Geometry: Semester 2 Practice Final
"Unofficial" Worked-Out Solutions by Earl Whitney

1. Wrapping a string around a trash can measures the circumference of the trash can. Assuming the trash can is circular, we get:

   Circumference formula: $C = 2\pi r = \pi d = 60\ inches$

   Divide by π: $\div \pi \quad \div \pi$

   $$d = \frac{60}{\pi} = 19.1\ inches$$

   Answer B

2. First find the radius, then find the area.

   Circumference formula: $C = 2\pi r = 100\pi\ feet$

   Divide by 2π: $\div 2\pi \quad \div 2\pi$

   $$r = 50\ feet$$

   Area formula: $A = \pi r^2 = 50^2\ \pi\ feet = 2{,}500\pi\ ft^2$

   Answer D

3. Calculate the radius, then the area, then the remaining pizza.

   Calculate the radius: $Radius = \frac{Diameter}{2} = \frac{18}{2} = 9\ inches$

   Area of entire pizza: $A = \pi r^2 = 9^2\ \pi\ in^2 = 81\pi\ in^2$

   Remaining pizza ($= \frac{1}{3}\ of\ pizza$): $A_{remaining} = \frac{1}{3} \cdot 81\pi\ in^2 = 85\ in^2$

   Answer B

4. Find the circumference, then the radius, then the area.

   Circumference: $C = \frac{360^\circ}{60^\circ} \cdot 10\ ft = 60\ ft$

   Calculate the radius: $C = 2\pi r = 60\ ft$

   Divide by 2π: $\div 2\pi \quad \div 2\pi$

   $$r = \frac{30}{\pi}\ feet$$

   Area of sector $= \frac{60^\circ}{360^\circ} \cdot A_{circle}$: $A = \frac{60^\circ}{360^\circ}\pi r^2 = \frac{1}{6} \cdot \pi \cdot \left(\frac{30}{\pi}\right)^2 feet = \frac{150}{\pi}\ ft^2$

   Answer A

5. Find the circumference, then the arc measure, and finally the arc length.

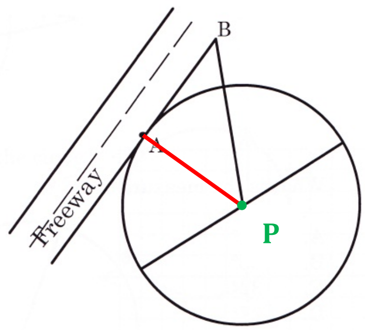
Circumference formula: $C = 2\pi r = 2\pi \cdot 45\ cm = 90\pi\ cm$

