2013-2014
Algebra 2 Semester 1

Instructional Materials for the WCSD Math Common Finals

The Instructional Materials are for student and teacher use and are aligned to the Math Common Final blueprint for this course. When used as test practice, success on the Instructional Materials does not guarantee success on the district math common final.

Students can use these Instructional Materials to become familiar with the format and language used on the district common finals. Familiarity with standards vocabulary and interaction with the types of problems included in the Instructional Materials can result in less anxiety on the part of the students.

Teachers can use the Instructional Materials in conjunction with the course guides to ensure that instruction and content is aligned with what will be assessed. <u>The Instructional Materials are not representative of the depth or full range of learning that should occur in the classroom.</u>

Note from Earl. Many of the solutions in this document use techniques presented in the Algebra Handbook, which is available on the www.mathguy.us website. If you have trouble following any of the techniques used, try looking in the handbook for pages that deal with the issue you are struggling with.

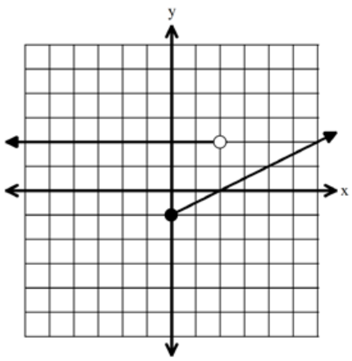
I solve the problems in this test using the quickest method available in most cases. Occasionally, I also make comments about some of the math involved in an effort to enhance your understanding of what is going on in the problem.

Multiple Choice: *Identify the choice that best completes the statement or answers the question.*

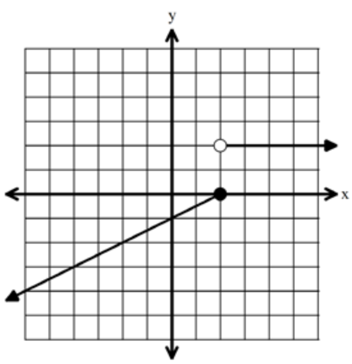
1. What graph represents the piecewise function?

$$f(x) = \begin{cases} \frac{1}{2}x - 1, & x \geq 2 \\ 2, & x < 2 \end{cases}$$

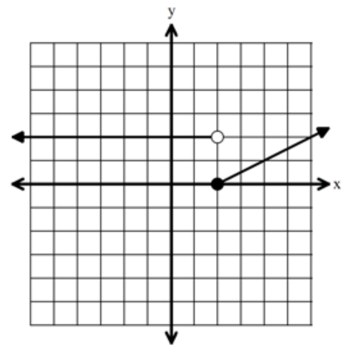
A.



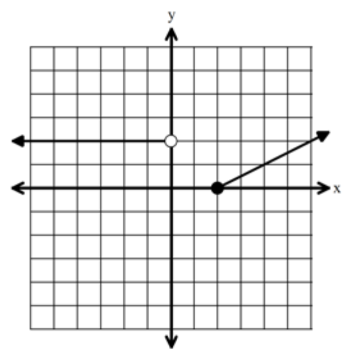
B.



C.



D.



First, note that the breakpoint for the function occurs at $x = 2$. In terms of the above graphs, this leaves out **A** (which is not a function) and **D** (which has two breaks, at $x = 0$ and $x = 2$).

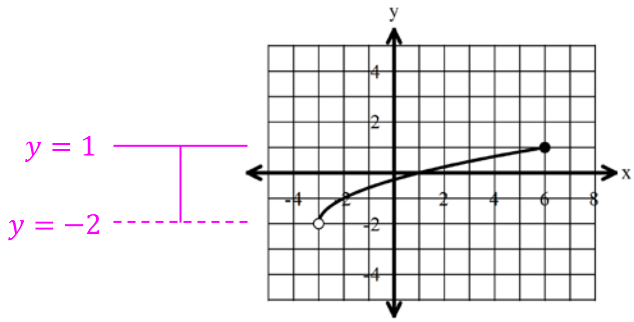
Next, notice that following about $f(x)$:

- The piece of $f(x)$ that is flat (since $f(x) = 2$) is the piece to the left of $x = 2$.
- The piece of $f(x)$ with a slope $m = \frac{1}{2}$ (since $f(x) = \frac{1}{2}x - 1$) is the piece to the right of $x = 2$.

Of the remaining answers, **B** and **C**, this occurs in **C** and not in **B**.

Answer C

2. Which of the following is a proper description of the range of the function shown:



A. $R: \{y|-3 < y \leq 6\}$

B. $(-3, 6)$

C. $R: \{y|-2 \leq y \leq 1\}$

D. $(-2, 1]$

Recall the following definitons:

- The **Domain** of a function is the set of all values of x for which it is possible to calculate a y-value.
- The **Range** is the set of y-values that function takes on for all x in the function's Domain.

To show the range, I like to locate a function's bottom and/or top y-values and draw a line between them off to the side of the graph. This in indicated by the vertical lines in the figure above. (Note: if the range is unlimited in either the up or down direction, I would show an arrow in that direction instead of a line.)

Notice that I drew a solid magenta line for $y = 1$, indicating that there is a closed dot at the point $(6, 1)$. I drew a dashed magenta line for $y = -2$, indicating that there is an open dot at the point $(-3, -2)$.

Based on this, the range is the set of all real numbers from -2 to 1, excluding -2 because of the open dot at the point $(-3, -2)$.

- In **set notation**, the range would be shown as: $\boldsymbol{R}: \{\boldsymbol{y}|-\mathbf{2} < \boldsymbol{y} \leq \mathbf{1}\}$
- In **interval notation**, the range would be shown as: $(-\mathbf{2}, \mathbf{1}]$

Answer D

3. Three students were chosen to show their solutions for solving the equation $y = a(x - h) + k$ for x. Their work is shown below. Determine which students were correct.

Student #1	Student #2	Student #3
$y = a(x-h)+k$	$y = a(x-h)+k$	$y = a(x-h)+k$
$y - k = a(x-h)$	$\frac{y}{a} = (x-h) + k$	$\frac{y}{a} = (x-h) + \frac{k}{a}$
$\frac{(y-k)}{a} = x - h$	$\frac{y}{a} - k = x - h$	$\frac{y}{a} - \frac{k}{a} = x - h$
$\frac{(y-k)}{a} + h = x$	$\frac{y}{a} - k + h = x$	$\frac{y}{a} - \frac{k}{a} + h = x$

A. Student #1 and Student #2

B. Student #2 and Student #3

C. Student #1 and Student #3

D. All students were correct

Student 1 is correct.

Student 2 forgot to divide the constant $\boldsymbol{k}$ by $\boldsymbol{a}$ in step 2.

Student 3 is correct. (Note that the solution shown could be simplified to that shown for Student 1.)

