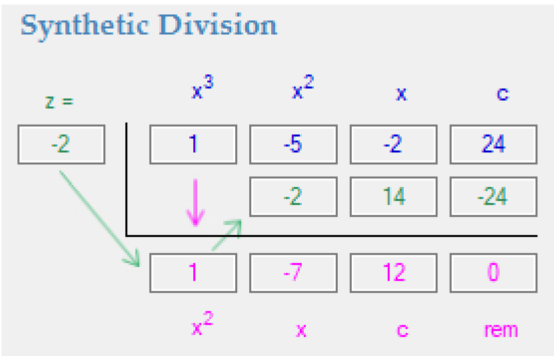
Synthetic divisions in this worksheet were performed using the Algebra App for PCs that is available at www.mathguy.us/PCApps.php.

1) Given the polynomial $f(x) = x^3 - 5x^2 - 2x + 24$ and factor $(x + 2)$, factor completely.

Given that $(x + 2)$ is a factor, change the sign to get a root of -2. Then, we can use synthetic division:



Note that the result of the division is provided in magenta in the gray rectangle above. The "c" refers to the constant term. "rem" refers to the remainder.

Result: $f(x) = x^3 - 5x^2 - 2x + 24 = (x + 2)\,(x^2 - 7x + 12)$

Then factor the quadratic term: $(x^2 - 7x + 12)$ to get the final solution:

$$= (x + 2)\,(x - 3)(x - 4)$$

2) What is the remainder in the division $(4x^3 - 6x^2 + 8x - 2) \div (2x - 1)$

We need to use long division for this problem because the coefficient of x is not one.

$$\begin{array}{r|l} & 2x^2 - 2x + 3 \\ \hline 2x - 1 & 4x^3 - 6x^2 + 8x - 2 \\ & \underline{4x^3 - 2x^2} \\ & \quad -4x^2 + 8x - 2 \\ & \quad \underline{-4x^2 + 2x} \\ & \qquad\qquad 6x - 2 \\ & \qquad\qquad \underline{6x - 3} \\ & \qquad\qquad\qquad 1 \end{array}$$

So, the remainder is: **1**

For #3 –10, perform the indicated operation:

3) $(5x^3 - x + 3) + (x^3 - 9x^2 + 4x)$

I like to add vertically, leaving space for missing terms.

$$\begin{array}{r} 5x^3 \qquad\quad - 1x + 3 \\ +\ 1x^3 - 9x^2 + 4x \qquad \\ \hline 6x^3 - 9x^2 + 3x + 3 \end{array}$$

4) $(x^3 + 4x^2 - 5x) - (4x^3 + x^2 - 7)$

I also like to subtract vertically, but to avoid confusion, I distribute the negative sign on the second term (i.e., change the signs of the second term) and then add.

Original Problem

$$\begin{array}{r} 1x^3 + 4x^2 - 5x \qquad \\ -\ (4x^3 + 1x^2 \qquad - 7) \\ \hline \end{array}$$

→

Revised Problem

$$\begin{array}{r} 1x^3 + 4x^2 - 5x \qquad \\ +\ \ -4x^3 - 1x^2 \qquad + 7 \\ \hline \mathbf{-3x^3 + 3x^2 - 5x + 7} \end{array}$$

5) $(x - 1)(2x + 3)^2$

First, I will square the $(2x + 3)$ term using FOIL:

$$(2x + 3)^2 = (2x + 3)(2x + 3) = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$$

Then, I will multiply the $(x - 1)$ term by the result using the stacking method. You can use another method if you like, but you should get the same answer.

$$\begin{array}{r} 4x^2 + 12x + 9 \\ \cdot \quad x - 1 \\ \hline -4x^2 - 12x - 9 \\ 4x^3 + 12x^2 + 9x \qquad \\ \hline \mathbf{4x^3 + 8x^2 - 3x - 9} \end{array}$$

6) $(4x^4 - 7x^3 + 15x - 7) - (-4x^2 - 10x)$

I like to subtract vertically, but to avoid confusion, I distribute the negative sign on the second term (i.e., change the signs of the second term) and then add.

