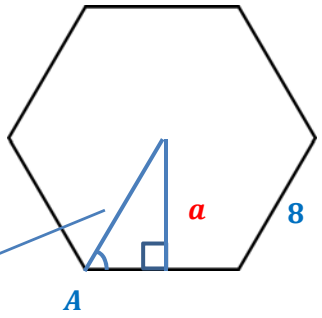
Geometry: 10.5 Part II Worksheet
"Unofficial" Worked-Out Solutions by Earl Whitney

Note: some of the problems state that the answer should be written as a simplified radical.

1) Area of a regular hexagon with a side $= 8\ m$. (use radical)

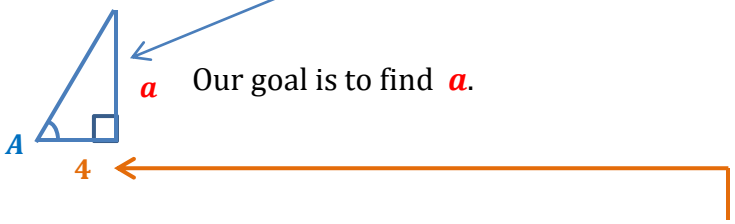


Step 1: How many sides? 6

Step 2: Find the perimeter: $P = 6 \cdot 8\ m = 48\ m$

Step 3: Find the apothem:

Create the "little guy" triangle:



Our goal is to find a.

The length of the base of the "little guy" triangle is: $8 \div 2 = 4\ m$

The sum of the angles in the figure (upper right) is: $(6-2) \cdot 180^\circ = 720^\circ$

Each angle of the figure measures: $720^\circ \div 6 = 120^\circ$

$m\angle A = 120^\circ \div 2 = 60^\circ$

Then, the "little guy" triangle is a 30°- 60°- 90° triangle. So, $a = 4\sqrt{3}$.

Step 4: Calculate the area:

$$Area = \frac{1}{2}\ a\ P = \frac{1}{2} \cdot 4\sqrt{3} \cdot 48 = 96\sqrt{3}\ m^2$$

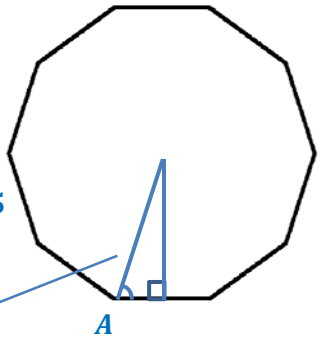
Step 5: (Optional) Compare result to the area of a square with side $2a$.

The area in Step 4 is $96 \cdot \sqrt{3} \sim 166.3$

This should be a little less than a square with side $2a = 8\sqrt{3}$

Rectangle area is: $8\sqrt{3} \cdot 8\sqrt{3} = 64 \cdot 3 = 192$ ✓

2) Area of a regular decagon with a perimeter $= 50\ in$.

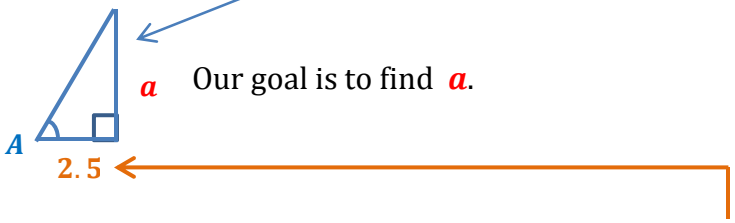


Step 1: How many sides? 10

Step 2: Find the length of a side: $s = 50 \div 10 = 5$

Step 3: Find the apothem:

Create the "little guy" triangle:



Our goal is to find a.