Answer C

4. Which of the following is NOT an equivalent form of the line represented in the table:

x	y
-2	-4
-1	-1
0	2
1	5

Differences

3

3

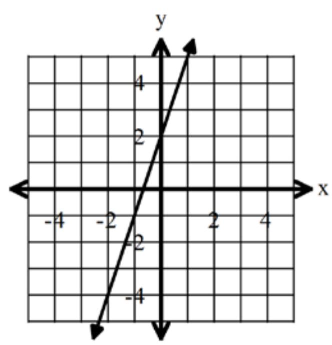
3

A. $y = 2x + 3$

B. $y - 2 = 3(x - 0)$

C. $y = 3(x - 0) + 2$

D.



The x-values in the table increment by 1. The differences in the y-values are all 3.

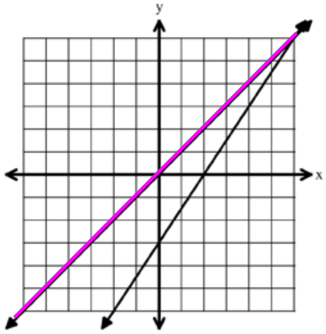
Any time the x-values increment by 1 and there is a constant difference in y-values, the table represents a linear function whose slope is equal to the constant difference in y-values.

All of the answers above have a slope of 3 except for **A**.

Answer A

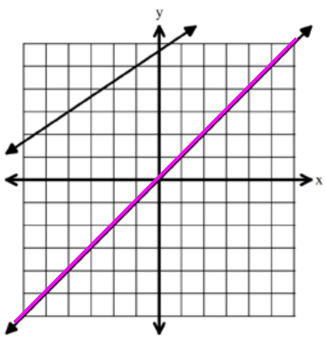
5. Graph the line $y = \frac{2}{3}(x-4)+3$ with its parent function. Then tell how the function is transformed from the parent function

A.



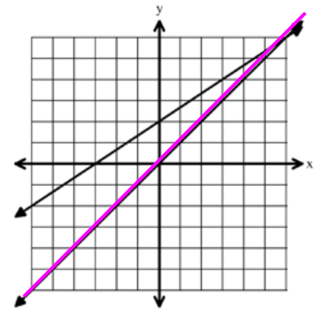
The graph is shifted right 4 units and up 3 units. The slope makes the graph steeper than the parent function.

B.



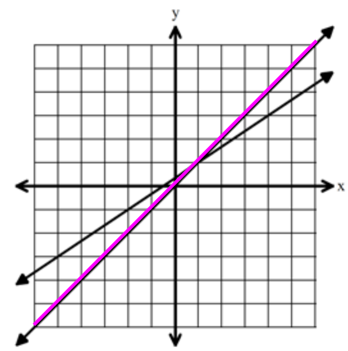
The graph is shifted left 4 units and up 3 units. The slope makes the graph less steep than the parent function.

C.



The graph is shifted up 4 units and right 3 units. The slope makes the graph less steep than the parent function.

D



The graph is shifted right 4 units and up 3 units. The slope makes the graph less steep than the parent function.

The equation given, $y = \frac{2}{3}(x-4)+3$, is in point slope form.

Point-slope form is $y = m(x-x_0)+y_0$ where (x_0, y_0) is a point on the line.

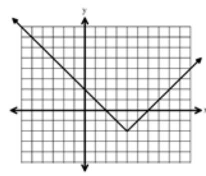
Note that the parent function, $y = x$, is given correctly in all four graphs. So, this problem boils sown to finding the line with slope $m = \frac{2}{3}$ that goes through the point $(4, 3)$.

- The black lines in both **A** and **D** go through the point $(4, 3)$.
- In **A,** $m > 1$ (it is steeper than the parent function).
- In **D,** $m < 1$ (it is less steep than the parent function).

The slope we want is $m = \frac{2}{3}$, which is < 1. **Answer D**

6. Which equation is obtained after the translation of the graph up 2 units and left 6 units?

   A. $f(x) = |x - 2|$
   B. $f(x) = |x| - 2$
   C. $f(x) = |x + 2|$
   D. $f(x) = |x| + 2$



The general form of an absolute value function is: $y = a|x - h| + k$, where (h, k) is the vertex of the function.

The function shown has a vertex at $(4, -2)$, and a slope of $a = 1$, so it's form would be: $y = |x - 4| - 2$.

Now, let's translate it.

- Translation left 6 units means "subtract 6 from h." So, our new $h = 4 - 6 = -2$.
- Translation up 2 units means "add 2 to k." So, our new $k = -2 + 2 = 0$.

These translations generate the equation $y = |x - (-2)| + 0$, or $\boldsymbol{y = |x + 2|}$

Answer C

7. Given $f(x) = \frac{5}{4}(x - 4)^3 + 2$, identify the name of the parent function and describe how the graph is transformed from the parent function.

   A. Rational Function with a vertical stretch, translated down 4 units and right 2 units
   B. Cubic Function with a vertical stretch, translated right 4 units and up 2 units
   C. Quadratic Function with a vertical compression, translated left 4 units and up 2 units
   D. Linear Function with a vertical compression, translated left 4 units and up 2 units

Note the exponent of 3 on the x-term, which is enough to tell us that the **answer is B**.

Let's go on anyway. One form of a cubic function is: $y = a(x - h)^3 + k$, where (h, k) is a point on the curve. (h, k) also defines the translation from the parent function:

- h is the number of units the function is translated to the right (a negative value of h means the translation is to the left).
- k is the number of units the function is translated up (a negative value of k means the translation is down).

The value of m determines whether there is a compression $(|a| < 1)$ or a stretch $(|a| > 1)$.

So, the equation given has a stretch $\left(|a| = \frac{5}{4} > 1\right)$, and translation 4 units right $(h = 4)$ and 2 units up $(k = 2)$.

Answer B

8. Which of following functions does NOT represent the parabola with a vertex at $(1, 4)$ and x-intercepts $(-1, 0)$ and $(3, 0)$.

A. $f(x) = -x^2 + x + 4$

B. $f(x) = -(x-1)^2 + 4$

C. $f(x) = -x^2 + 2x + 3$

D. $f(x) = -(x+1)(x-3)$

Let's look at the answers:

A. is a quadratic in general form.

B. is a quadratic with vertex $(1, 4)$. Recall that vertex form is $y = a(x-h)^2 + k$, where (h, k) is the vertex of the function. This matches what we are given about $f(x)$.

C. is a quadratic in general form.

D. is a quadratic with roots $\{-1, 3\}$. Recall that intercept form is $y = a(x - r_1)(x - r_2)$, where r_1 and r_2 are the intercepts of $f(x)$. This matches what we are given about $f(x)$.