Original Problem

$$\begin{array}{r} 4x^4 - 7x^3 \qquad + 15x - 7 \\ - \qquad (-4x^2 - 10x) \\ \hline \end{array}$$

Revised Problem

$$\begin{array}{r} 4x^4 - 7x^3 \qquad + 15x - 7 \\ + \qquad 4x^2 + 10x \\ \hline \mathbf{4x^4 - 7x^3 + 4x^2 + 25x - 7} \end{array}$$

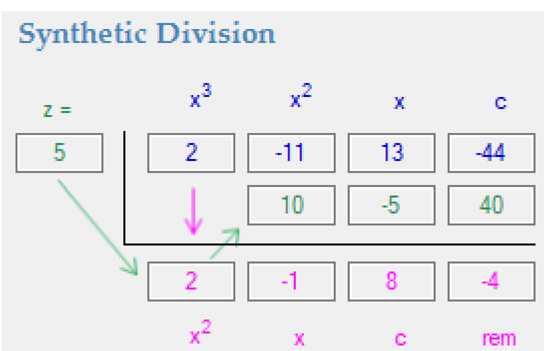
7) $(x - 6)(5x^2 + x - 8)$

I will use the stacking method for this multiplication:

$$\begin{array}{r} 5x^2 + 1x - 8 \\ \cdot \qquad x - 6 \\ \hline -30x^2 - 6x + 48 \\ 5x^3 + 1x^2 - 8x \qquad \\ \hline \mathbf{5x^3 - 29x^2 - 14x + 48} \end{array}$$

8) $(2x^3 - 11x^2 + 13x - 44) \div (x - 5)$

Let's use synthetic division for this. Given that $(x - 5)$ is a factor, change the sign to get a root of 5.



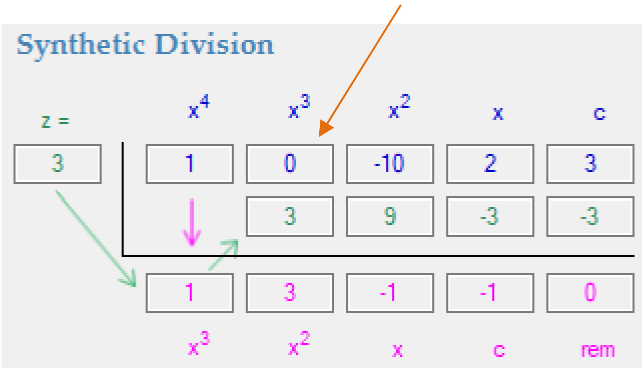
The result of the division is provided in magenta in the gray rectangle above. The "c" refers to the constant term. "rem" refers to the remainder. Result:

$$(2x^3 - 11x^2 + 13x - 44) \div (x - 5) \;=\; \mathbf{2x^2 - x + 8 + \frac{-4}{x - 5}}$$

Note: the remainder (-4) is placed over the divisor $(x - 5)$ in the final solution.

9) $(x^4 - 10x^2 + 2x + 3) \div (x - 3)$

Let's use synthetic division for this. Given that $(x - 3)$ is a factor, change the sign to get a root of 3. *Note that there is no x^3 term in the dividend (i.e., the first expression), so we must use 0 as a placeholder for this term in the synthetic division.*



The result of the division is provided in magenta in the gray rectangle above. The "c" refers to the constant term. "rem" refers to the remainder. Result:

$$(x^4 - 10x^2 + 2x + 3) \div (x - 3) \;=\; \boldsymbol{x^3 + 3x^2 - x - 1}$$

10) $(x - 2)(x + 6)(x - 4)$

First, I will multiply the first two terms using FOIL:

