Algebra 2 Unit 2 Part 2 Practice Test

Name:________________________

#1-16: Find the solutions, or zeroes or roots, to each function or equation by using the best method (square rooting, factoring, or the quadratic formula). Simplify all answers using *i* if needed.

Each problem can be solved in multiple ways. For each, I select a method that I think is both relatively quick and most likely to be free of errors.

1) $\frac{1}{2}x^2 + 5 = 41$

There is no x-term in this problem, so I will solve it by isolating x.

$$\frac{1}{2}x^2 + 5 = 41$$
$$-5 \quad -5$$
$$\frac{1}{2}x^2 = 36$$
$$\cdot 2 \quad \cdot 2$$
$$x^2 = 72$$

$$\sqrt{x^2} = \sqrt{72}$$
$$x = \pm\sqrt{36}\cdot\sqrt{2}$$
$$x = \pm 6\sqrt{2}$$

2) $x^2 + 6x - 27 = 0$

This one looks factorable. Let's try it.

$$x^2 + 6x - 27 = 0$$
$$(x+9)(x-3) = 0$$

Consider each factor separately.

$$x + 9 = 0, \quad so \;\; x = -9$$
$$x - 3 = 0, \quad so \;\; x = 3$$

3) $x^2 + 5x - 99 = 3x$

We need to get all terms on one side of the equal sign before factoring.

$$x^2 + 5x - 99 = 3x$$
$$-3x \quad -3x$$
$$x^2 + 2x - 99 = 0$$

Now, factor.

$$(x+11)(x-9) = 0$$

Consider each factor separately.

$$x + 11 = 0, \quad so \;\; x = -11$$
$$x - 9 = 0, \quad so \;\; x = 9$$

4) $-(x-1)^2 - 3 = -7$

This problem has no separate x-term, so I will solve it by isolating x.

$$
\begin{aligned}
-(x-1)^2 - 3 &= -7 \\
+3 \quad & \quad +3 \\
\hline
-(x-1)^2 &= -4 \\
\cdot(-1) \quad & \quad \cdot(-1) \\
\hline
(x-1)^2 &= 4
\end{aligned}
$$

$$\sqrt{(x-1)^2} = \sqrt{4}$$

$$x - 1 = \pm 2$$

Consider each factor separately.

$$x - 1 = 2, \quad so \;\; x = 3$$

$$x - 1 = -2, \quad so \;\; x = -1$$

5) $-2(x-6)^2 - 45 = 53$

This problem has no separate x-term, so I will solve it by isolating x.

$$
\begin{aligned}
-2(x-6)^2 - 45 &= 53 \\
+45 \quad & \quad +45 \\
\hline
-2(x-6)^2 &= 98 \\
\div(-2) \quad & \quad \div(-2) \\
\hline
(x-6)^2 &= -49
\end{aligned}
$$

$$\sqrt{(x-6)^2} = \sqrt{-49}$$

$$
\begin{aligned}
x - 6 &= \pm 7i \\
+6 \quad & \quad +6 \\
\hline
x &= 6 \pm 7i
\end{aligned}
$$

6) $f(x) = 2x^2 - 4x - 1$

This expression does not look factorable. To be sure, I will check the discriminant.

$$a = 2, \;\; b = -4, \;\; c = -1 \qquad \Delta = b^2 - 4ac = 16 - 4(2)(-1) = 16 + 8 = 24$$

Since 24 is not a perfect square, we must use another technique. Usually in cases like this, the best approach is the quadratic formula. Also, note that the discriminant, which we already calculated, is what goes under the radical in the quadratic equation. Then,

$$
\begin{aligned}
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} &= \frac{4 \pm \sqrt{24}}{2(2)} \\
&= \frac{4 \pm \left(\sqrt{4} \cdot \sqrt{6}\right)}{4} \\
&= \frac{4 \pm 2\sqrt{6}}{4} = \frac{2 \pm \sqrt{6}}{2}
\end{aligned}
$$

7) $-4x^2 + 25 = 0$

There is no x-term in this problem, so I will solve it by isolating x.

$$\begin{aligned} -4x^2 + 25 &= 0 \\ -25 \quad & \quad -25 \\ \hline -4x^2 &= -25 \\ \div(-4) \quad & \quad \div(-4) \\ \hline x^2 &= \frac{25}{4} \end{aligned}$$

$$\sqrt{x^2} = \sqrt{\frac{25}{4}}$$

$$x = \pm\frac{\sqrt{25}}{\sqrt{4}} = \pm\frac{5}{2}$$

8) $-3(x+4)^2 - 18 = 6$

This problem has no separate x-term, so I will solve it by isolating x.

