3} = 324 \cdot 3 = \mathbf{972\ mm^2}$ vs $\mathbf{841.8\ mm^2}$ ✓

9) A regular pentagon has each side = 8 cm. Find the area of the regular pentagon, rounded to one decimal place.

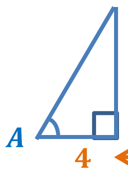
Step 1: How many sides? 5

Step 2: Find the perimeter: $P = 5 \cdot 8 \text{ cm} = 40 \text{ cm}$



Step 3: Find the apothem:

Create the "little guy" triangle:



Our goal is to find $\boldsymbol{a}$.

The length of the base of the "little guy" triangle is: $\mathbf{8} \div 2 = \mathbf{4\ cm}$

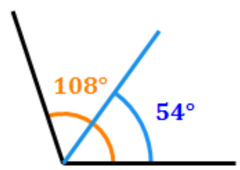
The sum of the angles in the figure (upper right) is: $(5 - 2) \cdot 180^\circ = 540^\circ$

Each angle of the figure measures: $540^\circ \div 5 = 108^\circ$

$m\angle A = 108^\circ \div 2 = 54^\circ$

Then, $\tan 54° = \dfrac{a}{4} \quad \Rightarrow \quad \boldsymbol{a} = 4 \tan 54° \sim \mathbf{5.5055}$.

(keep lots of decimals in your answers until the final calculation)



Step 4: Calculate the area:

$$Area = \frac{1}{2}\, a\, P = \frac{\mathbf{1}}{\mathbf{2}} \cdot \mathbf{5.5055} \cdot \mathbf{40} = \mathbf{110.1\ cm^2}$$

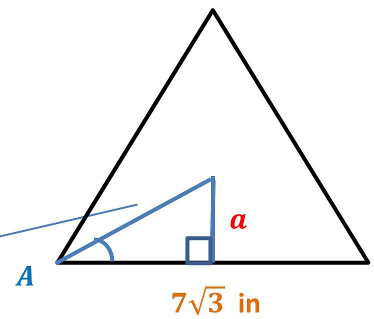
Step 5: (Optional) Compare result to the area of a square with side $2a$.

The area in Step 4 is $\mathbf{110.1\ cm^2}$

The area of a square with side $2a \sim 11.01$ should be a little more than this.

Square area is: $Area = 11.01 \cdot 11.01 = \mathbf{121.2\ cm^2}$ vs $\mathbf{110.1\ cm^2}$ ✓

10) An equilateral triangle has a side of $7\sqrt{3}$ inches. Find the area of the equilateral triangle. Leave your answer as a radical.

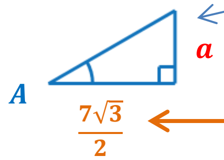


Step 1: How many sides? 3

Step 2: Find the perimeter: $P = 3 \cdot \mathbf{7\sqrt{3}}\ \text{mm} = 21\sqrt{3}\ \text{in}$

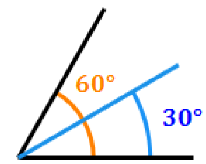
Step 3: Find the apothem:

Create the "little guy" triangle:



Our goal is to find $\boldsymbol{a}$.

The length of the base of the "little guy" triangle is: $\mathbf{7\sqrt{3}} \div 2 = \frac{\mathbf{7\sqrt{3}}}{\mathbf{2}}\ \mathbf{in}$



The sum of the angles in a triangle is 180°

Each angle of the figure measures: $180^\circ \div 3 = 60^\circ$

$m\angle A = 60^\circ \div 2 = 30^\circ$

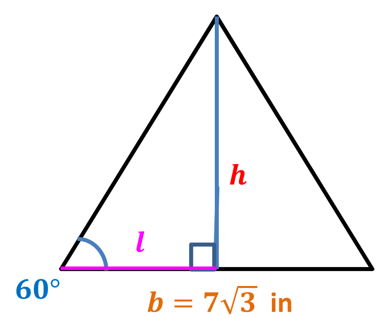
Then, the "little guy" triangle is a 30˚- 60˚- 90˚ triangle. So, $\boldsymbol{a} = \frac{\mathbf{7\sqrt{3}}}{\mathbf{2}} \div \mathbf{\sqrt{3}} = \frac{\mathbf{7\sqrt{3}}}{\mathbf{2\sqrt{3}}} = \frac{\mathbf{7}}{\mathbf{2}}$.