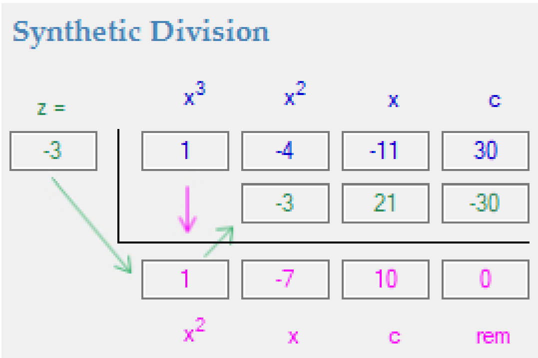
$$(x - 2)(x + 6) = x^2 + 6x - 2x - 12 = x^2 + 4x - 12$$

Then, I will multiply the $(x - 4)$ term by the result using the stacking method. You can use another method if you like, but you should get the same answer.

$$x^2 + 4x - 12$$
$$\cdot \quad x - 4$$
$$-4x^2 - 16x + 48$$
$$x^3 + 4x^2 - 12x$$
$$\boldsymbol{x^3 \qquad - 28x + 48} \quad = \quad \boldsymbol{x^3 - 28x + 48}$$

11) Find the other zeros of $f(x)$ given that $f(x) = x^3 - 4x^2 - 11x + 30$ *and* $x = -3$ is a zero of $f(x)$.

We will factor this function in order to find its zeros. First, use synthetic division with -3 as a root.



The result of the division is provided in magenta in the gray rectangle above. The "c" refers to the constant term. "rem" refers to the remainder. Result:

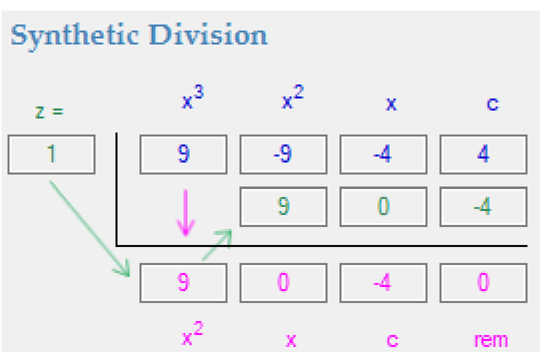
$$x^2 - 7x + 10 \quad = \quad (x-2)(x-5)$$

The other zeros, then, are found by setting the factored expression equal to zero.

$$(x-2)(x-5) = 0 \qquad \Rightarrow \qquad \boldsymbol{x = \{2, 5\}}$$

12) Given the polynomial $f(x) = 9x^3 - 9x^2 - 4x + 4$ and factor $(x-1)$, factor completely.

Use synthetic division with 1 as a root.



The result of the division is provided in magenta in the gray rectangle above. The "c" refers to the constant term. "rem" refers to the remainder. Result:

$$9x^3 - 9x^2 - 4x + 4 \quad = \quad (x-1)(9x^2 - 4)$$

Continue by factoring the quadratic term, which is a difference of squares:

$$9x^3 - 9x^2 - 4x + 4 \quad = \quad (x-1)(9x^2 - 4) \quad = \quad \boldsymbol{(x-1)(3x-2)(3x+2)}$$

For #13 – 19, factor completely:

13) $x^3 - 27$

Recall the formula for a difference of cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Then,

$$x^3 - 27 = (x)^3 - (3)^3 = (x - 3)(x^2 + 3x + 3^2)$$
$$= \boldsymbol{(x - 3)(x^2 + 3x + 9)}$$

14) $2x^3 + 3x^2 - 8x - 12$

I would first try to factor by grouping. I notice that the ratio of the first two coefficients is the same as the ratio of the last two coefficients (i.e., $\frac{2}{3} = \frac{-8}{-12}$). Therefore, I <u>can</u> factor by grouping.

$$2x^3 + 3x^2 - 8x - 12 = (2x^3 + 3x^2) - (8x + 12)$$ Notice the change of sign
$$= x^2(2x + 3) - 4(2x + 3)$$
$$= (x^2 - 4)(2x + 3)$$ Next, factor the quadratic term
$$= \boldsymbol{(x - 2)(x + 2)(2x + 3)}$$