The length of the base of the "little guy" triangle: $5 \div 2 = 2.5\ in$

The sum of the angles in the figure (upper right) is: $(10 - 2) \cdot 180^\circ = 1{,}440^\circ$

Each angle of the figure measures: $1{,}440^\circ \div 10 = 144^\circ$

$m\angle A = 144^\circ \div 2 = 72^\circ$

Then, $\tan 72^\circ = \dfrac{a}{2.5}$

So, $a = 2.5 \cdot \tan 72^\circ = 2.5 \cdot 3.0777 = 7.694\ in$

Step 4: Calculate the area:

$$Area = \frac{1}{2}\, a\, P = \frac{1}{2} \cdot 7.694 \cdot 50 = 192.4\ in^2$$

Step 5: (Optional) Compare result to the area of a square with side $2a$.

The area in Step 4 is 192.4

This should be a little less than a square with side $2a = 2 \cdot 7.694 = 15.388$

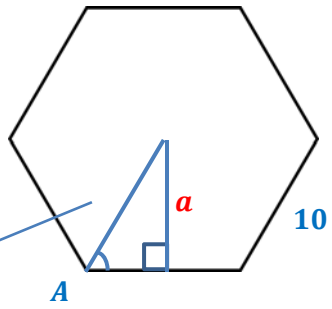
Rectangle area is: $15.388 \cdot 15.388 = 236.8$ ✓

3) Area of a regular hexagon with a side $= 10\ cm$. (use radical)

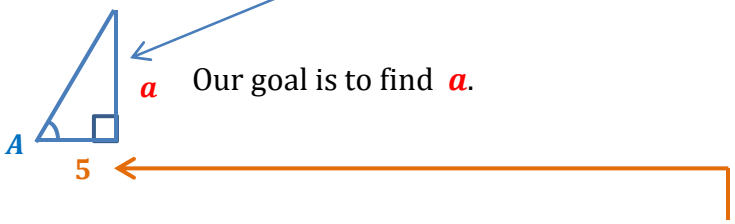
Step 1: How many sides? 6

Step 2: Find the perimeter: $P = 6 \cdot 10\ cm = 60\ cm$

Step 3: Find the apothem:



Create the "little guy" triangle:



Our goal is to find a.

The length of the base of the "little guy" triangle is: $10 \div 2 = 5\ cm$

The sum of the angles in the figure (upper right) is: $(6-2) \cdot 180^\circ = 720^\circ$

Each angle of the figure measures: $720^\circ \div 6 = 120^\circ$

$m\angle A = 120^\circ \div 2 = 60^\circ$

Then, the "little guy" triangle is a 30°- 60°- 90° triangle. So, $a = 5\sqrt{3}\ cm$.

Step 4: Calculate the area:

$$Area = \frac{1}{2}\ a\ P = \frac{1}{2} \cdot 5\sqrt{3} \cdot 60 = 150\sqrt{3}\ cm^2$$

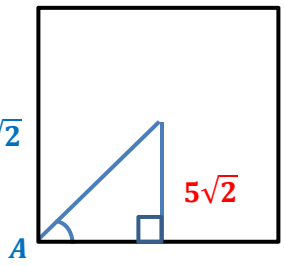
Step 5: (Optional) Compare result to the area of a square with side $2a$.

The area in Step 4 is $150 \cdot \sqrt{3} \sim 259.8$

This should be a little less than a square with side $2a = 10\sqrt{3}$

Rectangle area is: $10\sqrt{3} \cdot 10\sqrt{3} = 100 \cdot 3 = 300$ ✓

4) Area of a square with apothem $= 5\sqrt{2}\ in.$



Step 1: How many sides? 4

Step 2: Find the length of a side: $s = 2 \cdot 5\sqrt{2} = 10\sqrt{2}$

Method A: Using the Apothem Formula

Step 3: Find the perimeter: $P = 4 \cdot 10\sqrt{2}\ in = 40\sqrt{2}\ in$

Step 4: Calculate the area:

$$\boldsymbol{Area = \frac{1}{2}\ a\ P = \frac{1}{2} \cdot 5\sqrt{2} \cdot 40\sqrt{2} = \frac{1}{2} \cdot 200 \cdot 2 = 200\ in^2}$$

Method B: Using the Square Area Formula

Step 3: Calculate the area:

$$\boldsymbol{Area = s^2 = \left(10\sqrt{2}\right)^2 = 100 \cdot 2 = 200\ in^2}$$

5) Area of a equilateral (i.e., regular) triangle with radius = 8 *mm*.