Arc measure: $m\ \widehat{AB} = 2 \cdot 20^{\circ} = 40^{\circ}$

Arc length: $l = \frac{40^{\circ}}{360^{\circ}} \cdot 90\pi\ cm = 10\pi\ cm$

Answer B

6. Use the Pythagorean Theorem or your knowledge of Pythagorean Triples.



Let P be the center of the circle. Then,

$$AB^2 + AP^2 = BP^2$$

$$80^2 + AP^2 = 100^2$$

$$6,400 + AP^2 = 10,000$$

$$-6,400 \qquad -6,400$$

$$AP^2 = 3,600$$

$$AP = 60\ miles$$

$$Diameter = 2 \cdot AP = 120\ miles$$

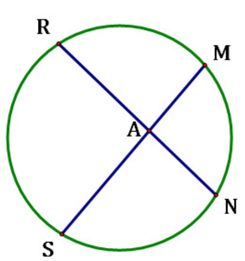
Answer C

7. The measure of the inscribed angle is half the measure of the arc .

$$\frac{1}{2} \cdot 220^{\circ} = 110^{\circ}$$

Answer C

8. Vertex inside the circle.



$RA \cdot AN = SA \cdot AM$

$$6x = 8 \cdot 9$$

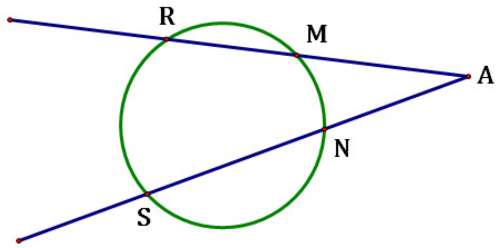
$$6x = 72$$

$$\div 6 \qquad \div 6$$

$$x = 12$$

Answer A

9. Vertex outside the circle



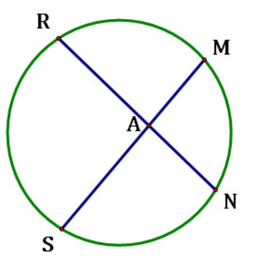
$$x = \frac{60^\circ - 10^\circ}{2}$$

$$x = 25^\circ$$

Answer C

$$m\angle A = \frac{1}{2}\left(m\,\widehat{RS} - m\,\widehat{MN}\right)$$

10. Vertex inside the circle.



$$m\angle A = \frac{1}{2}\left(m\,\widehat{RS} + m\,\widehat{MN}\right)$$

$$54^\circ = \frac{x + 76^\circ}{2}$$

$\cdot 2 \qquad \cdot 2$

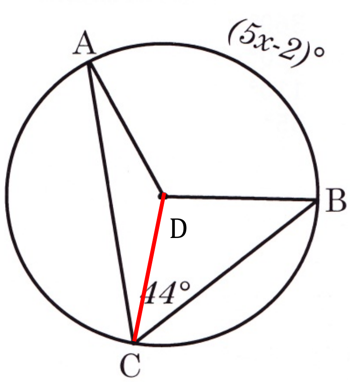
$108^\circ = x + 76^\circ$

$-76^\circ \qquad - 76^\circ$

$32^\circ = x$

Answer C

11. This one is more difficult than most.
After you calculate x, look for **congruent triangles.**



Picture is drawn poorly.
Don't be fooled!

$m\,\widehat{AB} = 2 \cdot m\angle C$

$5x - 2 = 2 \cdot 44$

$5x - 2 = 88$

$+2 \quad +2$

$5x \quad = 90$

$\div 5 \quad \div 5$

$x = 18$

Answer A

$\overline{AD} \cong \overline{BD}$

$\overline{DC} \cong \overline{DC}$

$\overline{CA} \cong \overline{CB}$

$\Delta ADC \cong \Delta BDC$

$\angle ADC \cong \angle BDC$

$\widehat{AC} \cong \widehat{BC}$

Let y = $m\,\widehat{BC}$

$2y + 88^\circ = 360^\circ$

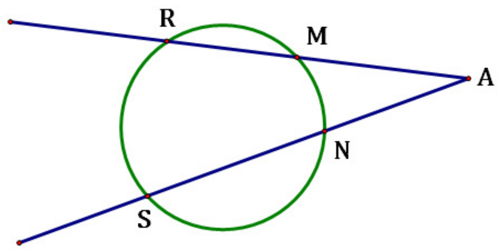
$-88^\circ \quad - 88^\circ$

$2y \quad = 272^\circ$

$\div 2 \quad \div 2$

$y = 136^\circ$

12. Vertex outside the circle



$AM \cdot AR = AN \cdot AS$

$x \cdot (x+5) = 2 \cdot (2+10)$

$x^2 + 5x = 24$

$-24 \quad -24$

$x^2 + 5x - 24 = 0$

$(x-3)(x+8) = 0$

$x \in \{3, -8\}$

However, $x \neq -8$ because lengths cannot be negative. Therefore,

$x = 3$

Answer C

13. The equation of a circle is:

$$(x-h)^2 + (y-k)^2 = r^2$$

where, (h, k) is the center of the circle, and r is the length of the radius.

In this problem, $h = -9$, $k = 2$, and $r = \sqrt{49} = 7$

Answer A

14. The center of this circle is at $(-2, 3)$ and its radius is $r = 4$. So the equation is:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x+2)^2 + (y-3)^2 = 16$$

Answer B

15. To get the perimeter, add the values for the five sides. I like to add vertically:

$2x - 3$
$x + 5$
$17x$
$x + 2$
$3x - 5$
$\overline{24x - 1}$

Answer A

16. Subtract the small rectangle's area from the large rectangle's area.

$$A_{walkway} = A_{large\ rectangle} - A_{small\ rectangle}$$

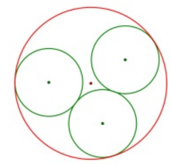
$$A_{walkway} = (5x-7)(8x-4) - (x^2+5x)$$

$$A_{walkway} = 40x^2 - 20x - 56x + 28 - x^2 - 5x$$

$$A_{walkway} = 39x^2 - 81x + 28 \ ft^2$$

Answer D

17. Insufficient information is given for this problem. So, let us assume that the centers of the circles are on the same line. This is the missing piece of information and is most likely what is intended for this problem. An alternative drawing, using the information given, is shown at right. This will clearly generate a different answer than the one intended by this problem.



