Algebra 2 Unit 2 Practice Test **Name:________________________**

For #1-4, graph. Include the vertex, x- & y-intercepts, axis of symmetry, min/max value, positive negative, & domain & range. Write the domain & range and positive & negative in interval notation

1. $y = -(x-2)^2 + 4$		**2.** $y = -x^2 - 12x - 27$	
Vertex: $(2, 4)$	Axis of Sym: $x = 2$	Vertex: $(-6, 9)$	Axis of Sym: $x = -6.$
x-intercepts: $(4, 0), (0, 0)$	y-intercept: $(0, 0)$	x-intercepts: $(-3, 0), (-9, 0)$	y-intercept: $(0, -27)$
min/max Value: Max value = 4	D: $(-\infty, +\infty)$ R: $(-\infty, 4]$	min/max Value: Max value = 9	D: $(-\infty, +\infty)$ R: $(-\infty, 9]$
Pos: $(0, 4)$	Neg: $(-\infty, 0) \cup (4, +\infty)$	Pos: $(-9, -3)$	Neg: $(-\infty, -9) \cup (-3, +\infty)$
For step-by-step solutions, see the following pages.		For step-by-step solutions, see the following pages.	
3. $y = (x+3)^2 - 9$		**4.** $y = -5x^2 + 5$	
Vertex: $(-3, -9)$	Axis of Sym: $x = -3$	Vertex: $(0, 5)$	Axis of Sym: $x = 0.$
x-intercepts: $(0, 0), (-6, 0)$	y-intercept: $(0, 0)$	x-intercepts: $(1, 0), (-1, 0)$	y-intercept: $(0, 5)$
min/max Value: Min value = -9	D: $(-\infty, +\infty)$ R: $[-9, +\infty)$	min/max Value: Max value = 5	D: $(-\infty, +\infty)$ R: $(-\infty, 5]$
Pos: $(-\infty, -6) \cup (0, +\infty)$	Neg: $(-6, 0)$	Pos: $(-1, 1)$	Neg: $(-\infty, -1) \cup (1, +\infty)$
For step-by-step solutions, see the following pages.		For step-by-step solutions, see the following pages.	

Problem 1: $\boldsymbol{y = -(x-2)^2 + 4}$

This function is presented in vertex form:

$$y = a(x-h)^2 + k$$

We see that: $a = -1,\ h = 2,\ k = 4$

We can immediately extract the vertex as:

$$(h, k) = (2, 4).$$

From this we also know:

(a) The axis of symmetry, which goes through the vertex, has form: $x = h$.
$$x = 2$$

(b) The function is an upside-down U, and so it has a maximum value equal to k.
Max value = 4

(c) Also, since the function is an upside-down U, its range goes from negative infinity to k.
Range: $(-\infty, 4]$

(d) The domain is always $(-\infty, +\infty)$

Next, let's find the y-intercept by setting $x = 0$:

$$y = -(0-2)^2 + 4 = -4 + 4 = 0$$

So, the y-intercept is: (0, 0).

Next, let's find the x-intercepts by setting $y = 0$:

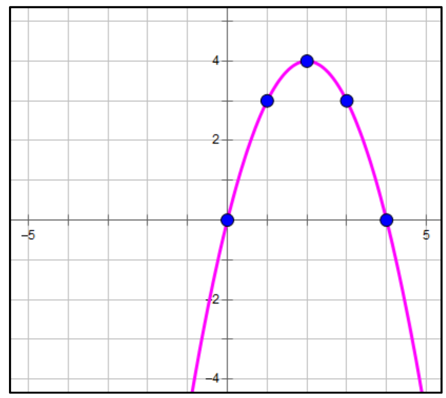
$$\begin{aligned} -(x-2)^2 + 4 &= 0 \\ +(x-2)^2 \quad &= +(x-2)^2 \\ \hline 4 &= (x-2)^2 \\ \sqrt{4} &= \sqrt{(x-2)^2} \\ \pm 2 &= x - 2 \\ +2 \quad &= \quad +2 \\ \hline 2 \pm 2 &= x \end{aligned}$$

$$x = 2 + 2 = 4 \ \ or \ \ x = 2 - 2 = 0$$

So, the x-intercepts are: $(4, 0), (0, 0)$.