$$\begin{aligned} -3(x+4)^2 - 18 &= 6 \\ +18 \quad & \quad +18 \\ \hline -3(x+4)^2 &= 24 \\ \div(-3) \quad & \quad \div(-3) \\ \hline (x+4)^2 &= -8 \end{aligned}$$

$$\sqrt{(x+4)^2} = \sqrt{-8}$$

$$\begin{aligned} x + 4 &= \pm 2i\sqrt{2} \\ -4 \quad & \quad -4 \\ \hline x &= -4 \pm 2i\sqrt{2} \end{aligned}$$

9) $3x^2 + 5x - 3 = 4x - 8$

We need to get all terms on one side of the equal sign before factoring.

$$\begin{aligned} 3x^2 + 5x - 3 &= 4x - 8 \\ -4x + 8 \quad & \quad -4x + 8 \\ \hline 3x^2 + x + 5 &= 0 \end{aligned}$$

This expression does not look factorable. To be sure, I will check the discriminant.

$a = 3, \ b = 1, \ c = 5 \qquad \Delta = b^2 - 4ac = 1 - 4(3)(5) = 1 - 60 = -59$

Since -59 is negative, we must use another technique. Usually in cases like this, the best approach is the quadratic formula. Also, note that the discriminant, which we already calculated, is what goes under the radical in the quadratic equation. Then,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{-59}}{2(3)}$$

$$= \frac{-1 \pm i\sqrt{59}}{6}$$

10) $x^2 + 50 = 2$

There is no x-term in this problem, so I will solve it by isolating x.

$$\begin{aligned} x^2 + 50 &= \quad 2 \\ -50 &\quad -50 \\ \hline x^2 \quad &= -48 \\ \sqrt{x^2} \quad &= \sqrt{-48} \end{aligned}$$

$$\begin{aligned} x &= \pm\sqrt{-48} \\ x &= \pm\, i\,\sqrt{16}\,\sqrt{3} \\ x &= \pm\, 4i\,\sqrt{3} \end{aligned}$$

11) $16x^2 - 14x = 0$

This one is factorable because there is no constant term.

$$16x^2 - 14x = 0$$

$$2x(8x - 7) = 0$$

Consider each factor separately.

$$x = 0, \quad so \;\; x = 0$$

$$8x - 7 = 0, \quad so \;\; x = \frac{7}{8}$$

12) $3x^2 + 4x + 12 = 15$

We need to get all terms on one side of the equal sign before factoring.

$$\begin{aligned} 3x^2 + 4x + 12 &= \quad 15 \\ -15 &\quad -15 \\ \hline 3x^2 + 4x - \;\; 3 &= \quad\; 0 \end{aligned}$$

This expression does not look factorable. To be sure, I will check the discriminant.

$a = 3, \;\; b = 4, \;\; c = -3 \qquad \Delta = b^2 - 4ac = 16 - 4(3)(-3) = 16 + 36 = 52$

Since 52 is not a perfect square, we must use another technique. Usually in cases like this, the best approach is the quadratic formula. Also, note that the discriminant, which we already calculated, is what goes under the radical in the quadratic equation. Then,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{52}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{4}\;\sqrt{13}}{6}$$

$$= \frac{-4 \pm 2\,\sqrt{13}}{6} = \frac{-2 \pm \sqrt{13}}{3}$$

13) $x^2 + 8x + 6 = 3x$

We need to get all terms on one side of the equal sign before factoring.

$$\begin{array}{rcl} x^2 + 8x + 6 & = & 3x \\ -3x & & -3x \\ \hline x^2 + 5x + 6 & = & 0 \end{array}$$

This expression looks factorable. Just for fun, let's check the discriminant.

$a = 1, \ b = 5 \ \ c = 6 \qquad \Delta = b^2 - 4ac = 25 - 4(1)(6) = 25 - 24 = 1$

Since 1 is a perfect square, we can factor this expression.

$x^2 + 5x + 6 = 0$

$(x + 2)(x + 3) = 0$

Consider each factor separately.

$x + 2 = 0, \quad so \ \ x = -2$

$x + 3 = 0, \quad so \ \ x = -3$

14) Write the equation of a parabola in vertex form that has a vertex at (-3, 1) and goes through the point (-1, 2).

We want vertex form and we have all of the values we need except for a, which we must find.

We are given the following values: $h = -3, \ \ k = 1, \ \ x = -1, \ \ y = 2$. Substitute them into the form of the equation requested (vertex form) and calculate the value of a.

$y = a(x - h)^2 + k$

$2 = a(-1 - (-3))^2 + 1$

$2 = a \cdot 4 + 1$

$1 = 4a$

$\frac{1}{4} = a$

So the equation is as follows:

$y = \frac{1}{4}(x + 3)^2 + 1$

15) Write $y = 3x^2 - 18x + 7$ in vertex form. Then state the vertex, axis of symmetry and max/min.

We could complete the square, but let's use the fact that $h = -\frac{b}{2a}$ instead.