15) $2x^4 + 16x$

There appears to be a Common Factor in this expression. Let's start by factoring it out.

$$2x^4 + 16x = 2x(x^3 + 8)$$

Recall the formula for a sum of cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Then,

$$2x^4 + 16x = 2x(x^3 + 8) = 2x\,[(x)^3 + (2)^3] = 2x\,[(x + 2)(x^2 - 2x + 2^2)]$$
$$\boldsymbol{2x\,(x + 2)\,(x^2 - 2x + 4)}$$

16) $6x^3 + 4x^2 - 16x$

There appears to be a Common Factor in this expression. Let's start by factoring it out.

$$6x^3 + 4x^2 - 16x = 2x(3x^2 + 2x - 8)$$

Next, factor the quadratic term. Note that it is possible to factor because the discriminant, $\Delta = b^2 - 4ac = 2^2 - 4(3)(-8) = 100$ is a perfect square. So,

$$6x^3 + 4x^2 - 16x = 2x(3x^2 + 2x - 8) = \mathbf{2x(3x-4)(x+2)}$$

17) $x^4 - 10x^2 + 9$

In this problem, we have x^4 and x^2 instead of the usual x^2 and x. So when we factor, our x-terms will be x^2 instead of x.

$$x^4 - 10x^2 + 9 = (x^2 - 9)(x^2 - 1)$$

Now, we are back to normal. We see two terms that are both differences of squares.

$$x^4 - 10x^2 + 9 = (x^2 - 9)(x^2 - 1) = (x-3)(x+3)(x-1)(x+1)$$
$$= \mathbf{(x-3)(x+3)(x-1)(x+1)}$$

18) $2x^4 - 4x^3 + 3x - 6$

I would first try to factor by grouping. I notice that the ratio of the first two coefficients is the same as the ratio of the last two coefficients (i.e., $\frac{2}{-4} = \frac{3}{-6}$). Therefore, I can factor by grouping.

$$2x^4 - 4x^3 + 3x - 6 = (2x^4 - 4x^3) + (3x - 6)$$
$$= 2x^3(x-2) + 3(x-2)$$
$$= (2x^3 + 3)(x-2)$$
$$= \mathbf{(2x^3 + 3)(x-2)}$$

19) List all possible roots for $f(x)$: $f(x) = 3x^3 + 2x^3 - x^2 + 18$

$$\text{Possible Roots} = \pm\frac{\text{Factors of 18}}{\text{Factors of 3}} = \pm\frac{1, 2, 3, 6, 9, 18}{1, 3}$$

$$= \pm\mathbf{1}, \pm\mathbf{2}, \pm\mathbf{3}, \pm\mathbf{6}, \pm\mathbf{9}, \pm\mathbf{18}, \pm\frac{\mathbf{1}}{\mathbf{3}}, \pm\frac{\mathbf{2}}{\mathbf{3}}$$

Note: there are more fractions with 3 in the denominator, but they are duplicates, and so they need not be written again. For example, $\pm\frac{9}{3} = \pm 3$, which is already listed.

20) Write the equation of the parabola in vertex form.

Vertex: (-2, 4); Point: (1, -5)

We want vertex form and we have all of the values we need except for a, which we must find.

We are given the following values: $h = -2$, $k = 4$, $x = 1$, $y = -5$. Substitute them into the form of the equation requested (vertex form) and calculate the value of a.

$y = a(x - h)^2 + k$

$-5 = a(1 - (-2))^2 + 4$

$-5 = a \cdot 9 + 4$

$-9 = 9a$

$-1 = a$

So, the equation is as follows:

$y = -(x + 2)^2 + 4$

21) Write an equation for the parabola in factored form with x-intercepts at -3 and 4 and passes through (-1, 5).

Intercept form is $y = a(x-p)(x-q)$, where p and q are the intercepts.

Substituting in the intercept values (-3 and 4), our equation will have the following form:

$y = a(x+3)(x-4)$ Notice the sign changes!