In order to determine where the function is positive or negative, let's graph it:

- First plot the vertex, $(2, 4)$.
- Next, plot (over 1, down 1) and (over 2, down 4) on both sides of the vertex.
- Finally, draw a smooth curve through the five points.



Look at the graph now to see that the function is positive between the x-intercepts and negative outside them. Note: use parentheses, ( ), on all positive and negative intervals.

Positive: $(0, 4)$

Negative: $(-\infty, 0) \cup (4, +\infty)$

Problem 2: $\boldsymbol{y = -x^2 - 12x - 27}$

This function is presented in standard form:

$$y = ax^2 + bx + c$$

The vertex exists at: $x = -\frac{b}{2a} = -\frac{-12}{2(-1)}$

$$= -6$$

To get the y-value of the vertex, substitute the x-value into the original equation:

$$y = -(-6)^2 - 12(-6) - 27 = 9$$

So, the vertex is: $(h, k) = (-6, 9)$.

We now know: $a = -1,\ h = -6,\ k = 9$

From this we also know:

(a) The axis of symmetry, which goes through the vertex, has form: $x = h$.
$x = -6$

(b) The function is an upside-down U, and so it has a maximum value equal to k.
Max value = 9

(c) Also, since the function is an upside-down U, its range goes from negative infinity to k.
Range: $(-\infty, 9]$

(d) The domain is always $(-\infty, +\infty)$

Next, let's find the y-intercept by setting $x = 0$:

$$y = -(0)^2 - 12(0) - 27 = -27$$

So, the y-intercept is: $(0, -27)$.

Next, let's find the x-intercepts by setting $y = 0$:

$$-x^2 - 12x - 27 = 0$$

Multiply by -1 to make the solution easier.

$$x^2 + 12x + 27 = 0$$

Factor the resulting equation.

$$(x + 3)(x + 9) = 0$$

Set each factor equal to zero and solve to find the zeros (x-intercepts) of the function.

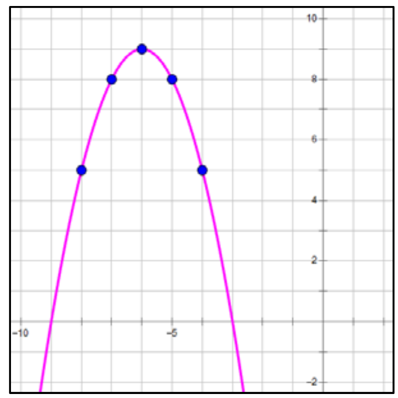
$$x + 3 = 0, \quad so\ \ x = -3$$

$$x + 9 = 0, \quad so\ \ x = -9$$

So, the x-intercepts are: $(-3, 0), (-9, 0)$.

In order to determine where the function is positive or negative, let's graph it:

- First plot the vertex, $(-6, 9)$.
- Next, plot (over 1, down 1) and (over 2, down 4) on both sides of the vertex.
- Finally, draw a smooth curve through the five points.



Look at the graph now to see that the function is positive between the x-intercepts and negative outside them. Note: use parentheses, ( ), on all positive and negative intervals.

Positive: $(-9, -3)$

Negative: $(-\infty, -9) \cup (-3, +\infty)$

Problem 3: $\boldsymbol{y = (x+3)^2 - 9}$

This function is presented in vertex form:

$$y = a(x-h)^2 + k$$

We see that: $a = 1,\ h = -3,\ k = -9$

We can immediately extract the vertex as:

$$(h, k) = (-3, -9).$$

From this we also know:

(e) The axis of symmetry, which goes through the vertex, has form: $x = h$.
$$x = -3$$
(f) The function is a regular U, and so it has a minimum value equal to k.
Min value = -9
(g) Also, since the function is a regular U, its range goes from k to infinity.
Range: $[-9, +\infty)$
(h) The domain is always $(-\infty, +\infty)$

Next, let's find the y-intercept by setting $x = 0$:

$$y = (0+3)^2 - 9 = 9 - 9 = 0$$

So, the y-intercept is: (0, 0).