In this equation, $a = 3$, $b = -18$, $c = 7$. Then,

$$h = -\frac{b}{2a} = -\frac{-18}{2(3)} = 3$$

To find the value of k, substitute $x = h$ into the equation. This finds the y-value associated with the x-value of h, which is k.

$$y = 3x^2 - 18x + 7$$

$$k = 3h^2 - 18h + 7 = 3(3)^2 - 18(3) + 7$$

$$= 27 - 54 + 7 = -20$$

So the equation is as follows (note: we know that $a = 3$ from above):

$$y = 3(x-3)^2 - 20$$

The axis of symmetry, which goes through the vertex, has form: $x = h$.

$$x = 3$$

The function is a regular U, and so it has a minimum value equal to k.

Minimum value = -20

16) Find the y-intercept and the vertex of the quadratic function $y = x^2 + 4x - 1$. Then use that information to graph the function.

In this equation, $a = 1$, $b = 4$, $c = -1$. Then,

The y-intercept is $c = -1$. So, the point $(0, -1)$ is on the graph.

$$h = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$$

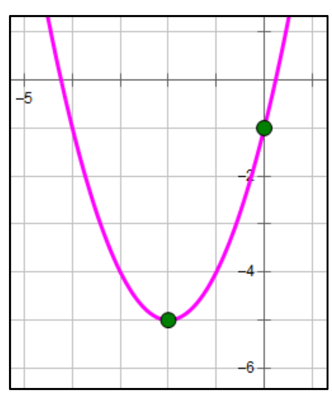
To find the value of k, substitute $x = h$ into the equation. This finds the y-value associated with the x-value of h, which is k.

$$y = x^2 + 4x - 1$$

$$k = h^2 + 4h - 1 = (-2)^2 + 4(-2) - 1$$

$$= 4 - 8 - 1 = -5$$

So, the vertex of the equation is $(-2, -5)$.



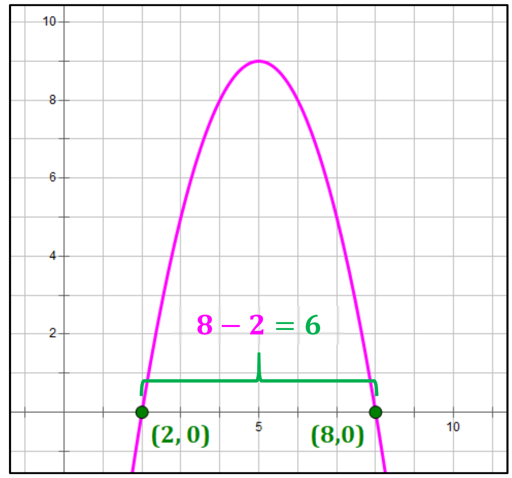
17) The graph of $h(x) = -x^2 + 10x - 16$ models the height, in feet, of one of the arches at the entrance of a parking structure. What is the width of the parking structure at the base?

The width at the base of the parking structure is the difference between the two x-intercepts. So, let's find them and find their difference. Begin by setting $h(x) = 0$.

$$h(x) = -x^2 + 10x - 16$$

$$h(x) = -(x^2 - 10x + 16)$$

$$h(x) = -(x - 8)(x - 2)$$



Consider each factor separately (the negative sign has no impact on the x-intercepts).

$$x - 8 = 0, \quad so \;\; x = 8$$

$$x - 2 = 0, \quad so \;\; x = 2$$

The width of the base, then is the difference between 2 and 8: $8 - 2 = 6$ feet.

(I hope you have a small car if you want to park in this garage!)

18) Write an equation in intercept/factored form that has x-intercepts at -1 and 3 and goes through the point (1, 4).

Intercept form is $y = a(x - p)(x - q)$, where p and q are the intercepts.

Substituting in the intercept values, our equation will have the following form:

$y = a(x + 1)(x - 3)$ Notice the sign changes!

We need to find the value of a. To do this, substitute the values of $x = 1$ and $y = 4$ into this equation and then solve for a.

$$4 = a(1 + 1)(1 - 3)$$

$$4 = a(2)(-2)$$

$$4 = -4a$$

$$-1 = a$$

Substituting the value of a back into the equation gives our solution:

$$y = -(x + 1)(x - 3)$$

19) Compare the axis of symmetry and the minimum values for the two functions below.

$$h(x) = 2(x+3)(x-7) \qquad\qquad j(x) = x^2 - 4x - 21$$

Determine which of the following statements is correct.

A. The functions $h(x)$ and $j(x)$ have the same axis of symmetry, but the minimum value of $h(x)$is less than the minimum value of $j(x)$.

B. The functions $h(x)$ and $j(x)$ have the same axis of symmetry, but the minimum value of $h(x)$is greater than the minimum value of $j(x)$.

