

Illustration of Different Answers to Integration Problems

$$\int \tan x \sec^2 x \, dx$$

METHOD 1: setting $u = \tan x$

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

$$\int \tan x \sec^2 x \, dx = \int \tan x \sec^2 x \, dx = \int u \, du = \frac{1}{2} u^2 + C = \frac{1}{2} \tan^2 x + C$$

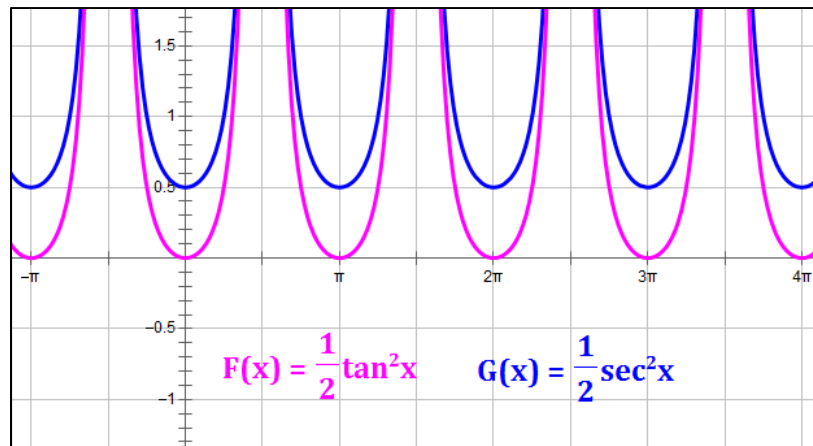
METHOD 2: setting $u = \sec x$

$$u = \sec x$$

$$du = \sec x \tan x \, dx$$

$$\int \tan x \sec^2 x \, dx = \int \sec x (\sec x \tan x \, dx) = \int u \, du = \frac{1}{2} u^2 + C = \frac{1}{2} \sec^2 x + C$$

Conclude: The two functions $F(x) = \frac{1}{2} \tan^2 x$ and $G(x) = \frac{1}{2} \sec^2 x$ have the same derivative and, therefore, the same slopes for all values of x . See the graphs of these two functions below. Note that they differ only by a constant (i.e., $\frac{1}{2}$). This is because from Trigonometry, we know: $\sec^2 x = \tan^2 x + 1$.



$$\int \sin x \cos x \, dx$$

METHOD 1: setting $u = \sin x$

$$u = \sin x$$

$$du = \cos x \, dx$$

$$\int \sin x \cos x \, dx = \int \sin x \cos x \, dx = \int u \, du = \frac{1}{2} u^2 + C = \frac{1}{2} \sin^2 x + C$$

METHOD 2: setting $u = \cos x$

$$u = \cos x$$

$$du = -\sin x \, dx$$

$$\int \sin x \cos x \, dx = -\int -\sin x \cos x \, dx = -\int u \, du = -\frac{1}{2} u^2 + C = -\frac{1}{2} \cos^2 x + C$$

METHOD 3: recalling that $\sin 2x = 2 \sin x \cos x$ and setting $u = 2x$

$$u = 2x$$

$$du = 2 \, dx, \quad dx = \frac{1}{2} \, du$$

$$\begin{aligned} \int \sin x \cos x \, dx &= \int \left(\frac{1}{2} \sin 2x \right) dx = \frac{1}{2} \int \sin 2x \, dx = \frac{1}{2} \int \sin u \cdot \frac{1}{2} \, du = \frac{1}{4} \int \sin u \, du \\ &= -\frac{1}{4} \cos u + C = -\frac{1}{4} \cos 2x + C \end{aligned}$$