Next, let's find the x-intercepts by setting $y = 0$:

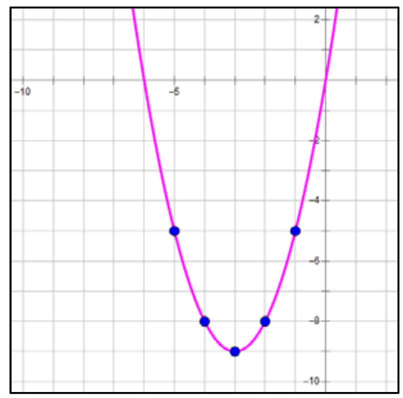
$$(x+3)^2 - 9 = 0$$
$$+9 = +9$$
$$(x+3)^2 = 9$$
$$\sqrt{(x+3)^2} = \sqrt{9}$$
$$x + 3 = \pm 3$$
$$-3 = -3$$
$$x = -3 \pm 3$$

$$x = -3 + 3 = 0 \ \ or \ \ x = -3 - 3 = -6$$

So, the x-intercepts are: $(0, 0), (-6, 0)$.

In order to determine where the function is positive or negative, let's graph it:

- First plot the vertex, $(-3, -9)$.
- Next, plot (over 1, up 1) and (over 2, up 4) on both sides of the vertex.
- Finally, draw a smooth curve through the five points.



Look at the graph now to see that the function is negative between the x-intercepts and positive outside them. Note: use parentheses, ( ), on all positive and negative intervals.

Positive: $(-\infty, -6) \cup (0, +\infty)$

Negative: $(-6, 0)$

Problem 4: $\boldsymbol{y = -5x^2 + 5}$

This function is presented in standard form:

$$y = ax^2 + bx + c$$

Note that there is no "x" term, so $b = 0$.

The vertex exists at: $x = -\frac{b}{2a} = -\frac{0}{2(-5)}$

$$= 0$$

To get the y-value of the vertex, substitute the x-value into the original equation:

$$y = -5(0)^2 + 5 = 5$$

So, the vertex is: $(h, k) = (0, 5)$.

We now know: $a = -5,\ h = 0,\ k = 5$

From this we also know:

(e) The axis of symmetry, which goes through the vertex, has form: $x = h$.
$x = 0$

(f) The function is an upside-down U, and so it has a maximum value equal to k.
Max value = 5

(g) Also, since the function is an upside-down U, its range goes from negative infinity to k.
Range: $(-\infty, 5]$

(h) The domain is always $(-\infty, +\infty)$

Next, let's find the y-intercept by setting $x = 0$:

$$y = -5(0)^2 + 5 = 5$$

So, the y-intercept is: $(0, 5)$.

Next, let's find the x-intercepts by setting $y = 0$:

$$-5x^2 + 5 = 0$$

Divide by -5 to make the solution easier.

$$x^2 - 1 = 0$$

Factor the resulting equation.

$$(x - 1)(x + 1) = 0$$

Set each factor equal to zero and solve to find the zeros (x-intercepts) of the function.

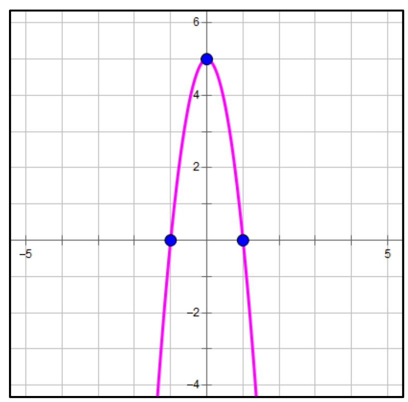
$x - 1 = 0, \quad so\ \ x = 1$

$x + 1 = 0, \quad so\ \ x = -1$

So, the x-intercepts are: $(1, 0), (-1, 0)$.