So, **B** and **D** do represent the parabola in question.

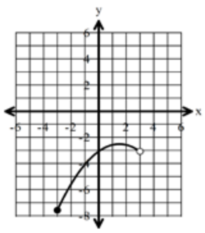
It remains to see which of **A** or **C** does not represent the parabola. Since answer **D** is representative of our function, let's multiply it out and see what we get. You can do this with either the FOIL Method or the Box Method, whichever you are more comfortable with. I will use FOIL.

$$f(x) = -(x+1)(x-3) = -(x^2 - 3x + x - 3) = -x^2 + 2x + 3 \quad \text{(which is answer C)}$$

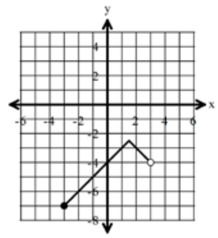
So, the equation that does **NOT** represent the parabola defined above is **A**. **Answer A**

9. Which of the following is the graph of $f(x) = -\left|x - \frac{3}{2}\right| - \frac{5}{2}$ over the domain $[-3, 3)$

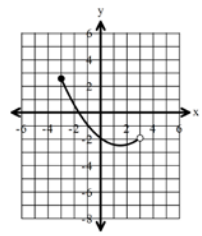
A.



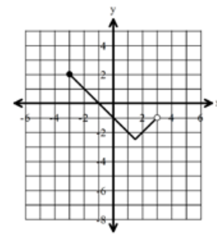
B.



C.



D.



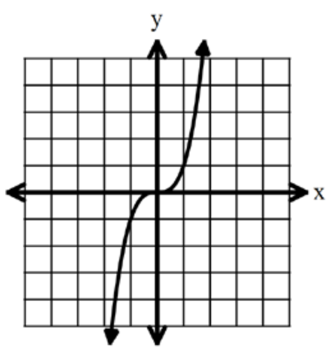
The general form of an absolute value function is: $y = a|x - h| + k$, where (h, k) is the vertex of the function. It has a "V-shape," not a "U-shape."

When $a > 0$, the function has a regular V-shape. When $a < 0$, the function has an inverted V-shape. For this problem, $a = -1 < 0$, so the function has an inverted V-shape.

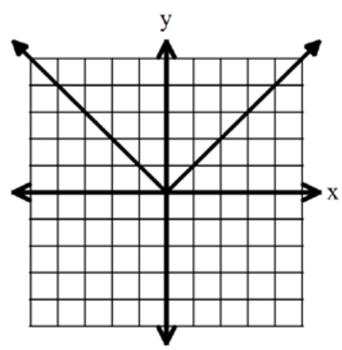
Answer B

10. Identify each graph as being a quadratic function, absolute value function, cubic function, or rational function.

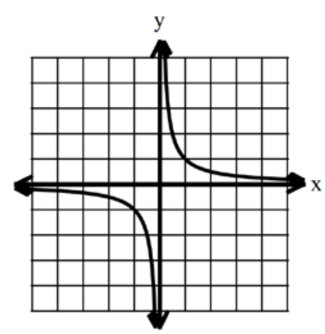
Graph A



Graph B



Graph C



A. Graph A: Absolute Value
Graph B: Quadratic
Graph C: Cubic

B. Graph A: Cubic
Graph B: Quadratic
Graph C: Rational

C. Graph A: Rational
Graph B: Absolute Value
Graph C: Cubic

D. Graph A: Cubic
Graph B: Absolute Value
Graph C: Rational

There is not much to say here, except for the forms represented in the graphs. (Incidentally, it looks like all of the graphs shown are parent functions, so I will identify what each one is.)

A. is a cubic function: $y = x^3$

B. is an absolute value function: $y = |x|$

C. is a rational function: $y = \frac{1}{x}$

Answer D

You will need the quadratic formula for the next couple of questions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{provides solutions to the equation:} \quad ax^2 + bx + c = 0$$

11. What are the solutions to the quadratic equation, $3x^2 + 7x + 11 = 5x + 7$?

A. $x = \dfrac{-2 \pm 2i\sqrt{11}}{3}$

C. $x = \dfrac{-1 \pm i\sqrt{11}}{3}$

B. $x = \dfrac{-1 \pm 2i\sqrt{11}}{3}$

D. $x = \dfrac{\pm i\sqrt{11}}{3}$

First, combine all terms on one side of the equation.

Original Equation:	$3x^2 + 7x + 11 = \ 5x + 7$
Subtract $(5x + 7)$:	$-5x - \ 7 \quad -5x - 7$
Result:	$3x^2 + 2x + \ 4 = \ 0$

Next, use the quadratic formula with $a = 3$, $b = 2$, $c = 4$

$$x = \frac{-(2) \pm \sqrt{2^2 - 4(3)(4)}}{2(3)} = \frac{-2 \pm \sqrt{-44}}{6} = \frac{-2 \pm 2i\sqrt{11}}{6} = \frac{-1 \pm i\sqrt{11}}{3}$$

Answer C

12. What are the solutions to the quadratic equation, $y^2 + 2y = 9 + 5y$?

A. $y = \dfrac{3 \pm 3i\sqrt{3}}{2}$

C. $y = \dfrac{3 \pm 3i\sqrt{5}}{2}$

B. $y = \dfrac{-3 \pm 3\sqrt{5}}{2}$

D. $y = \dfrac{3 \pm 3\sqrt{5}}{2}$

First, combine all terms on one side of the equation.

Original Equation:	$y^2 + 2y \quad = \ 9 + 5y$
Subtract $(5y + 9)$:	$-5y - 9 \quad -9 - 5y$
Result:	$y^2 - 3y - 9 = \ 0$

Next, use the quadratic formula with $a = 1$, $b = -3$, $c = -9$

$$y = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)} = \frac{3 \pm \sqrt{45}}{2} = \frac{3 \pm 3\sqrt{5}}{2}$$

Answer D

13. Simplify: $\sqrt{-25}\cdot\sqrt{-36}$

A. $30i$

B. -30

C. 30

D. 900

$\sqrt{-25}\cdot\sqrt{-36} = 5i\cdot 6i = 30i^2 = 30\cdot(-1) = -30$

Answer B

14. Simplify: $(11+i)+(3-15i)$

A. $14-14i$

B. $-4+4i$

C. $12-12i$

D. $14+16i$

For addition of complex numbers, I like to line things up vertically.

$$\begin{array}{rr} 11 + & i \\ 3 - & 15i \\ \hline \mathbf{14} - & \mathbf{14i} \end{array}$$