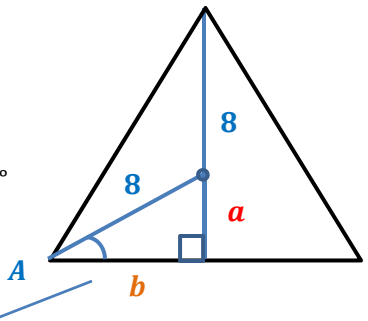
Step 1: How many sides? 3

Step 2: Find the measure of $\angle A$:

The sum of the angles in a triangle = 180°

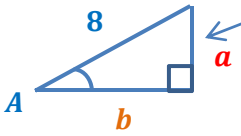
Each angle of the figure measures: $180 \div 3 = 60°$

$m\angle A = 60° \div 2 = 30°$



Step 3: Find the apothem:

Create the "little guy" triangle:



Our goals are to find ***a*** and b.

Note that the "little guy" triangle is a 30°- 60°- 90° triangle.
So, $\boldsymbol{a = 8 \div 2 = 4\ mm}$.

Step 4: Find the perimeter:

The length of the base of the "little guy" triangle is: $\boldsymbol{b = 4 \cdot \sqrt{3} = 4\sqrt{3}\ mm}$

The length of a side of the main triangle is: $\boldsymbol{2 \cdot 4\sqrt{3} = 8\sqrt{3}\ mm}$

Perimeter then is $P = 3 \cdot 8\sqrt{3}\ mm = \boldsymbol{24\sqrt{3}\ mm}$

Method A: Using the Apothem Formula

Step 5: Calculate the area:

$$\boldsymbol{Area = \frac{1}{2}\ a\ P = \frac{1}{2} \cdot 4 \cdot 24\sqrt{3} = 48\sqrt{3}\ mm^2 \sim 83.1\ mm^2}$$

Method B: Using the Triangle Area Formula

Step 5: Calculate the area:

$$\boldsymbol{Area = \frac{1}{2}\ b\ h = \frac{1}{2} \cdot 8\sqrt{3} \cdot (8 + 4) = 48\sqrt{3}\ mm^2 \sim 83.1\ mm^2}$$

6) Area of a regular hexagon with a perimeter $= 72\ in$ and apothem $= 6\sqrt{3}$. (use radical)

$$\boldsymbol{Area = \frac{1}{2}\ a\ P = \frac{1}{2} \cdot 6\sqrt{3} \cdot 72 = 216\sqrt{3}\ in^2}$$

7) Area of a regular pentagon with apothem $= 7\ cm$. (1 decimal)

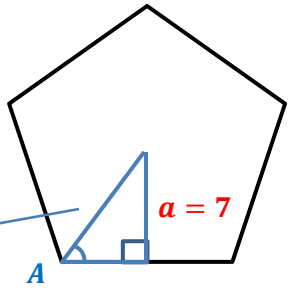
Step 1: How many sides? 5

Step 2: Find the measure of $\angle A$:

The sum of the angles in the figure (lower right) is: $(5-2) \cdot 180^\circ = 540^\circ$

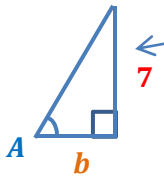
Each angle of the figure measures: $540 \div 5 = 108^\circ$

$m\angle A = 108^\circ \div 2 = 54^\circ$



Step 3: Find the length of the base of the pentagon:

Create the "little guy" triangle:



Our goal is to find b.