In order to determine where the function is positive or negative, let's graph it:

- First plot the vertex, $(0, 5)$.
- Next, plot (over 1, down 5) on both sides of the vertex. That's enough points since the stretch is large.
- Finally, draw a smooth curve through the three points.



Look at the graph now to see that the function is positive between the x-intercepts and negative outside them. Note: use parentheses, ( ), on all positive and negative intervals.

Positive: $(-1, 1)$

Negative: $(-\infty, -1) \cup (1, +\infty)$

Factor the following:

5. $x^2 - 2x - 8$

$(x-4)(x+2)$

6. $3x^3 + 12x^2 + 9x$

First, factor out the greatest common factor of the coefficients (3) and the lowest power of x (x^1).

$3x(x^2 + 4x + 3)$

$3x(x+1)(x+3)$

7. $x^2 - 10x + 16$

$(x-8)(x-2)$

8. $-x^2 - 15x - 56$

First, factor out the greatest common factor of the coefficients (-1).

$-(x^2 + 15x + 56)$

$-(x+7)(x+8)$

Simplify the following:

9. (2x + 3)(x – 7)

Use FOIL or the Box Method. I use FOIL.

$F\colon\ 2x \cdot x = 2x^2$

$O\colon\ 2x \cdot (-7) = -14x$

$I\colon\ 3 \cdot x = 3x$

$L\colon\ 3 \cdot (-7) = -21$

$2x^2 - 14x + 3x - 21 \quad = \quad 2x^2 - 11x - 21$

10. $5x(x2\ +\ 3x\ +\ 4)$

Just distribute:

$5x \cdot x^2 \quad + \quad 5x \cdot 3x \quad + \quad 5x \cdot 4 \quad = \quad 5x^3 + 15x^2 + 20x$

11. Write an equation for the parabola with x-intercepts at -1 and 3 and passes through (1, 4).

Since we know the intercepts, I suggest we use intercept form to solve the problem.

$y = a(x - p)(x - q)$, where p and q are the intercepts.

Substituting in the intercept values, our equation will have the following form:

$y = a(x + 1)(x - 3)$ Notice the sign changes!

We need to find the value of a. To do this, substitute the values of $x = 1$ and $y = 4$ into this equation and then solve for a.

$4 = a(1 + 1)(1 - 3)$

$4 = a(2)(-2)$

$4 = -4a$

$-1 = a$

Substituting the value of a back into the equation gives our solution:

$y = -(x + 1)(x - 3)$

12. Write a system of equations or matrix that could be used to write an equation in standard form of a parabola passing through the points (-2, 4), (5, 1) and (9, 6)

The easiest method for this problem is to substitute each set of x and y values into the standard equation form of a quadratic equation:

$y = ax^2 + bx + c$

Point: $(-2, 4)$: $4 = a(-2)^2 + b(-2) + c \Rightarrow 4 = 4a - 2b + c$

Point: $(5, 1)$: $1 = a(5)^2 + b(5) + c \Rightarrow 1 = 25a + 5b + c$

Point: $(9, 6)$: $6 = a(9)^2 + b(9) + c \Rightarrow 6 = 81a + 9b + c$

13. A parabola has a vertex of $(5, 6)$ and passes through the point $(10, -4)$. In the $y = a(x - h)^2 + k$ form of the parabola, what is the value of a ?

We are given the following values: $h = 5$, $k = 6$, $x = 10$, $y = -4$. Substitute them into the form of the equation provided (vertex form) and calculate the value of a.

$y = a(x - h)^2 + k$

$-4 = a(10 - 5)^2 + 6$

$-4 = 25a + 6$

$-10 = 25a$ → $-\frac{10}{25} = a$

$-\frac{2}{5} = a$

14. The graph $f(x) = x^2$ has a vertical compression of by a factor of ½, is shifted up 6, and right 5. What is the equation of the function after the transformation in vertex form?