Answer A

15. Simplify: $3i(6-5i)-4(2+3i)$

A. $23+6i$

B. $-8-9i$

C. $8-9i$

D. $7+6i$

$3i(6-5i)-4(2+3i)$

$=(18i-15i^2)-(8+12i)$

$=(15+18i)+(-8-12i)$

$$\begin{array}{rr} 15 + & 18i \\ -8 - & 12i \\ \hline \mathbf{7} + & \mathbf{6i} \end{array}$$

Answer D

16. Simplify: $(3-7i)^2$

A. $-40+0i$

B. $-40-42i$

C. $58+0i$

D. $58-42i$

$(3-7i)\cdot(3-7i)$

F: $3\cdot 3 = 9$

O: $3\cdot(-7i) = -21i$

I: $-7i\cdot 3 = -21i$

L: $-7i\cdot(-7i) = 49i^2 = -49$

Result: $(9-49)+(-21i-21i)$

$= \mathbf{-40-42i}$ **Answer B**

17. Simplify: $(4-5i)(4+5i)$

A. -9
B. 41
C. $16+25i$
D. $16-25i$

$(4-5i)\cdot(4+5i)$

F: $4\cdot 4=16$

O: $4\cdot 5i=20i$

I: $-5i\cdot 4=-20i$

L: $-5i\cdot 5i=-25i^2=25$

Result: $(16+25)+(20i-20i)$

$=41$ **Answer B**

18. Simplify: $\frac{6+2i}{2-i}$

A. $2-3i$
B. $2+3i$
C. $2+2i$
D. $3+2i$

$$=\frac{6+2i}{2-i}\cdot\frac{2+i}{2+i}$$

Multiply by the conjugate of the complex number in the denominator.

$$=\frac{6\cdot 2+6\cdot i+(2i)\cdot 2+(2i)\cdot i}{2\cdot 2+2\cdot i+(-i)\cdot 2+(-i)\cdot i}$$

$$=\frac{12+6i+4i+2i^2}{4+2i-2i-i^2}$$

$$=\frac{12-2+10i}{4+0+1}$$

$$=\frac{10+10i}{5}=2+2i$$

Answer C

19. Given the function, $f(x)=-(x-4)^2-3$, state whether the parabola opens up or down and the maximum or minimum value.

A. Opens down, Maximum value is -3
B. Opens down, Maximum value is 4
C. Opens up, Minimum value is 4
D. Opens up, Minimum value is -3

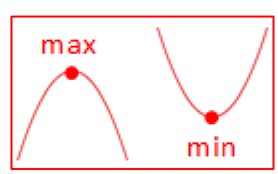
This parabola is in vertex form, which is: $y=a(x-h)^2+k$, where (h,k) is the vertex of the function. It has a "U-shape".

- When $a>0$, the function opens up.
- When $a<0$, the function opens down.

For this problem, $a<0$, so it opens down.

The y-value of the vertex (h,k) is of maximum or minimum of the function.

- When the function opens up, the function has a minimum equal to k.
- When the function opens down, the function has a maximum equal to k.



This function opens down, so it has a maximum equal to $k=-3$. **Answer A**

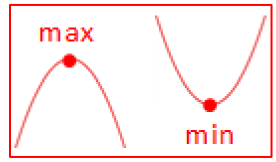
20. Given the function, $f(x) = x^2 + 2x + 7$, state whether the parabola opens up or down and the maximum or minimum value.

A. Opens up, Minimum value is 7

B. Opens up, Minimum value is 6

C. Opens down, Maximum value is 7

D. Opens down, Maximum value is 6



For a quadratic function in general form, $ax^2 + bx + c = 0$, the lead coefficient (i.e., the value of a) determines whether the function opens up or down.

- When $a > 0$, the function opens up and will have a minimum.
- When $a < 0$, the function opens down and will have a maximum.

For this function, $a = 1 > 0$, so the function opens up and will have a minimum.

The easiest way to determine the vertex of this function is to complete the square:

$$\begin{aligned} x^2 + 2x + 7 &= (x^2 + 2x + \_\_) \quad + 7 - \_\_ \\ &= (x^2 + 2x + \mathbf{1}) \quad + 7 - \mathbf{1} \\ &= (x+1)^2 + 6 \end{aligned}$$

which has vertex $(h, k) = (-1, 6)$

To get the missing constant term, divide the coefficient of x by 2, then square the result. In this case, $\left(\frac{2}{2}\right)^2 = \mathbf{1}$

The value of the minimum is $k = 6$. **Answer B**

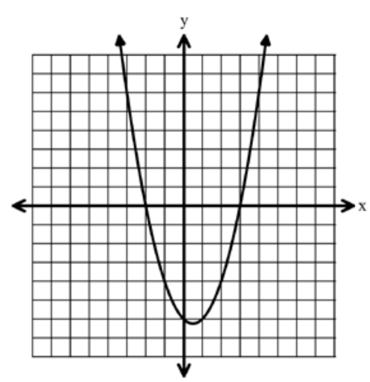
21. Which function is represented by the graph?

A. $f(x) = (x-2)(x+3)$

B. $f(x) = (x-2)(x-3)$

C. $f(x) = (x+2)(x-3)$

D. $f(x) = (x+2)(x+3)$



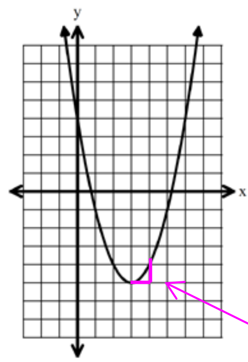
These equations are in intercept form, which is $y = a(x - r_1)(x - r_2)$, where r_1 and r_2 are the intercepts of the function.

This function has intercepts at $x = \{-2, 3\}$.
Note that all of the answers have $a = 1$.

The equation, then, is $y = 1(x - (-2))(x - 3) = (x + 2)(x - 3)$ **Answer C**

22. Which equation is represented by the graph?

A. $y = (x-3)^2 - 5$
B. $y = 2(x-3)^2 - 5$
C. $y = 2(x+3)^2 - 5$
D. $y = -(x+3)^2 - 5$



These equations are in vertex form, which is $y = a(x-h)^2 + k$, where (h, k) is the vertex of the function.

This function's vertex is $(3, 5)$, so its form is $y = a(x-3)^2 - 5$.

Both **A** and **B** have this form, so we need to determine the value of a. You can do this algebraically if you like, but an easy way to determine a on a graph is to move left or right one unit from the vertex and see how many units the curve moves up or down. The amount of movement up or down is the value of a.

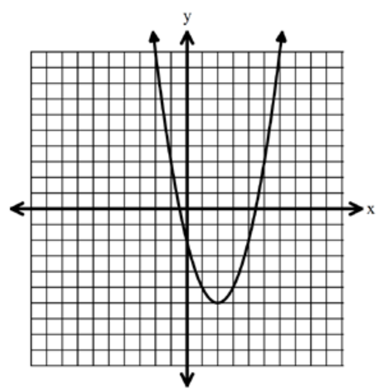
For this problem if we move left or right one unit, we also move up one unit, so $a = 1$.