Now, on with solving the problem the way the test intended. The radius of one small circle is $\frac{1}{6}$ of the diameter of the large circle. Then,

$$r_{small} = \frac{1}{6}\,(24x + 24) = 4x + 4$$

$$A_{small} = \pi r^2 = (4x+4)^2\pi = (16x^2 + 32x + 16)\pi \;\; in^2$$

Answer A

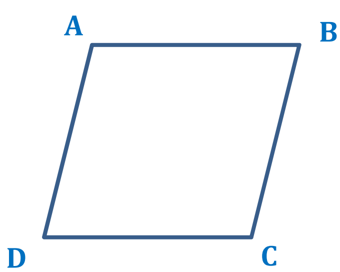
18. You might be tempted to calculate the height of the cone, but that is not necessary.

$$SA = \pi r^2 + \pi r l = (8x)^2\,\pi + (8x)(10x)\pi$$

$$= 64x^2\,\pi + 80x^2\pi = 144x^2\,\pi \;\; ft^2$$

Answer D

19. It's a good idea to draw the figure if it is not given to you.



$$m\angle A + m\angle B = 180^\circ$$

$$(3x + 6) + (2x + 14) = 180$$

$$5x + 20 = 180$$

$$-20 \quad -20$$

$$5x = 160$$

$$\div 5 \quad \div 5$$

$$x = 32$$

Then,

$$m\angle C = m\angle A$$

$$= (3x + 6)^\circ$$

$$= (3 \cdot (32) + 6)^\circ$$

$$= 102^\circ$$

Answer A

20. The length of GH, which is given in this problem, is not used.

$$m\angle G = m\angle J$$

$$46 = 3x + 10$$

$$-10 \quad -10$$

$$36 = 3x$$

$$\div 3 \quad \div 3$$

$$12 = x$$

Then,

$$HJ = FG$$

$$= (x + 7)$$

$$= (12 + 7)$$

$$= 19$$

Answer A

21. Set up simultaneous equations as follows:

$\boldsymbol{x = 6y}$ because opposite sides of a rectangle are congruent (i.e., have the same measure)

$\boldsymbol{2x + 8y + 10 = 90}$ because each angle of a rectangle is 90˚

Substitute $x = 6y$ into the second equation and calculate y.

$$\boldsymbol{2 \cdot (6y) + 8y + 10 = 90}$$

$$\begin{array}{lcr} 20y + 10 & = & 90 \\ -10 & & -10 \\ \hline 20y & = & 80 \\ \div 20 & & \div 20 \\ \hline y & = & 4 \end{array}$$

Finally, calculate x from the value of y based on the first equation.

$x = 6y$

$x = 6 \cdot 4 = 24$ **Answer B**

22. Consider each answer:

A. Three consecutive sides are congruent in a parallelogram. FALSE. Not even close.

B. Two pair of opposite sides are congruent in a parallelogram. TRUE.

C. Consecutive angles add to 180˚ in a parallelogram. TRUE.

D. The sums of the lengths of consecutive sides are equal in a parallelogram. TRUE.

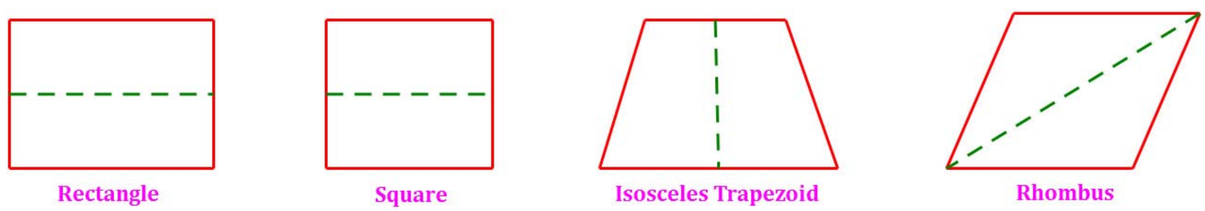
Answer A

23. Consider each answer:

Definitions:

Rotational symmetry - it is possible to rotate the image and get a result that looks the same. The *order* of a rotational symmetry is the number of positions the shape can take (within a 360˚ rotation) and look the same.

