For #1 – 3, evaluate the expressions.

In evaluating logarithmic expressions, it is often helpful to set the expression equal to x, and then use the "first-last-middle" rule to convert the logarithmic expression to an exponential expression, which is easier to think about. *Note: the "first-last-middle" rule requires that the logarithmic or exponential portion be on the left-hand side of the equation.*

1) $\log_4 64$

In the log expression, $\log_4 64 = x$, **first** is "4", **last** is "x" and **middle** is "64." We put these in the exponential expression, from left to right, to get: $4^x = 64$, then solve.

$\log_4 64 = x$ converts to: $4^x = 64$ $\boldsymbol{x = 3}$

2) $\ln e^{-2}$ (remember that: ln is shorthand for: $\log_e$)

$\ln e^{-2} = \log_e e^{-2} = x$ converts to: $e^x = e^{-2}$ $\boldsymbol{x = -2}$

3) $\log_8 \frac{1}{64}$

$\log_8 \frac{1}{64} = x$ converts to: $8^x = \frac{1}{64}$ $\boldsymbol{x = -2}$

4) Simplify: $\log_7 49 + \ln(e^{12}) - \log_3 243$

$\log_7 49 + \ln(e^{12}) - \log_3 243$

$= 2 + 12 - 5 \quad \boldsymbol{= 9}$

Note: use the first-last-middle approach on each term of this expression if it helps.

For #5 – 7, write the expression in exponential form.

5) $\log_5 125 = 3$

In the log expression, $\log_5 125 = 3$, **first** is "5", **last** is "3" and **middle** is "125." We put these in the exponential expression, from left to right, to get: $\boldsymbol{5^3 = 125}$

6) $\log_6 \frac{1}{36} = -2$

In the log expression, $\log_6 \frac{1}{36} = -2$, **first** is "6", **last** is "-2" and **middle** is "$\frac{1}{36}$." We put these in the exponential expression, from left to right, to get: $\boldsymbol{6^{-2} = \frac{1}{36}}$

7) $\log_{64} 1024 = \frac{5}{3}$

In the log expression, $\log_{64} 1024 = \frac{5}{3}$, **first** is "64", **last** is "$\frac{5}{3}$" and **middle** is "1024." We put these in the exponential expression, from left to right, to get: $\mathbf{64^{5/3} = 1024}$

For #8 – 9, expand the expressions.

8) $\log\left(\frac{3x^4}{7y^3}\right)$

Steps to laying this out:

Step 1: write log of all of the items in parentheses in the original problem:

$\log 3 \quad \log x \quad \log 7 \quad \log y$

Step 2: add the exponents from the original problem as coefficients of each log:

$\log 3 \quad 4\ \log x \quad \log 7 \quad 3\ \log y$

Step 3: add the signs ("+" for items in the numerator; "−" for items in the denominator):

$\log 3 + 4\ \log x - \log 7 - 3\ \log y$

$$\log\left(\frac{3x^4}{7y^3}\right) = \log 3 + 4\ \log x - \log 7 - 3\ \log y$$

9) $\log\left(\frac{x^5y^2}{3z^4}\right)$

Steps to laying this out:

Step 1: write log of all of the items in parentheses in the original problem:

$\log x \quad \log y \quad \log 3 \quad \log z$

Step 2: add the exponents from the original problem as coefficients of each log:

$5 \log x \quad 2 \log y \quad \log 3 \quad 4 \log z$

Step 3: add the signs ("+" for items in the numerator; "−" for items in the denominator):

$5 \log x + 2 \log y - \log 3 - 4 \log z$

$$\log\left(\frac{x^5y^2}{3z^4}\right) = 5 \log x + 2 \log y - \log 3 - 4 \log z$$

For #10 – 12, use the change-of-base formula to evaluate. Round to nearest thousandth.

The change of base formula is: $\log_a b = \dfrac{\log b}{\log a} = \dfrac{\ln b}{\ln a}$.

You can use either $\log_{10}$ or $\ln$ on your calculator to change the base. I typically use $\ln$ because that is what is used most in the real world mathematics. $\log_{10}$ is used in some applications, but $\ln$ is used much more frequently.

10) $\log_8 5$

$$\log_8 5 = \frac{\ln 5}{\ln 8} = \mathbf{0.774}$$

11) $\log_2 6$

$$\log_2 6 = \frac{\ln 6}{\ln 2} = \mathbf{2.585}$$

12) $\log_5 7$

$$\log_5 7 = \frac{\ln 7}{\ln 5} = \mathbf{1.209}$$

For #13 – 14, <u>condense</u> the expressions.

exponents

13) $4\log_3 2 - 5\log_3 x + \log_3 y \quad = \log_3\left(\frac{2^4\, y}{x^5}\right) \quad = \mathbf{\log_3\left(\frac{16\, y}{x^5}\right)}$

"−" indicates term goes in denominator

"+" indicates term goes In numerator

exponents

14) $3\log_5 4 - \log_5 x - 6\log_5 y \quad = \log_5\left(\frac{4^3}{xy^6}\right) \quad = \mathbf{\log_5\left(\frac{64}{xy^6}\right)}$

"−" indicates term goes in denominator

For #15 -21 , solve the equation. Check for extraneous solutions. Round to the nearest hundredth when necessary.