Step 4: Calculate the area:

$$Area = \frac{1}{2}\, a\, P = \frac{\mathbf{1}}{\mathbf{2}} \cdot \frac{\mathbf{7}}{\mathbf{2}} \cdot \mathbf{21\sqrt{3}} = \frac{\mathbf{147\sqrt{3}}}{\mathbf{4}}\ \mathbf{in^2}$$

Alternative: Find the height and then use the formula $A = \frac{1}{2}bh$

The length of the base of the left triangle is: $\boldsymbol{l} = \mathbf{7\sqrt{3}} \div 2 = \frac{\mathbf{7\sqrt{3}}}{\mathbf{2}}\ \mathbf{in}$



The sum of the angles in a triangle is 180°

Each angle of the figure measures: $180^\circ \div 3 = 60^\circ$

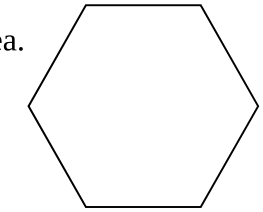
Then, the left triangle is a 30˚- 60˚- 90˚ triangle.

So, $h = \boldsymbol{l} \cdot \sqrt{3} = \frac{7\sqrt{3}}{2} \cdot \sqrt{3} = \frac{7\sqrt{3}\cdot\sqrt{3}}{2} = \frac{7\cdot 3}{2} = \frac{21}{2}$.

Calculate the area of the entire triangle:

$$Area = \frac{1}{2}bh = \frac{\mathbf{1}}{\mathbf{2}} \cdot \mathbf{7\sqrt{3}} \cdot \frac{\mathbf{21}}{\mathbf{2}} = \frac{\mathbf{147\sqrt{3}}}{\mathbf{4}}\ \mathbf{in^2}$$

11) A regular hexagon has an apothem of 15 inches. Each side is $10\sqrt{3}$. Find the area.

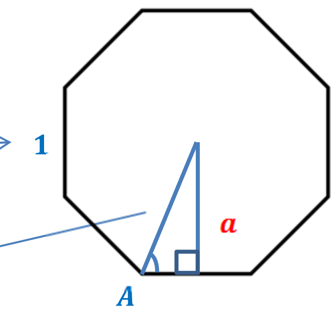


We are given: $a = 15$ $s = 10\sqrt{3}$

Perimeter $= 6s = 6 \cdot 10\sqrt{3} = 60\sqrt{3}$

So, $Area = \frac{1}{2}ap = \frac{1}{2}\,(15)\,(60\sqrt{3}) \;=\; \mathbf{450\sqrt{3}\ in^2}$

12) A regular octagon has a perimeter of 8 cm. Find the area of the regular octagon. Round your answer to the nearest tenth.

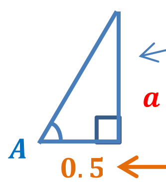


Step 1: How many sides? 8 (note also that we are given $\boldsymbol{P = 8}$)

Step 2: Find the length of a side: $s = \mathbf{8} \div 8 = 1$

Step 3: Find the apothem:

Create the "little guy" triangle:



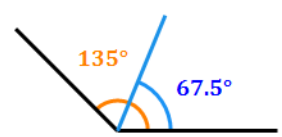
Our goal is to find $\boldsymbol{a}$.

The length of the base of the "little guy" triangle is: $1 \div 2 = \mathbf{0.5\ cm}$

The sum of the angles in the figure (upper right) is: $(8 - 2) \cdot 180^\circ = 1{,}080^\circ$

Each angle of the figure measures: $1{,}080^\circ \div 8 = 135^\circ$

$m\angle A = 135^\circ \div 2 = 67.5^\circ$



Then, $\tan 67.5^\circ = \dfrac{\boldsymbol{a}}{\mathbf{0.5}}$

So, $\boldsymbol{a} = \mathbf{0.5} \cdot \tan 67.5^\circ = \mathbf{1.2071\ cm}$

Step 4: Calculate the area:

$$Area = \frac{1}{2}\,a\,P = \frac{1}{2} \cdot \mathbf{1.2071} \cdot \mathbf{8} = \mathbf{4.8\ cm^2}$$

Step 5: (Optional) Compare result to the area of a square with side $2a$.