Line symmetry – it is possible to draw a line so that the image looks the same when reflected over the line. In the figures below, lines of symmetry are drawn as dashed green segments.



All of the figures have line symmetry, as shown above, regarding rotational symmetry:

A. A rectangle has rotational symmetry of order 2 (0˚ and 180˚ rotations).

B. A square has rotational symmetry of order 4 (0˚, 90˚, 180˚ and 270˚ rotations).

C. An isosceles trapezoid does not have rotational symmetry.

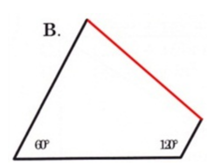
D. A rhombus has rotational symmetry of order 2 (0˚ and 180˚ rotations).

Answer C

24. Let's look at each of these.

A. A pair of consecutive angles adds up to 180˚, making the two horizontal segments parallel. The two 36˚ angles make the vertical segments parallel. A quadrilateral with two pair of parallel sides is a parallelogram.

B. I have drawn a red line segment to replace a portion of the diagram without violating the information given in the problem. Clearly, this is not a parallelogram.



C. The vertical sides are both parallel and congruent. The figure is, therefore, a parallelogram.

D. A quadrilateral with two pair of congruent sides is a parallelogram.

Answer B

25. To get the sum of the angles in terms of x, add the six values. I like to add vertically:

$$\begin{array}{r} x \\ 4x + 4 \\ 2x + 2 \\ 3x + 3 \\ x + 1 \\ 5x + 5 \\ \hline \mathbf{16x + 15} \end{array}$$

The sum of the angles in a polygon with n sides is given by the formula:

$$S = (n-2) \cdot 180^\circ$$

So, $S = (6-2) \cdot 180^\circ = 720^\circ$

Then, $\mathbf{16x + 15^\circ} = 720^\circ$

$$\begin{array}{rr} -15^\circ & -15^\circ \\ \hline 16x & = 705^\circ \\ \div 16 & \div 16 \\ \hline x \sim 44^\circ & \end{array}$$

Answer B

26. The sum of the exterior angles of any polygon is 360°.

In a regular pentagon, each exterior angle measures: $\frac{360^\circ}{5} = 72^\circ$.

Answer B

27. The sum of the angles in a polygon with n sides is given by the formula:

$$S = (n-2) \cdot 180^\circ$$

So, $S = (15-2) \cdot 180^\circ = 2{,}340^\circ$

Then, each angle in a regular 15-gon is:

$$\frac{2{,}340^\circ}{15} = 156^\circ$$

Answer D

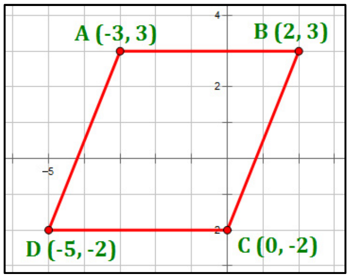
28. The sum of the exterior angles of any polygon is 360°.

For each exterior angle to be 45°, we have: $\frac{360^\circ}{n} = 45^\circ$.

So, $n = 8$ and the answer is an octagon.

Answer C

29. First draw it, then consider the lengths of the legs.



Leg lengths:

$$AB = 2 - (-3) = 5$$

$$CD = 0 - (-5) = 5$$

$$BC = \sqrt{(2-0)^2 + (3-(-2))^2} = \sqrt{29}$$

$$DA = \sqrt{(-3-(-5))^2 + (3-(-2))^2} = \sqrt{29}$$

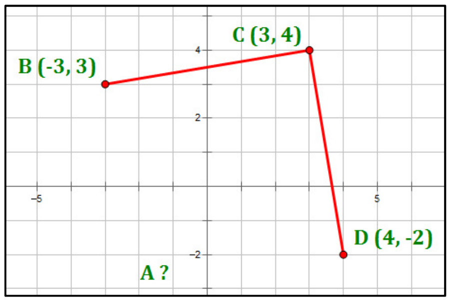
Therefore, the figure has two pair of opposite legs that are congruent, so the figure must be a parallelogram.