Consider each piece separately and how it relates to the vertex form: $f(x) = a(x-h)^2 + k$

There is no mention of a reflection over the x-axis, so a must be positive

Vertical compression of by a factor of $\frac{1}{2}$: implies $a = \frac{1}{2}$

Shifted up 6: implies $k = 6$

Shifted right 5: implies $h = 5$

Be careful here, and think in terms of the vertex. The vertex is (5, 6).

Put it all together into the vertex form to get:

$$f(x) = \frac{1}{2}(x-5)^2 + 6$$

15. Describe in words how the graph of $g(x) = -5(x+2)^2 - 3$ would be transformed from the parent function $f(x) = x^2$.

Consider each piece separately and how it relates to the vertex form: $g(x) = a(x-h)^2 + k$

a is negative, so there is a reflection over the x-axis

The magnitude of a is 5 so there is a vertical stretch by a factor of 5

Next, think about the vertex. The vertex is $(-2, -3)$.

- $h = -2$ implies a horizontal shift left 2 units
- $k = -3$ implies a vertical shift down 3 units

16. Where does the function $f(x) = -\frac{1}{2}(x+4)^2 - 3$ cross the y-axis? (Hint: put in standard form first)

Ignoring the hint, we know that the y-intercept occurs when $x = 0$.

Substituting $x = 0$ into the equation gives us the required value of y:

$$y = -\frac{1}{2}(0+4)^2 - 3$$

$$= -\frac{1}{2}(16) - 3$$

$$= -8 - 3$$

$= -11$, so $f(x)$ crosses the y-axis at the point $(0, -11)$.

17. A parabola has a vertex of $(-2, 10)$ and passes through the point $(-3, 6)$.

Write the equation in $y = a(x-h)^2 + k$ form?

We want vertex form and we have all of the values we need except for a, which we must find.

We are given the following values: $h = -2$, $k = 10$, $x = -3$, $y = 6$. Substitute them into the form of the equation provided (vertex form) and calculate the value of a.

$$y = a(x-h)^2 + k$$

$$6 = a(-3-(-2))^2 + 10$$

$$6 = a(-1)^2 + 10$$

$$-4 = a$$

So the equation is as follows:

$$y = -4(x+2)^2 + 10$$

18. If $f(x) = x^2 - 14x + 39 = a(x-h)^2 + k$, then what is the value of h *and* k? (vertex)

Recall that $h = -\frac{b}{2a} = -\frac{-14}{2(1)} = 7$

Then, to get k (the y-value of the vertex), substitute h (the x-value of the vertex) into the original equation:

$$k = (7)^2 - 14(7) + 39 = -10$$

So, the vertex is: $(h, k) = (7, -10)$.

19. Identify the zeros of the quadratic function y = -(x + 1)(x – 5)

Set each factor equal to zero and solve to find the zeros (x-intercepts) of the function.

$$x + 1 = 0, \quad so \;\; x = -1$$

$$x - 5 = 0, \quad so \;\; x = 5$$

20. Identify the zeros of the quadratic function $y = 2x^2 + 17x + 8$

To find the zeros, we must factor, then solve for the values that make each factor zero.

I will use the **AC Method** to factor the function. I like the AC Method better than guessing when the leading coefficient is not 1, and it always works (if the function is factorable):

Step by Step method for factoring: $Ax^2 + Bx + C$

- Step 1. Multiply together A and C, and consider multipliers that result in AC.

$$y = 2x^2 + 17x + 8$$

$$AC = 2 \cdot 8 = 16$$

The multipliers that result in 16 are: $1 \cdot 16$, $2 \cdot 8$, $4 \cdot 4$.

- Step 2. Find a pair that adds to B. If you cannot find such a pair then the trinomial does not factor.