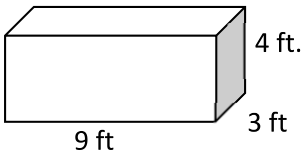
The area in Step 4 is $\mathbf{4.8\ cm^2}$

The area of a square with side $2a \sim 2.4142$ <u>should be</u> a little more than this.

Square area is: $Area = 2.4142 \cdot 2.4142 = \mathbf{5.8\ cm^2}$ vs $\mathbf{4.8\ cm^2}$ ✓

Find the surface area for each solid.

13)

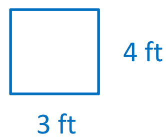


The two main options to deal with this problem are to use a formula, which works if you have it available, or to deconstruct the shape. We will illustrate the deconstruction method in Problems 13 and 14.

Front and Back

$Area = 9 \cdot 4 = 36 \text{ ft}^2$

Two Sides



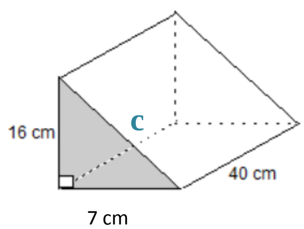
$Area = 3 \cdot 4 = 12 \text{ ft}^2$

Top and Bottom

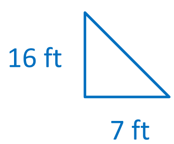
$Area = 9 \cdot 3 = 27 \text{ ft}^2$

Total Surface Area $= 2 \cdot 36 + 2 \cdot 12 + 2 \cdot 27 = \mathbf{150 \text{ ft}^2}$

14)



Front and Back Triangular Faces



$Area = \frac{1}{2} \cdot 16 \cdot 7$

$= 56 \text{ cm}^2$

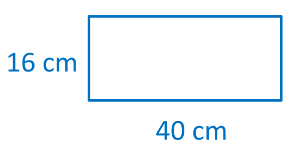
Find length of hypotenuse:

$7^2 + 16^2 = c^2$

$305 = c^2$

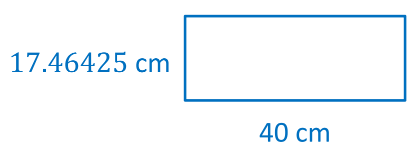
$c \sim 17.46425$

Left Rectangular Face



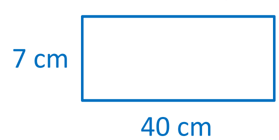
$Area = 40 \cdot 16 = 640 \text{ cm}^2$

Right Rectangular Face



$Area = 40 \cdot 17.46425 = 698.57 \text{ cm}^2$

Bottom Rectangle



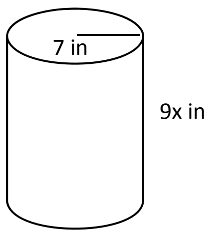
$Area = 40 \cdot 7 = 280 \text{ cm}^2$

Total Surface Area $= 2 \cdot 56 + 640 + 698.57 + 280 = \mathbf{1,730.57 \text{ cm}^2}$

We will use formulas to calculate the surface area in the balance of these exercises.

Find the surface area in terms of x. Leave pi in the answer.

15)

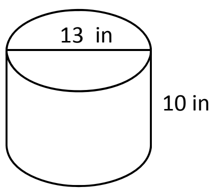


The formula for the surface area of a cylinder is $SA = 2\pi rh + 2\pi r^2$, where r is the radius of the circular faces and h is the height of the cylinder.

For this problem, $r = 7$ and $h = 9x$. So,

$$SA = 2\pi rh + 2\pi r^2 = 2\pi(7)(9x) + 2\pi(7)^2$$
$$= (126\pi x + 98\pi)\ \text{in}^2$$

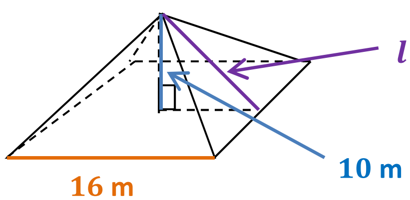
16)



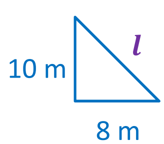
For this problem, $r = \frac{13}{2} = 6.5$ and $h = 10$. So,

$$SA = 2\pi rh + 2\pi r^2 = 2\pi(6.5)(10) + 2\pi(6.5)^2$$
$$= 214.5\pi\ \text{in}^2$$

17) Find the surface area given the square base of side equal to 16m, and height of 10m.