Answer D

Also, note:

- Since the legs are not all the same length, we do not have a rhombus.
- Since the angles are not 90˚, we do not have a rectangle or square.

30. First draw the information given, connect the points, and then find the location of A.



Consider the relative locations of the top two points:

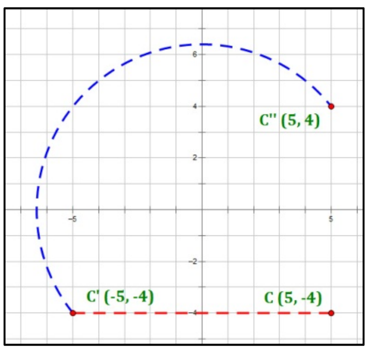
To get from C to B, you move left 6 and down 1.

Now, let's do that from D to get to A.

$$(4, -2) + \langle -6, -1 \rangle = (-2, -3)$$

Answer C

31. Do the required transformations on Point C only, since that is what the answer calls for.



The red segment reflects point C across the y-axis to point C'.

$$C(5, -4) \rightarrow C'(-5, -4)$$

The blue arc rotates C' through 180˚ to point C''.

$$C'(-5, -4) \rightarrow C''(5, 4)$$

Answer B

32. Look at the coordinates of the pre-image points and the image points:

$A(-2, 1)$ $A'(-6, 3)$

$B(1, 4)$ $B'(3, 12)$

$C(3, 2)$ $C'(9, 6)$

Note that the ratios of the coordinates of each image point are 3 times the coordinates of the corresponding pre-image point. Therefore, the scale factor is 3.

Answer D

Note: to do this problem, it is sufficient to divide one coordinate of one image point by the same coordinate in the corresponding pre-image point. For example, using C and C', we see that $6 \div 2 = 3$. Then, check a couple more coordinates to see if they have the same ratio. If so, you have found the scale factor.

33. Simply add 7 to the x-values and subtract 2 from the y-values of each vertex of the quadrilateral.

$A(1, -3)$	$B(2, -1)$	$C(5, -2)$	$D(3, -5)$
$+\langle 7, -2\rangle$	$+\langle 7, -2\rangle$	$+\langle 7, -2\rangle$	$+\langle 7, -2\rangle$
$A'(8, -5)$	$B'(9, -3)$	$C'(12, -4)$	$D'(10, -7)$

Answer D

34. A 90˚ clockwise rotation transforms $(a, b) \rightarrow (b, -a)$. Then add 3 to the resulting x-values and 2 to the resulting y-values.

Note: since the answers have different values for each point, it is sufficient to do the transformation of one point to obtain your answer. I have shown how to do all three points in order to illustrate the process if you get a problem with more similar answers on the final.

Starting points:	$G(-2, -3)$	$H(1, -5)$	$K(2, -1)$
90˚ clockwise rotation:	$G'(-3, 2)$	$H'(-5, -1)$	$K'(-1, -2)$
Translation:	$+\langle 3, 2\rangle$	$+\langle 3, 2\rangle$	$+\langle 3, 2\rangle$
Final Images:	$G''(0, 4)$	$H''(-2, 1)$	$K''(2, 0)$

Answer B

35. None of the information provided is relevant. All you need to see for this problem is that the shaded area is $\frac{1}{5}$ of the area of the entire pentagon. $P = \frac{1}{5}$.

Answer A

36. We need to calculate the areas of the square and the circle, and then divide.

The radius of the circle is 6, so the diagonal of the square is $2 \cdot 6 = 12$.

Note that a square is also a kite, so the area can be calculated using the kite formula:

$$A_{square} = \frac{1}{2} \cdot d_1 \cdot d_2 = \frac{1}{2} \cdot 12 \cdot 12 = 72$$

The area of the circle is based on your favorite formula:

$$A_{circle} = \pi r^2 = \pi \cdot 6^2 = 36\pi$$

The probability, then, is calculated as:

$$P = \frac{A_{square}}{A_{circle}} = \frac{72}{36\pi} = \frac{2}{\pi}$$

Answer D

37. We need to calculate the areas of the triangle and the trapezoid, and then divide.

Let "h" be the height of both the triangle and the trapezoid.