We need to find the value of a. To do this, substitute the point values of $x = -1$ and $y = 5$ into this equation and then solve for a.

$5 = a(-1+3)(-1-4)$

$5 = a(2)(-5)$

$5 = -10a$

$-\frac{1}{2} = a$

Substituting the value of a back into the equation gives our solution:

$y = -\frac{1}{2}(x+3)(x-4)$

22) Write $y = 2x^2 + 12x + 10$ in vertex form. Then state the vertex, axis of symmetry and min/max value.

We could complete the square, but let's use the fact that $h = -\frac{b}{2a}$ instead.

In this equation, $a = 2$, $b = 12$, $c = 10$. Then,

$$h = -\frac{b}{2a} = -\frac{12}{2(2)} = -3$$

To find the value of k, substitute $x = h$ into the equation. This finds the y-value associated with the x-value of h, which is k.

$y = 2x^2 + 12x + 10$

$k = 2h^2 + 12h + 10 = 2(-3)^2 + 12(-3) + 10$

$= 18 - 36 + 10 = -8$

So the equation is as follows (note: we know that $a = 2$ from above): $y = 2(x+3)^2 - 8$

The axis of symmetry, which goes through the vertex, has form: $x = h$. $x = -3$

The function is a regular U, and so it has a minimum value equal to k. Minimum value = -8

Solve the following equations:

23) $(x-1)^2 - 12 = -3$

This problem has no separate x-term, so I will solve it by isolating x.

$$\begin{aligned} (x-1)^2 - 12 &= -3 \\ +12 &\quad +12 \\ \hline (x-1)^2 &= 9 \\ \sqrt{(x-1)^2} &= \sqrt{9} \\ x-1 &= \pm 3 \end{aligned}$$

Consider each factor separately.

$x-1=3, \quad so \ x=4$

$x-1=-3, \quad so \ x=-2$

24) $x^2 - 2x - 3 = 0$

This one looks factorable. Let's try it.

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

Consider each factor separately.

$x+1=0, \quad so \ x=-1$

$x-3=0, \quad so \ x=3$

25) $3x^2 - 8x - 10 = 0$

This expression does not look factorable. To be sure, I will check the discriminant.

$a=3, \ b=-8, \ c=-10 \qquad \Delta = b^2 - 4ac = 64 - 4(3)(-10) = 64 + 120 = 184$

Since 184 is not a perfect square, we must use another technique. Usually in cases like this, the best approach is to use the quadratic formula. Also, note that the discriminant, which we already calculated, is what goes under the radical in the quadratic equation. Then,

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{8 \pm \sqrt{184}}{2(3)}$$

$$= \frac{8 \pm \left(\sqrt{4}\cdot\sqrt{46}\right)}{6}$$

$$= \frac{8 \pm 2\sqrt{46}}{6} = \frac{4 \pm \sqrt{46}}{3}$$

26) $x^2 = -81$

This problem simply requires us to take the square root of a negative number. Don't forget the $\pm$ and don't forget the i.

$$x^2 = -81$$
$$\sqrt{x^2} = \sqrt{-81}$$
$$\boldsymbol{x = \pm 9i}$$

27) $x^2 + 8 = -20$

This problem has no separate x-term, so I will solve it by isolating x.

$$x^2 + 8 = -20$$
$$-8 \quad -8$$
$$x^2 = -28$$

$$\sqrt{x^2} = \sqrt{-28}$$
$$x = \pm\sqrt{-28} = \pm i\sqrt{4}\sqrt{7}$$
$$x = \boldsymbol{\pm 2i\sqrt{7}}$$

28) $2x^2 = -126$

This problem has no separate x-term, so I will solve it by isolating x.

$$2x^2 = -126$$
$$\div 2 \quad \div 2$$
$$x^2 = -63$$

$$\sqrt{x^2} = \sqrt{-63}$$
$$x = \pm\sqrt{-63} = \pm i\sqrt{9}\sqrt{7}$$
$$x = \boldsymbol{\pm 3i\sqrt{7}}$$