The equation, then, is $y = 1(x-3)^2 - 5 = (x-3)^2 - 5$

Answer A

23. Which description explains how the graph of $f(x) = x^2 - 4x + 4$ is related to the graph of $g(x) = x^2 - 4x - 2$ shown here?

A. $f(x)$ is vertically stretched to make $g(x)$
B. $f(x)$ is translated down 6 units to make $g(x)$
C. $f(x)$ is translated 6 units left to make $g(x)$
D. $f(x)$ is compressed vertically to make $g(x)$



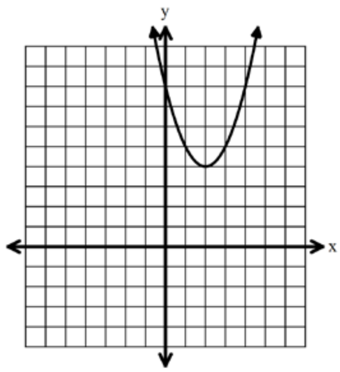
The lead coefficient of both functions is 1. Therefore, there is no stretch or compression.

Note that $g(x)$ is the same as $f(x)$, except for the constant term, which is 6 lower $((-2) - 4 = -6)$. A constant term which is 6 lower implies a downward translation of 6 units.

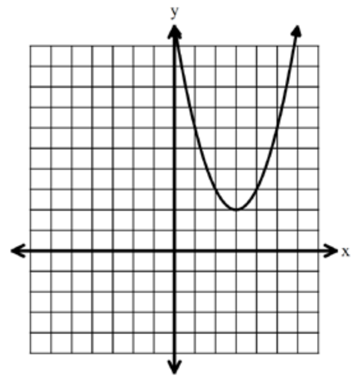
Answer B

24. Translate $y = x^2 - 4x + 6$ five (5) units to the left. What is the graph obtained after the translation?

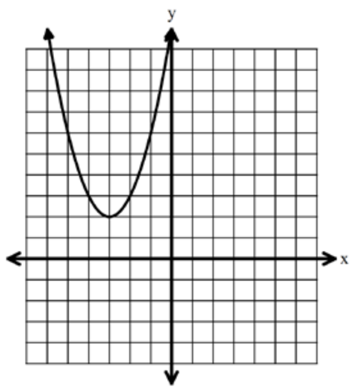
A.



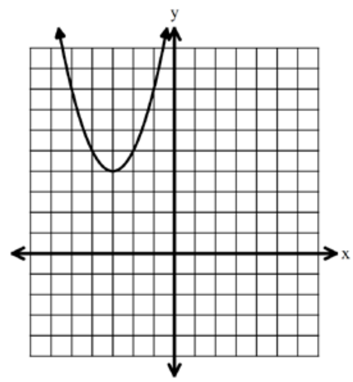
C.



B.



D.



Notice that the vertices in the graphs are at four different locations. So, if we can find the vertex after the translation, we have our answer.

The easiest way to determine the vertex of this function is to complete the square:

$$x^2 - 4x + 6 = (x^2 - 4x + \_\_) \quad + 6 - \_\_$$
$$= (x^2 - 4x + \mathbf{4}) \quad + 6 - \mathbf{4}$$
$$= (x - 2)^2 + 2$$

which has vertex $(h, k) = (2, 2)$

To get the missing constant term, divide the coefficient of x by 2, then square the result. In this case, $\left(\frac{-4}{2}\right)^2 = \mathbf{4}$

Finally, we need to translate the vertex 5 units to the left. Our new vertex becomes:

$(2 - 5, 2) = (\mathbf{-3}, \mathbf{2})$ **Answer B**

25. What is the end behavior for the function, $f(x) = (x^2 + 4x - 3)(3x^5 + 6x^3)$?

A. as $x \to -\infty, f(x) \to -\infty$ and as $x \to +\infty, f(x) \to +\infty$

B. as $x \to -\infty, f(x) \to +\infty$ and as $x \to +\infty, f(x) \to +\infty$

C. as $x \to -\infty, f(x) \to -\infty$ and as $x \to +\infty, f(x) \to -\infty$

D. as $x \to -\infty, f(x) \to +\infty$ and as $x \to +\infty, f(x) \to -\infty$

End Behavior of Polynomials		
Degree of Lead Term	**Lead Coefficient +**	**Lead Coefficient −**
Odd (e.g., $y = x, y = x^3$)	as $x \to -\infty,\ f(x) \to -\infty$ as $x \to +\infty,\ f(x) \to +\infty$	as $x \to -\infty,\ f(x) \to +\infty$ as $x \to +\infty,\ f(x) \to -\infty$
Even (e.g., $y = x^2, y = x^4$)	as $x \to -\infty,\ f(x) \to +\infty$ as $x \to +\infty,\ f(x) \to +\infty$	as $x \to -\infty,\ f(x) \to -\infty$ as $x \to +\infty,\ f(x) \to -\infty$

End behavior depends only on the lead term of a polynomial, so we can throw out the rest. For this problem, the lead term is: $(x^2) \cdot (3x^5) = 3x^7$.

The lead term has a **positive coefficient** and an **odd exponent**, so it will behave like the line $y = x$. Therefore, as $x \to -\infty,\ f(x) \to -\infty$**,** and as $x \to +\infty,\ f(x) \to +\infty$**.**

Answer A

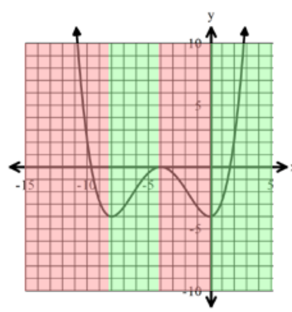
26. State where the function is increasing and decreasing.

A. Never Increasing
Decreasing: $(-\infty, +\infty)$

B. Increasing: $(-8, 0) \cap (0, +\infty)$
Decreasing: $(-\infty, -8)$

C. Increasing: $(-\infty, -8) \cup (0, 8)$
Decreasing: $(-8, 0)$

D. Increasing: $(-8, -4) \cup (0, +\infty)$
Decreasing: $(-\infty, -8) \cup (-4, 0)$



Follow the curve from left to right, and identify the x-values where the curve is increasing (going up). This curve increases from $x = -8$ to -4 and from $x = 0$ to $+\infty$. In interval notation, we write both of these intervals with a **"union"** sign between them:

The curve increases over the intervals: $(\mathbf{-8}, \mathbf{-4}) \cup (\mathbf{0}, +\infty)$**.**

Similarly, the curve decreases over the intervals: $(-\infty, \mathbf{-8}) \cup (\mathbf{-4}, \mathbf{0})$**.**

Answer D

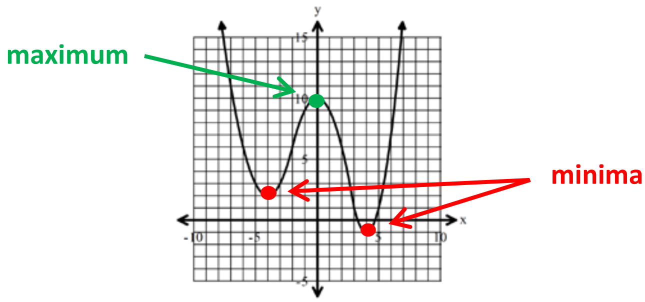
27. What are the values of the relative maxima and/or minima of the function graphed?