We want a pair of multipliers that adds to $B = 17$. That would be the pair: $1 \cdot 16$

- Step 3. Rewrite the middle term as a sum of terms whose coefficients are the chosen pair.

$$y = 2x^2 \quad + 17x \quad + 8$$

$$y = 2x^2 \quad + 1x + 16x \quad + 8$$

- Step 4. Factor by grouping.

$$y = (2x^2 + 1x) + (16x + 8)$$

$$= x(2x + 1) + 8(2x + 1)$$

$$= (x + 8)(2x + 1)$$

- Step 5: Set each factor equal to zero and solve to find the zeros of the function.

$$x + 8 = 0, \quad so \ \ x = -8$$

$$2x + 1 = 0, \quad so \ \ x = -\frac{1}{2}$$

21. Identify the x-intercepts of the function $y = x^2 - 7x + 12$

Thank goodness! An easy one at the end.

Factor and solve to find the x-intercepts of the function.

$$x^2 - 7x + 12 = 0$$

$$(x - 3)(x - 4) = 0$$

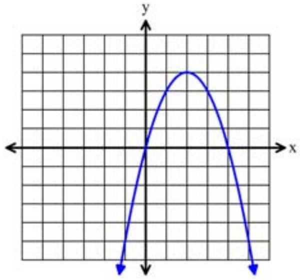
Consider each factor separately.

$$x - 3 = 0, \quad so \;\; x = 3$$

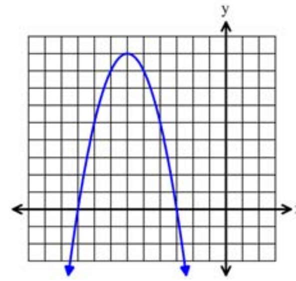
$$x - 4 = 0, \quad so \;\; x = 4$$

Practice Test Key Alg 2 Unit 2 part 2

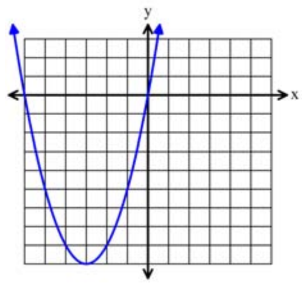
1. Vertex: (2, 4)
Axis: x = 2
x-int: 0, 4
y-int: (0,0)
Max @ 4
D: $(-\infty, \infty)$
R: $(-\infty, 4]$



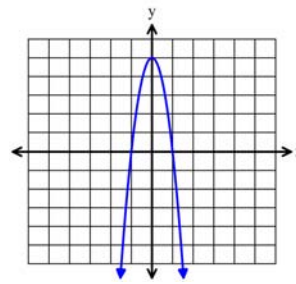
2. Vertex: (-6, 9)
Axis: x = -6
x-int: x= -9, -3
y-int: (0,-27)
Max @ 9
D: $(-\infty, \infty)$
R: $(-\infty, 9]$



3. Vertex: (-3, -9)
Axis: x = -3
x-int: 0, -6
y-int: (0,0)
Min @ -9
D: $(-\infty, \infty)$
R: $[-9, \infty)$



4. Vertex: (0, 5)
Axis: x = 0
x-int: -1, 1
y-int: (0, 5)
Max @ 5
D: $(-\infty, \infty)$
R: $(-\infty, 5]$



5. $(x-4)(x+2)$

6. $3x(x+3)(x+1)$

7. $(x-8)(x-2)$

8. $-(x+7)(x+8)$

9. $2x^2-11x-21$

10. $5x^3+15x^2+20x$

11. y = -(x + 1)(x - 3)

12. $\begin{aligned} 4a-2b+c&=4 \\ 25a+5b+c&=1 \\ 81a+9b+c&=6 \end{aligned}$ or $\begin{bmatrix} 4 & -2 & 1 & 4 \\ 25 & 5 & 1 & 1 \\ 81 & 9 & 1 & 6 \end{bmatrix}$

13. $a=-\frac{2}{5}$

14. $f(x)=\frac{1}{2}(x-5)^2+6$

15. Reflected, stretched, left two, down 3

16. $(0,-11)$

17. $y=-4(x+2)^2+10$

18. $(7,-10)$

19. $-1\ and\ 5$

20. $-\frac{1}{2}\ and -8$

21. 3 and 4