Then,

$$A_{triangle} = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 8 \cdot h = 4h$$

$$A_{trapezoid} = \frac{1}{2} \cdot (b_1 + b_2) \cdot h = \frac{1}{2} \cdot (23 + 8) \cdot h = 15.5h$$

The probability, then, is calculated as:

$$P = \frac{A_{triangle}}{A_{trapezoid}} = \frac{4h}{15.5h} = \frac{4}{15.5} = \frac{8}{31}$$

Answer B

38. We need to calculate the areas of the square and the circle, subtract and then divide.

The width of the square is 10, so the radius of the circle is 5. Then,

$$A_{square} = s^2 = 10^2 = 100$$

$$A_{circle} = \pi r^2 = \pi \cdot 5^2 = 25\pi$$

The probability, then, is calculated as:

$$P = \frac{A_{square} - A_{circle}}{A_{square}} = \frac{100 - 25\pi}{100} = \frac{100}{100} - \frac{25\pi}{100} = 1 - \frac{\pi}{4}$$

Answer C

39. Use the kite area formula and solve for DB.

Area Formula: $A = \frac{1}{2} \cdot d_1 \cdot d_2 = \frac{1}{2} \cdot AC \cdot DB$

Substitute values: $162x^2 = \frac{1}{2} \cdot 18x \cdot DB$

Multiply: $162x^2 = 9x \cdot DB$

Divide by $9x$: $\div 9x \quad \div 9x$

$18x = DB$

Answer A

40. Use the regular polygon area formula.

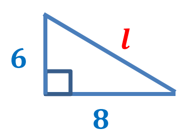
Area Formula: $A = \frac{1}{2} \cdot a \cdot P$

Substitute values: $A = \frac{1}{2} \cdot 10 \cdot \left(6 \cdot \frac{20\sqrt{3}}{3}\right) = 200\sqrt{3}\ cm^2$

Answer A

41. Calculate the slant height (l), then use the Surface Area formula.

Let's look at half of a cross section of the pyramid.



You might recognize this as a $3 - 4 - 5$ triangle with a multiplier of 2, giving a value of $l = 10$. If not, use the Pythagorean Theorem to calculate l.

Then the surface area is made up of four faces, each of whose area is:

$$A_{face} = \frac{1}{2} \cdot b \cdot l = \frac{1}{2} \cdot 16 \cdot 10 = 80\ ft^2$$

And one base with area:

$$A_{base} = b^2 = 16^2 = 256\ ft^2$$

So, then the total surface area is:

$$SA = A_{base} + 4 \cdot A_{face} = 256 + (4 \cdot 80)\ ft^2 = 576\ ft^2$$

Answer D

42. Use the formula for the surface area of a sphere.

$$SA = 4 \cdot \pi \cdot r^2 = 4 \cdot \pi \cdot 10^2 = 400\pi\ ft^2$$

Answer A

43. Use the formula for the volume of a cylinder:

$$V = \pi \cdot \mathbf{r^2} \cdot \mathbf{h} = \pi \cdot (\mathbf{3x-1})^2 \cdot (\mathbf{5x+4})$$

$$= \pi \cdot (\mathbf{9x^2 - 6x + 1}) \cdot (\mathbf{5x+4})$$

Multiplication of polynomials can be performed in columns.

$$\begin{array}{lrrr} & & \mathbf{9x^2} & \mathbf{-\ 6x + 1} \\ & & & \mathbf{5x + 4} \\ \hline \text{Product of 4 and } (9x^2-6x+1): & & 36x^2 & -\ 24x + 4 \\ \text{Product of } 5x \text{ and } (9x^2-6x+1): & 45x^3 & -\ 30x^2 & +\ 5x \\ \hline & 45x^3 & +\ 6x^2 & -\ 19x + 4 \end{array}$$