A. relative maxima: 0
relative minima: $-4, 4$

B. relative maxima: 10
relative minima: $-1, 2$

C. relative maxima: $3.3, 4.7$
relative minima: 0

D. relative maxima: $2, 10$
relative minima: -1



Relative maxima occur at any points where the curve "tops out" before decreasing again.

Relative minima occur at any points where the curve "bottoms out" before increasing again.

Based on these descriptions, we find one relative maximum at $(\mathbf{0}, \mathbf{10})$. We find two relative minima at $(\mathbf{-4}, \mathbf{2})$ and $(\mathbf{4}, \mathbf{-1})$. We report relative maxima and minima by their y-values:

Relative maximum $= 10$. Relative minima $= 2, -1$. **Answer B**

28. Factor: $x^4 - 13x^2 + 36$

A. $(x^2 - 9)(x^2 + 4)$

B. $(x - 3)(x + 3)(x + 4)(x - 4)$

C. $(x - 3)(x + 3)(x + 2)^2$

D. $(x - 3)(x + 3)(x + 2)(x - 2)$

Memorize the difference of squares formula!

$$a^2 - b^2 = (a + b)\ (a - b)$$

Then,

$$x^4 - 13x^2 + 36$$
$$= (x^2 - 4)(x^2 - 9)$$
$$= (x + 2)(x - 2)(x + 3)(x - 3)$$

Answer D

29. Factor: $64x^3 - 27$

A. $(4x - 3)(4x^2 + 12x + 9)$

B. $(4x - 3)(4x^2 + 12x - 9)$

C. $(4x - 3)(16x^2 + 12x + 9)$

D. $(4x - 3)(16x^2 - 12x - 9)$

This is a difference of cubes. Here are the formulas for the sum and difference of cubes. Memorize them!

$$a^3 + b^3 = (a + b)\ (a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)\ (a^2 + ab + b^2)$$

Then,

$$64x^3 - 27 = (4x)^3 - 3^3 \quad \text{Let: } a = 4x,\ b = 3.$$
$$= (4x - 3)\ (16x^2 + 4x \cdot 3 + 3^2)$$
$$= (\mathbf{4x + 3})\ (\mathbf{16x^2 + 12x + 9})$$

Answer C

Note: This problem could be solved simply by checking the lead coefficients and knowing the patterns in the formulas for sums and differences of cubes.

30. Solve: $3m^3 - 2m^2 - 5m = 0$

A. $m = -1, m = 0, m = \frac{5}{3}$ B. $m = -3, m = 0, m = 5$

C. $m = -\frac{5}{3}, m = 0, m = 1$ D. $m = -5, m = 0, m = 3$

Starting Equation: $3m^3 - 2m^2 - 5m = 0$

Factor out m: $m(3m^2 - 2m - 5) = 0$

Factor the remaining trinomial: $m(3m-5)(m+1) = 0$

Use the **AC Method** if you need to.

Break into separate equations: $m = 0$ $3m - 5 = 0$ $m + 1 = 0$

Identify solutions: $m = \left\{0, \frac{5}{3}, -1\right\}$ **Answer A**

31. Solve: $x^4 - 3x^2 = 10$

A. $x = 5, x = -2$ B. $x = \pm 5, x = \pm 2i$

C. $x = \pm\sqrt{5}, x = \pm\sqrt{2}$ D. $x = \pm\sqrt{5}, x = \pm i\sqrt{2}$

Starting Equation: $x^4 - 3x^2 = 10$

Subtract 10: $-10 \quad -10$

Result: $x^4 - 3x^2 - 10 = 0$

Factor the trinomial: $(x^2 - 5)(x^2 + 2) = 0$

Break into separate equations: $x^2 - 5 = 0$ $x^2 + 2 = 0$

Manipulate each equation: $x^2 = 5$ $x^2 = -2$

Identify solutions: $x = \{\pm\sqrt{5}, \pm i\sqrt{2}\}$ **Answer D**

32. Solve: $x^4 - 36 = 0$

A. $x = \pm\sqrt{6}, x = \pm 6i$ B. $x = \pm\sqrt{6}, x = \pm i\sqrt{6}$

C. $x = \pm 6, x = \pm 6i$ D. $x = \pm 6, x = \pm i\sqrt{6}$

Starting Equation: $x^4 - 36 = 0$

Factor the trinomial: $(x^2 - 6)(x^2 + 6) = 0$

Break into separate equations: $x^2 - 6 = 0$ $x^2 + 6 = 0$

Manipulate each equation: $x^2 = 6$ $x^2 = -6$

Identify solutions: $x = \{\pm\sqrt{6}, \pm i\sqrt{6}\}$ **Answer B**

33. One way to factor $x^6 + x^3 - 72$ is to rewrite the expression as $b^2 + b - 72$. What is the equivalent of b ?

A. $x + 9$

B. x^2

C. x

D. x^3

Notice that the middle term in the first equation is x^3, and in the second equation is b. Set these two equal to each other and you have the solution. $\boldsymbol{x^3 = b}$.

Answer D

34. What is the remainder in the division $(6x^3 - x^2 + 4x - 9) \div (2x - 3)$?

A. -15

B. -3

C. 3

D. 15

The easiest approach to this is to use synthetic division. First, note that the root implied by the divisor $(2x - 3)$ is $x = \frac{3}{2} = 1.5$.

$$\begin{array}{r|rrrr} 1.5 & 6 & -1 & 4 & -9 \\ & & 9 & 12 & 24 \\ \hline & 6 & 8 & 16 & 15 \end{array}$$

Answer D

35. Use synthetic or long division to find the quotient of $(2x^2 - 33x + 16) \div (x - 16)$.

A. $2x - 33 + \frac{16}{x-16}$

B. $2x - 1$

C. $2x - 1 + \frac{-32}{x-16}$

D. $2x + 1 + \frac{32}{x-16}$

The easiest approach to this is to use synthetic division. First, note that the root implied by the divisor $(x - 16)$ is $x = 16$.

$$\begin{array}{r|rrr} 16 & 2 & -33 & 16 \\ & & 32 & -16 \\ \hline & 2 & -1 & 0 \end{array}$$

The result is $\boldsymbol{2x - 1}$ **Answer B**

For a full explanation of synthetic division, see pages 122-123 of the Algebra Handbook or use the synthetic division section of the Algebra App, both of which are available at www.mathguy.us.

