## Illustration of Different Answers to Integration Problems

$$\int \tan x \sec^2 x \, dx$$

**METHOD 1:** setting $u = \tan x$

$u = \tan x$

$du = \sec^2 x \; dx$

$$\int \tan x \sec^2 x \, dx = \int \tan x \sec^2 x \, dx \; = \int u \, du \; = \frac{1}{2} u^2 + C \; = \boldsymbol{\frac{1}{2} \tan^2 x + C}$$

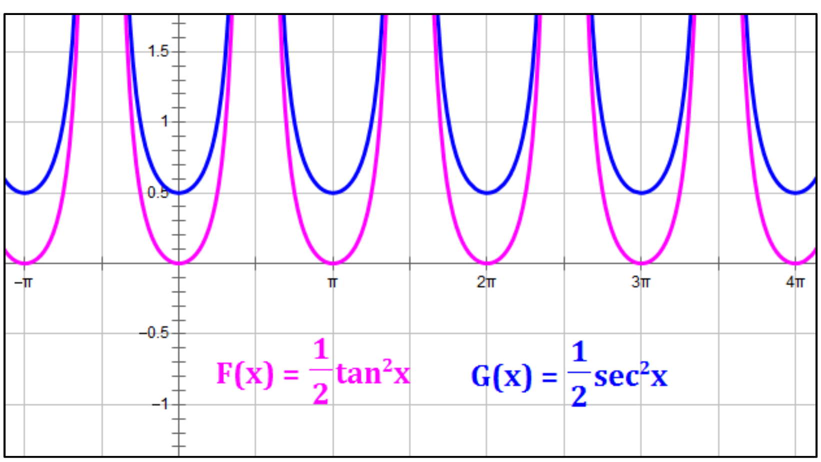
**METHOD 2:** setting $u = \sec x$

$u = \sec x$

$du = \sec x \tan x \; dx$

$$\int \tan x \sec^2 x \, dx = \int \sec x \; (\sec x \; \tan x \; dx) = \int u \, du \; = \frac{1}{2} u^2 + C \; = \boldsymbol{\frac{1}{2} \sec^2 x + C}$$

**Conclude:** The two functions $F(x) = \frac{1}{2}\tan^2 x$ and $G(x) = \frac{1}{2}\sec^2 x$ have the same derivative and, therefore, the same slopes for all values of $x$. See the graphs of these two functions below. Note that they differ only by a constant (i.e., $\frac{1}{2}$). This is because from Trigonometry, we know: $\sec^2 x = \tan^2 x + 1$.



$$\int \sin x \cos x \, dx$$

**METHOD 1:** setting $u = \sin x$

$u = \sin x$

$du = \cos x \, dx$

$$\int \sin x \cos x \, dx = \int \sin x \cos x \, dx \; = \int u \, du \; = \frac{1}{2} u^2 + C \; = \frac{\mathbf{1}}{\mathbf{2}} \mathbf{\sin^2 x + C}$$

**METHOD 2:** setting $u = \cos x$

$u = \cos x$

$du = -\sin x \, dx$

$$\int \sin x \cos x \, dx = -\int -\sin x \cos x \, dx = -\int u \, du \; = -\frac{1}{2} u^2 + C \; = -\frac{\mathbf{1}}{\mathbf{2}} \mathbf{\cos^2 x + C}$$

**METHOD 3:** recalling that $\sin 2x = 2 \sin x \cos x$ and setting $u = 2x$

$u = 2x$

$du = 2dx, \;\; dx = \frac{1}{2} du$

$$\int \sin x \cos x \, dx = \int \left(\frac{1}{2} \sin 2x\right) dx = \frac{1}{2} \int \sin 2x \, dx = \frac{1}{2} \int \sin u \cdot \frac{1}{2} du \; = \frac{1}{4} \int \sin u \, du$$

$$= -\frac{1}{4} \cos u + C = -\frac{\mathbf{1}}{\mathbf{4}} \mathbf{\cos 2x + C}$$