Find the Slant-Height



Note that the length of the base of the triangular semi-cross-section if the pyramid is $16 \div 2 = 8$.

Then,

$$8^2 + 10^2 = l^2$$
$$164 = l^2$$
$$l \sim 12.80625$$

The formula for the surface area of a pyramid is $SA = \frac{1}{2}pl + B$, where p is the perimeter of the base, l is the slant-height of a face, and B is the area of the base.

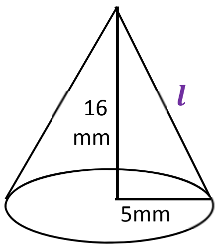
For this problem,

- $p = 4 \cdot 16 = 64,$
- $l = 12.80625$ (see box at left), and
- $B = 16^2 = 256$

$$SA = \frac{1}{2}pl + B = \frac{1}{2}(64)(12.80625) + 256 = 665.8\ \text{m}^2$$

Find the surface area for the following solids. Leave pi in the answer.

18)

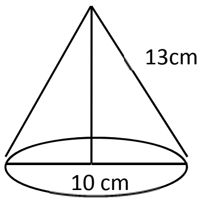


The formula for the surface area of a cone is $SA = \pi r l + \pi r^2$, where r is the radius of the circular base and l is the height of the cone.

For this problem, $r = 5$ and $l = \sqrt{5^2 + 16^2} = 16.763$. So,

$$SA = \pi r l + \pi r^2 = \pi(5)(16.763) + \pi(5)^2$$
$$= \mathbf{108.8\pi\ mm^2}$$

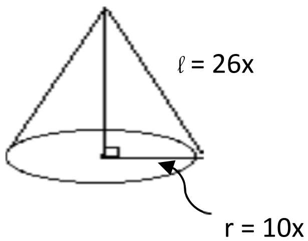
19)



For this problem, $r = \frac{10}{2} = 5$ and $l = 13$. So,

$$SA = \pi r l + \pi r^2 = \pi(5)(13) + \pi(5)^2$$
$$= \mathbf{90\pi\ cm^2}$$

20) Find the surface area of the cone in terms of x.

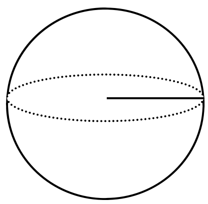


For this problem, $r = 10x$ and $l = 26x$. So,

$$SA = \pi r l + \pi r^2 = \pi(10x)(26x) + \pi(10x)^2$$
$$= \mathbf{360\pi x^2\ cm^2}$$

Find the surface area of the solids. Leave pi in the answer.

21) radius = 14 in

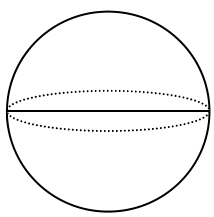


The formula for the surface area of a sphere is $SA = 4\pi r^2$, where r is the radius of the sphere.

For this problem, $r = 14$. So,

$$SA = 4\pi r^2 = 4\pi(14)^2$$
$$= \mathbf{784\pi\ in^2}$$

22) diameter = 14 cm

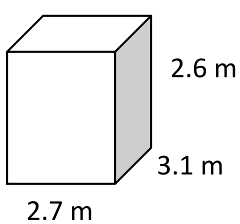


For this problem, $r = \frac{14}{2} = 7$. So,

$$SA = 4\pi r^2 = 4\pi(7)^2$$
$$= \mathbf{196\pi\ cm^2}$$

Find the Surface Area of the solid.

23)

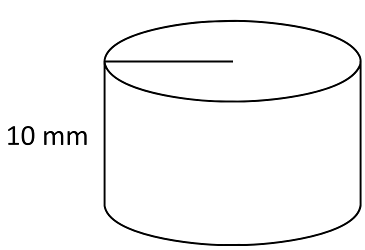


The formula for the surface area of a rectangular prism is $SA = 2(lw + lh + wh)$, where l, w and h are the dimensions of the prism.