36. Given $x = 5$ is a solution, use synthetic division to find the two remaining real solutions of $3x^3 - 35x^2 + 128x - 140 = 0$

A. $x = 2,\ x = \frac{14}{3}$

B. $x = 6,\ x = \frac{14}{3}$

C. $x = -2,\ x = \frac{14}{3}$

D. no other solutions

Note that one root is given to us directly: $x = 5$.

$$\begin{array}{r|rrrr} 5 & 3 & -35 & 128 & -140 \\ & & 15 & -100 & 140 \\ \hline & 3 & -20 & 28 & 0 \end{array}$$

The result is: $3x^2 - 20x + 28 = 0$

Split the middle term: $3x^2 - 6x - 14x + 28 = 0$

The split was determined using the **AC Method.**

Pair terms: $(3x^2 - 6x) - (14x - 28) = 0$

Factor pairs: $3x(x - 2) - 14(x - 2) = 0$

Collect terms: $(3x - 14)(x - 2) = 0$

Break into separate equations: $3x - 14 = 0 \quad x - 2 = 0$

Solutions for x in the equation: $\boldsymbol{x = \left\{\frac{14}{3},\ 2\right\}}$ **Answer A**

37. Given $(x - 9)$ is a factor of the polynomial, $f(x) = 16x^3 - 144x^2 - 81x + 729$, what are the remaining factors?

A. $(4x - 9)(4x + 9)$

B. $(16x^2 + 81)$

C. $(4x - 9)$

D. $(4x - 9)(4x - 9)$

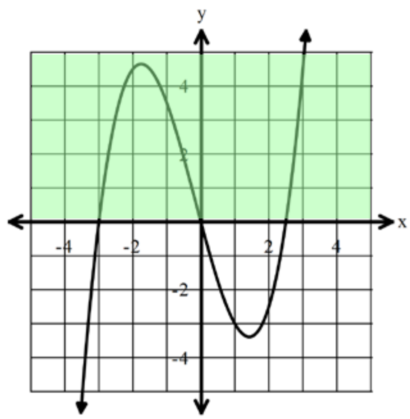
Note that the root implied by the divisor $(x - 9)$ is $x = 9$.

$$\begin{array}{r|rrrr} 9 & 16 & -144 & -81 & 729 \\ & & 144 & 0 & -729 \\ \hline & 16 & 0 & -81 & 0 \end{array}$$

The result is: $16x^2 - 81 = 0$

Factor the difference of squares: $\boldsymbol{(4x + 9)(4x - 9)}$ **Answer A**

38. The function $f(x) = \frac{1}{2}x^3 + \frac{1}{4}x^2 - \frac{15}{4}x$ is graphed below, over which intervals of x is the graph above the x-axis?



Easy Peezy. Above the x-axis is the green part in the figure at left. Where does the curve exist in this green area?

Answer: between $x = -3$ and $x = 0$, as well as above $x = 2.5$. In interval form, this is: $(-3, 0) \cup (2.5, +\infty)$.

Answer B

A. $(-\infty, +\infty)$

B. $(-3, 0) \cup (2.5, +\infty)$

C. $(-3, 2.5)$

D. $[-\infty, -3) \cup (2.5, +\infty]$

39. Identify the domain of the function $f(x) = \frac{2}{x-2} - 8$.

A. $D: \{x | x \neq 2\}$

B. $D: \{all\ real\ numbers\}$

C. $D: \{x | x \neq 0\}$

D. $D: \{x | x \neq -2\}$

To get values excluded from the domain, consider the denominator in the function.

The domain excludes any value of x where the denominator is zero.

Want denominator not equal to zero: $x - 2 \neq 0$

Add 2 to each side: $+2 \quad +2$

$x \neq 2$ is the domain **Answer A**

40. State the domain of the function: $y = \frac{x-6}{2x-8}$

A. *Domain is all real numbers except* $\frac{1}{2}$

B. *Domain is all real numbers except* 6

C. *Domain is all real numbers except* 8

D. *Domain is all real numbers except* 4

Want denominator not equal to zero: $2x - 8 \neq 0$

Add 8 to each side: $+8 \quad +8$

$2x \neq 8$

Divide by 2: $\div 2 \quad \div 2$

$x \neq 4$ is the domain **Answer D**

41. Simplify: $\frac{x^2-x-30}{2x^2-11x-6}$

A. $\frac{(x+5)(x+6)}{(2x+1)(x-6)}$

B. $\frac{(x+5)(x-6)}{(2x+1)(x+6)}$

C. $\frac{(x+5)}{(2x+1)}$

D. $\frac{(2x+1)}{(x+5)}$

Factor first and then simplify. You may look at the answers to help you do your factoring.

$$\frac{x^2-x-30}{2x^2-11x-6} = \frac{(x+5)(x-6)}{(2x+1)(x-6)} = \frac{(x+5)}{(2x+1)}$$

Answer C

42. Perform the indicated operation: $\frac{x^2-3x-10}{x^2+2x-3} \div \frac{x+5}{x+3}$

A. $\frac{(x-5)(x+2)}{(x-1)(x+5)}$

B. $\frac{(x-5)(x+3)(x+2)}{(x-1)(x-3)(x+5)}$

C. $\frac{(x+2)}{(x-1)}$

D. $\frac{(x-1)}{(x+2)}$

Regarding the second fraction, *"flip that guy and multiply*."

$$\frac{x^2-3x-10}{x^2+2x-3} \div \frac{x+5}{x+3} = \frac{x^2-3x-10}{x^2+2x-3} \cdot \frac{x+3}{x+5}$$

$$= \frac{(x-5)(x+2)}{(x-1)(x+3)} \cdot \frac{(x+3)}{(x+5)}$$

$$= \frac{(x-5)(x+2)}{(x-1)(x+5)}$$

Answer A

43. Perform the indicated operation: $\frac{7}{x-4} - \frac{11}{x-4}$

A. $\frac{-77}{x-4}$

C. $\frac{18}{x-4}$

B. $\frac{-77}{(x-4)^2}$

D. $\frac{-4}{x-4}$

This is one of the easiest problems on the test.
Don't expect one like this on the real final.

$$\frac{7}{x-4} - \frac{11}{x-4} = \frac{7-11}{x-4} = \frac{-4}{x-4}$$

Answer D

44. Perform the indicated operation: $\frac{4x+5}{x^2-25} + \frac{7}{x-5}$

A. $\frac{11x+40}{x^2-25}$

B. $\frac{11x+40}{x-5}$

C. $\frac{4x+12}{x^2-25}$

D. $\frac{4x+12}{x-5}$

Get a common denominator and then add.

$$\frac{4x+5}{x^2-25} + \frac{7}{x-5} = \frac{4x+5}{(x-5)(x+5)} + \frac{7}{(x-5)}$$

$$= \frac{4x+5}{(x-5)(x+5)} + \frac{7\,(x+5)}{(x-5)(x+5)}$$

$$= \frac{(4x+5)+(7x+35)}{(x-5)(x+5)}$$

$$= \frac{\mathbf{11x+40}}{\mathbf{x^2-25}}$$

Answer A

45. The area of a triangle is $3x^2 - 2x - 5$ square units and the base is equal to $x - 1$ units. Write an expression that can be used to represent the height of the triangle.