Then, $\tan 54^\circ = \frac{7}{b}$ So, $b = \frac{7}{\tan 54^\circ} = \frac{7}{1.3764} = 5.0858\ cm$

The length of a side of the main figure is: $2 \cdot 5.0858 = 10.1716\ cm$

Step 4: Find the perimeter of the pentagon:

Perimeter then is $P = 5 \cdot 10.1716\ cm = 50.858\ cm$

Step 5: Calculate the area:

$$\boldsymbol{Area = \frac{1}{2}\ a\ P = \frac{1}{2} \cdot 7 \cdot 50.858 = 178\ cm^2}$$

Step 6: (Optional) Compare result to the area of a square with side $2a$.

The area in Step 5 is 178

This should be a little less than a square with side $2a = 2 \cdot 7 = 14$

Rectangle area is: $14 \cdot 14 = 196$ ✓

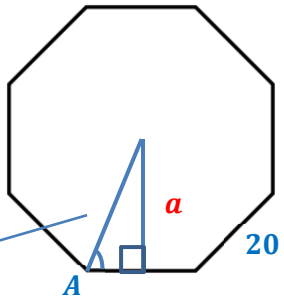
8) Area of a regular octagon with a side $= 20\ mm$. (1 decimal)

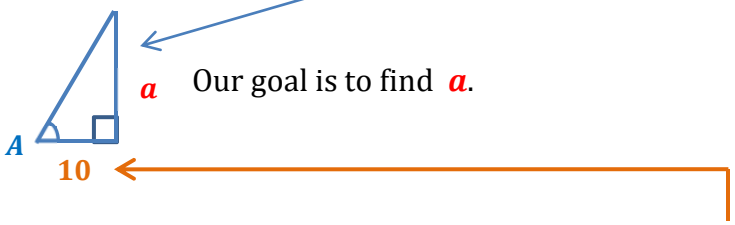
Step 1: How many sides? 8

Step 2: Find the perimeter: $P = 8 \cdot 20\ mm = 160\ mm$

Step 3: Find the apothem:

Create the "little guy" triangle:





Our goal is to find a.

The length of the base of the "little guy" triangle is: $20 \div 2 = 10\ mm$

The sum of the angles in the figure (upper right) is: $(8 - 2) \cdot 180^\circ = 1{,}080^\circ$

Each angle of the figure measures: $1{,}080^\circ \div 8 = 135^\circ$

$m\angle A = 135^\circ \div 2 = 67.5^\circ$

Then, $\tan 67.5^\circ = \frac{a}{10}$

So, $a = 10 \cdot \tan 67.5^\circ = 10 \cdot 2.4142 = 24.142\ mm$

Step 4: Calculate the area:

$$Area = \frac{1}{2}\, a\, P = \frac{1}{2} \cdot 24.142 \cdot 160 = 1{,}931.4\ mm^2$$

Step 5: (Optional) Compare result to the area of a square with side $2a$.

The area in Step 4 is 1,931.4

This should be a little less than a square with side $2a = 2 \cdot 24.142 = 48.284$

Rectangle area is: $48.284 \cdot 48.284 = 2{,}331$ ✓

9) Area of a regular pentagon with a perimeter $= 30\ mm$ and apothem $= 4.12\ mm$. (1 decimal)

$$\boldsymbol{Area = \frac{1}{2}\ a\ P = \frac{1}{2} \cdot 4.12 \cdot 30 = 61.8\ mm^2}$$

Note: a regular pentagon with perimeter $30\ mm$ should have an apothem of $4.13\ mm$, which would generate an area of $61.9\ mm^2$

10) Area of a regular hexagon when $\frac{1}{6}$ of it is $12\ in^2$.

$$\boldsymbol{Area = 6 \cdot 12 = 72\ in^2}$$

11) Circumference of a circle when area is 49π. (no units given)

$$Area = \pi r^2 = 49\pi \qquad Circumference = 2\pi r$$

$$r^2 = 49 \qquad Circumference = 2\pi \cdot 7$$

$$r = 7 \qquad \boldsymbol{Circumference = 14\pi}$$