C. The functions $h(x)$ and $j(x)$ do not have the same axis of symmetry, and the minimum value of $h(x)$is less than the minimum value of $j(x)$.

D. The functions $h(x)$ and $j(x)$ do not have the same axis of symmetry, and the minimum value of $h(x)$is greater than the minimum value of $j(x)$.

Though it may be difficult to see at first, this question is basically asking for the values of h and k for the two functions. Recall that $x = h$ is the axis of symmetry and k is the value of the maximum or minimum.

$h(x) = 2(x+3)(x-7)$

The x-intercepts of $h(x)$ are $-3, 7$.

h is halfway between the x-intercepts, so

$$h = \frac{-3+7}{2} = \frac{4}{2} = 2$$

So, the axis of symmetry is: $x = 2$

To find the value of k, substitute $x = 2$ into the equation.

$$y = 2(x+3)(x-7)$$
$$k = 2(2+3)(2-7)$$
$$= 2(5)(-5) = -50$$

Since a is positive, the minimum of $h(x)$ is -50.

$j(x) = x^2 - 4x - 21$

Then,

$$h = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2$$

So, the axis of symmetry is: $x = 2$

To find the value of k, substitute $x = 2$ into the equation.

$$y = x^2 - 4x - 21$$
$$k = 2^2 - 4(2) - 21$$
$$= 4 - 8 - 21 = -25$$

Since a is positive, the minimum of $f(x)$ is -25.

Based on the above work, functions $h(x)$ and $f(x)$ have the same axis of symmetry; further, the minimum value of $h(x)$ is less than the minimum value of $f(x)$. **Answer A**

Simplify the following expressions:

20) $(6-4i)(-2+5i)$

F: $(6)(-2) = -12$

O: $(6)(5i) = 30i$

I: $(-4i)(-2) = 8i$

L: $(-4i)(5i) = -20i^2 = 20$

$-12 + 20 + 30i + 8i \quad = \quad 8 + 38i$

21) $(3-8i)(3+8i)$

F: $(3)(3) = 9$

O: $(3)(8i) = 24i$

I: $(-8i)(3) = -24i$

L: $(-8i)(8i) = -64i^2 = 64$

$9 + 64 + 24i - 24i \quad = \quad 73$

22) $(11-4i)^2 \quad = \quad (11-4i)(11-4i)$

F: $(11)(11) = 121$

O: $(11)(-4i) = -44i$

I: $(-4i)(11) = -44i$

L: $(-4i)(-4i) = 16i^2 = -16$

$121 - 16 - 44i - 44i \quad = \quad 105 - 88i$

23) $2i(i-5) + 6(2-7i)$

Distribute then add:

$$2i(i-5) \quad + \quad 6(2-7i)$$
$$= 2i^2 - 10i \quad + \quad 12 - 42i$$
$$= -2 - 10i \quad + \quad 12 - 42i$$
$$= -2 + 12 - 10i - 42i \quad = \quad 10 - 52i$$

24) $-i(3i+2)-(5i+11)$

Distribute then add:

$$-i(3i+2) - (5i+11)$$
$$= -3i^2 - 2i \ - 5i - 11$$
$$= 3 - 2i \ - 5i - 11$$
$$= 3 - 11 - 2i - 5i \quad = \quad -8 - 7i$$

25) $(8-i)(-1-12i)$

F: $(8)(-1) = -8$

O: $(8)(-12i) = -96i$

I: $(-i)(-1) = +1i$

L: $(-i)(-12i) = 12i^2 = -12$

$$-8 - 12 - 96i + i \quad = \quad -20 - 95i$$

Simplify the following by using the conjugate:

26) $\dfrac{5i}{2-3i}$

Multiply by the ratio of the denominator's conjugate to itself.

$$\frac{5i}{2-3i} \cdot \frac{2+3i}{2+3i} = \frac{10i+15i^2}{4-6i+6i-9i^2} = \frac{-15+10i}{4+9} = \frac{-15+10i}{13}$$

27) $\dfrac{7-4i}{1+6i}$

Multiply by the ratio of the denominator's conjugate to itself.

$$\frac{7-4i}{1+6i} \cdot \frac{1-6i}{1-6i} = \frac{7-42i-4i+24i^2}{1-6i+6i-36i^2} = \frac{7-24-42i-4i}{1+36} = \frac{-17-46i}{37}$$

28) $\dfrac{2+2i}{-3+i}$

Multiply by the ratio of the denominator's conjugate to itself.

$$\frac{2+2i}{-3+i} \cdot \frac{-3-i}{-3-i} = \frac{-6-2i-6i-2i^2}{9+3i-3i-i^2} = \frac{-6+2-2i-6i}{9+1} = \frac{-4-8i}{10} = \frac{-2-4i}{5}$$