A. $h = \frac{1}{2}(3x^2 - 2x - 5)(x - 1)$

B. $h = \frac{1}{2}(x - 1)$

C. $h = \frac{2(x-1)}{3x^2-2x-5}$

D. $h = \frac{2(3x^2-2x-5)}{x-1}$

Recall that the area of a triangle is $A = \frac{1}{2}bh$, where b is the length of the base and h is the height of the triangle. Then, substitute the known quantities into the equation:

$$A = \frac{1}{2}bh$$

$$3x^2 - 2x - 5 = \frac{1}{2}(x-1)h$$

Multiply both sides by 2 and divide both sides by $(x - 1)$ to get:

$$\frac{2(3x^2-2x-5\,)}{(x-1)} = \boldsymbol{h}$$

Answer D

46. Identify any x-values for which the expression below is undefined:

$$\frac{x^2+14x+45}{25x^2-9}-\frac{2x+3}{x+7}$$

A. The expression is undefined at $x=-9$ and 7

B. The expression is undefined at $x=9$ and -7

C. The expression is undefined at $x=-9, 9$ and 7

D. The expression is undefined at $x=-\frac{3}{5},\frac{3}{5}$ and -7

The expression is undefined at x-values that make any denominator is zero. Let's deal with each denominator separately, then combine the results.

First Fraction

Want denominator not equal to zero: $25x^2-9 \neq 0$

Add 9 to each side: $+9 \quad +9$

$25x^2 \neq 9$

Divide by 25: $\div 25 \quad \div 25$

$x^2 \neq \frac{9}{25}$

Take square roots: $x \neq \pm\sqrt{\frac{9}{25}}$

Simplify $x \neq \pm\frac{3}{5}$

Second Fraction

Want denominator not equal to zero: $x+7 \neq 0$

Sutract 7 from each side: $-7 \quad -7$

$x \neq -7$

Combining these, we get: $x \neq \pm\frac{3}{5}, 7$

Answer D

47. Simplify: $\frac{1}{1-x}+\frac{x}{x-1}$

A. 1

B. $\dfrac{x+1}{x-1}$

C. $\dfrac{x+1}{1-x}$

D. $\dfrac{x+1}{(x-1)^2}$

Notice that the denominators are the opposites of each other. That is, $x-1=-(1-x)$.

So, let's multiply the second equation by $\frac{-1}{-1}$ to get common denominators.

$$\frac{1}{1-x}+\frac{x}{x-1}$$

$$=\frac{1}{1-x}+\left(\frac{x}{x-1}\cdot\frac{-1}{-1}\right) = \frac{1}{1-x}+\frac{-x}{1-x} = \frac{1-x}{1-x} = \mathbf{1}$$

Answer A

Note: this is true as long as $x \neq 1$.

48. If each of the following expressions is defined, which is equivalent to $x-1$?

A. $\dfrac{(x+1)(x-1)}{(x-1)}$

B. $\dfrac{(x-1)(x+2)}{x+1}\cdot\dfrac{x+1}{x+2}$

C. $\dfrac{(x+1)(x+2)}{x-2}\div\dfrac{x+2}{x-2}$

D. $\dfrac{x+1}{x+2}+\dfrac{x-1}{x+2}$

Look at each answer to see how it can be simplified. Let's do each, just for practice. We don't know which of these might be on the final!

A. $\frac{(x+1)(x-1)}{(x-1)} = x+1$

B. $\frac{(x-1)(x+2)}{(x+1)}\cdot\frac{(x+1)}{(x+2)} = x-1$ **Answer B**

C. $\frac{(x+1)(x+2)}{(x-2)}\div\frac{(x+2)}{(x-2)} = \frac{(x+1)(x+2)}{(x-2)}\cdot\frac{(x-2)}{(x+2)} = x+1$

D. $\frac{(x+1)}{(x+2)}+\frac{(x-1)}{(x+2)} = \frac{(x+1)+(x-1)}{(x+2)} = \frac{2x}{x+2}$

49. What would be the next logical step in simplifying the expression below?

Step 1	$\frac{6x-3}{x^2-x-12}-\frac{x}{x+3}$
Step 2	$\frac{6x-3}{(x+3)(x-4)}-\frac{x}{x+3}$
Step 3	$\frac{6x-3}{(x+3)(x-4)}-\frac{x}{x+3}\cdot\frac{x-4}{x-4}$
Step 4	$\frac{(6x-3)-x(x-4)}{(x+3)(x-4)}$

A. $\frac{6x+3+x^2+4}{(x+3)(x-4)}$

B. $\frac{6x-3+x^2-4}{(x+3)(x-4)}$

C $\frac{6x-3-x^2-4}{(x+3)(x-4)}$

D. $\frac{6x-3-x^2+4x}{(x+3)(x-4)}$

The expression has been simplified to the point where it has a common denominator and the numerator needs to be simplified. The next step would be to multiply $-x$ by $(x-4)$:

$$\frac{(6x-3)-x(x-4)}{(x+3)(x-4)}=\frac{(6x-3)-x^2+4x}{(x+3)(x-4)}$$

Answer D

50. What is the Lowest Common Denominator for

$$\frac{5}{x+3}-\frac{4x+1}{x^2+8x+15}-\frac{8x}{x+15}$$

A. $x^2+8x+15$

B. $(x+3)(x+5)(x+15)$

C. x^2+15

D. $(x+3)(x+3)(x+5)(x+15)$

Factor the denominators and make sure every term in any of them is included in the common denominator.

- First term: denominator = $(x+3)$
- Second term: denominator = $x^2+8x+15=(x+3)(x+5)$
- Third term: denominator = $(x+15)$

Including the terms from each of these gives a common denominator of:

$(x+3)(x+5)(x+15)$

Answer B