For this problem, $l = 2.7$, $w = 3.1$, $h = 2.6$. So,

$$SA = 2lw = 2lh + 2wh = 2 \cdot [(2.7)(3.1) + (2.7)(2.6) + (3.1)(2.6)]$$
$$= \mathbf{46.9\ m^2}$$

24) radius = 6 mm

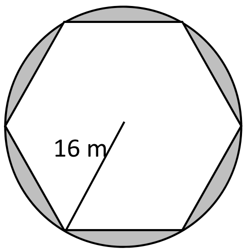


The formula for the surface area of a cylinder is $SA = 2\pi rh + 2\pi r^2$, where r is the radius of the circular faces and h is the height of the cylinder.

For this problem, $r = 6$ and $h = 10$. So,

$$SA = 2\pi rh + 2\pi r^2 = 2\pi(6)(10) + 2\pi(6)^2$$
$$= \mathbf{192\pi\ mm^2}$$

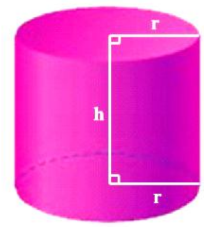
25) Find the surface area of both the right hexagonal prism and the cylinder whose bases are drawn below. Notice that both figures have the same radius. Assume the hexagon is regular and that the height of both solids is 10 m.



Surface Area of the Cylinder: $SA = 2\pi rh + 2\pi r^2$

For this problem, $r = 16$ and $h = 10$. So,

$$SA = 2\pi rh + 2\pi r^2 = 2\pi(16)(10) + 2\pi(16)^2$$
$$= \mathbf{832\pi\ m^2 \sim 2,613.8\ m^2}$$



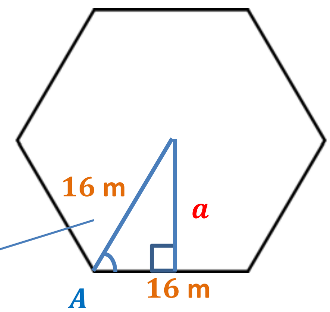
Surface Area of the Hexagonal Prism

First, find the area of the Hexagonal Base

Step 1: How many sides? 6

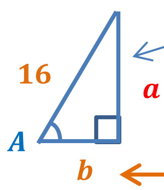
Step 2: Find the perimeter:

Note that the length of a side of a hexagon is the same as its radius. So, $P = 6 \cdot 16\text{ mm} = 96\text{ m}$



Step 3: Find the apothem:

Create the "little guy" triangle:

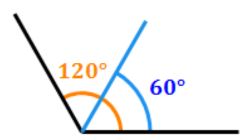


Our goal is to find $\boldsymbol{a}$.

The length of the base of the "little guy" triangle is: $\boldsymbol{b} = \mathbf{16} \div 2 = \mathbf{8\ m} = \frac{1}{2}(\text{hypotenuse})$

Therefore, the "little guy" triangle is a 30˚- 60˚- 90˚ triangle.

So, $\boldsymbol{a} = \mathbf{8} \cdot \sqrt{3} = \mathbf{8\sqrt{3}}$.



Step 4: Calculate the area of each hexagonal base:

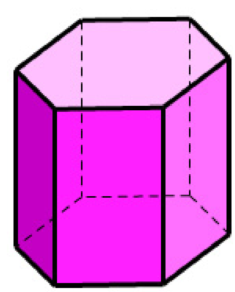
$$A_{base} = \frac{1}{2}\, a\, P = \frac{1}{2} \cdot \mathbf{8\sqrt{3}} \cdot \mathbf{96} = \mathbf{384\sqrt{3}} \sim \mathbf{665.1\ m^2}$$

Next, find the area of each rectangular face:

$$A_{face} = length \cdot width = 16 \cdot 10 = \mathbf{160\ m^2}$$

Then, find the surface area of the regular hexagonal prism:

The prism has 2 bases and 6 faces, so $SA = 2A_{base} + 6A_{face}$, where A_{base} is the area of a hexagonal base and A_{face} is the area of a rectangular face.



$$SA = 2(\mathbf{665.1}) + 6(\mathbf{160}) = \mathbf{2,290.2\ m^2}$$