Result is: $V = (45x^3 + 6x^2 - 19x + 4)\pi\ cm^3$

Answer C

44. Use the formula for the volume of a pyramid:

$$V = \frac{1}{3} \cdot \boldsymbol{l} \cdot \mathbf{w} \cdot \mathbf{h} = \frac{1}{3} \cdot (\mathbf{3x+5}) \cdot (\mathbf{3x+5}) \cdot (\mathbf{6x+6})$$

$$= (\mathbf{9x^2 + 30x + 25}) \cdot (\mathbf{2x+2})$$

Multiplication of polynomials can be performed in columns.

$$\begin{array}{lrrrr} & & \mathbf{9x^2 +} & \mathbf{30x} & \mathbf{+ 25} \\ & & & \mathbf{2x +} & \mathbf{2} \\ \hline \text{Product of 2 and } (9x^2+30x+25): & & 18x^2 + & 60x & + 50 \\ \text{Product of } 2x \text{ and } (9x^2+30x+25): & 18x^3 & + 60x^2 + & 50x & \\ \hline & 18x^3 & + 78x^2 + & 110x & + 50 \end{array}$$

Result is: $V = (18x^3 + 78x^2 + 110x + 50)\ ft^3$

Answer C

45. Use the formula for the volume of a sphere.

$$V = \frac{4}{3} \cdot \pi \cdot r^3 = \frac{4}{3} \cdot \pi \cdot (3x)^3 = 36\pi x^3\ ft^2$$

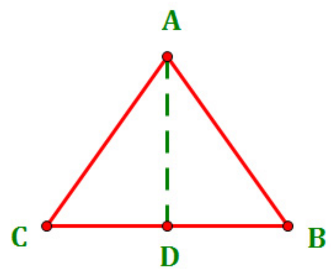
Answer D

46. Volume ratios are the cubes of the side length ratios.

$$Ratio = 5^3 : 8^3 = 125 : 512$$

Answer C

47. Draw a picture, and the answers can be evaluated by inspection.



ΔABC is equilateral

$A.\ \angle B \cong \angle C$ TRUE

$B.\ AB = DC$ FALSE

$C.\ AD = BC$ FALSE

$D.\ \angle ABC \cong \angle ADC$ FALSE

Answer A

Note: Answer A on the test states that $\angle B = \angle C$, which is FALSE. The angles are not equal; they are congruent. Their measures are equal, however, because they are both 60˚. I believe the test creator intended answer A to state $\angle B \cong \angle C$, because all of the other answers are FALSE.

48. Review the picture, and evaluate the answers by inspection.

$A.\ CD = AC$ FALSE

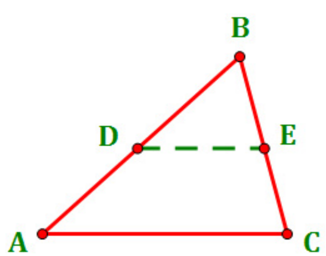
$B.\ \angle ABC \cong \angle ADC$ FALSE

$C.\ BD = DC$ TRUE, because the triangle is isosceles

$D.\ \angle C \cong \angle ADC$ FALSE

Answer C

49. Draw a picture, and the answers can be evaluated by inspection.



$A.\ \overline{DE} // \overline{AC}$ TRUE

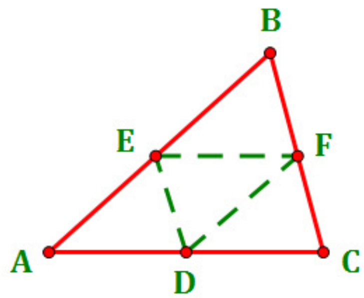
$B.\ DE = \frac{1}{2}(AC)$ TRUE

$C.\ \angle BAC \cong \angle BDE$ TRUE

$D.\ \angle C \cong \angle A$ FALSE

Answer D

50. Draw a picture, and use it to find the perimeter of ΔDFC. Note that there are four small congruent triangles in the illustration below.



$$DF = \frac{1}{2}AB = \frac{1}{2}(12x + 8) = \mathbf{6x + 4}$$

$$FC = DE = \mathbf{8x + 4}$$

$$CD = AD = \mathbf{10}$$

$$P = DF + FC + CD = (6x + 4) + (8x + 4) + 10$$

$$= 14x + 18$$

Answer B