15) $\log_6(x-1) = 2$

In the log expression, $\log_6(x-1) = 2$, **first** is "6", **last** is "2" and **middle** is "$(x-1)$." We put these in the exponential expression, from left to right, to get: $6^2 = x - 1$, then solve.

$\log_6(x-1) = 2$ converts to: $6^2 = x - 1$

$36 = x - 1$

$\mathbf{37 = x}$

Check the solutions in the original equation:

$\log_6(x-1) = 2$

Try $x = 37$: $\log_6(37-1) = 2$

$\log_6(36) = 2$ ✓

Solution: $x = 37$

16) $3^{0.2x} = 7$

Original equation: $3^{0.2x} = 7$

Take the $\log_3$ of both sides: $0.2x = \log_3 7$

Convert $\log_3 7$: $0.2x = \frac{\ln 7}{\ln 3}$

Multiply by 5: $x = \frac{5\ \ln 7}{\ln 3}$

Simplify $x = 8.856$

Check the solutions in the original equation:

$3^{0.2x} = 7$

Try $x = 8.856$: $3^{0.2 \cdot 8.856} = 7$

$3^{1.7712} = 7$ ✓

Solution: $x = 8.856$

17) $e^{0.06t} = 0.4$

Original equation: $e^{0.06t} = 0.4$

Take the $\ln$ of both sides: $0.06t = \ln 0.4$

Divide by 0.06: $t = \frac{\ln 0.4}{0.06}$

Simplify $t = -15.272$

Check the solutions in the original equation:

$e^{0.06t} = 0.4$

Try $t = -15.272$: $e^{0.06 \cdot (-15.272)} = 0.4$

$e^{-0.91632} = 0.4$ ✓

Solution: $x = -15.272$

18) $4^{-0.03x} + 5 = 8$

Original equation: $4^{-0.03x} + 5 = 8$

Subtract 5: $4^{-0.03x} = 3$

Take the $\log_4$ of both sides: $-0.03x = \log_4 3$

Convert $\log_4 3$: $-0.03x = \frac{\ln 3}{\ln 4}$

Divide by (-0.03): $x = -\frac{\ln 3}{(0.03)\ \ln 4}$

Simplify $x = -26.416$

Check the solutions in the original equation:

$4^{-0.03x} + 5 = 8$

Try $x = -26.416$: $4^{-0.03 \cdot (-26.416)} + 5 = 8$

$4^{0.79248} + 5 = 8$ ✓

Solution: $x = -26.416$

19) $\ln(x+9) = \ln(2x-7)$

When equal terms have the same logarithmic base (in this problem the base is e), set the objects of the logarithms equal.

Original equation: $\ln(x+9) = \ln(2x-7)$

Extract the objects of the logarithms: $x+9 = 2x-7$

Subtract x: $9 = x-7$

Add 7: $16 = x$

Check the solutions in the original equation:

$\ln(x+9) = \ln(2x-7)$

Try $x = 16$: $\ln(16+9) = \ln(2\cdot 16-7)$

$\ln(25) = \ln(25)$ ✓

Solution: $x = 16$

20) $3\log_8 x - 5 = 4$

Original equation: $3\log_8 x - 5 = 4$

Add 5: $3\log_8 x = 9$

Divide by 3: $\log_8 x = 3$

Take 8 to the power of both sides: $8^{\log_8 x} = 8^3$

Simplify: $x = 512$

Check the solutions in the original equation:

$3\log_8 x - 5 = 4$

Try $x = 512$: $3\cdot\log_8 512 - 5 = 4$

$3\cdot 3 - 5 = 4$ ✓

Solution: $x = 512$

21) $\log_4(3x+16) = \log_4 x + \log_4(x+9)$

Original equation: $\log_4(3x+16) = \log_4 x + \log_4(x+9)$

Condense terms: $\log_4(3x+16) = \log_4[x(x+9)]$

Extract the objects of the logarithms: $3x+16 = x(x+9)$

Simplify: $3x+16 = x^2+9x$

Subtract $(3x+16)$: $0 = x^2+6x-16$

Factor the trinomial: $0 = (x-2)(x+8)$

Separate the factors: $(x-2)=0$ and $(x+8)=0$

Solve the two equations: $x = \{2, -8\}$

Check the solutions in the original equation:

$\log_4(3x+16) = \log_4 x + \log_4(x+9)$

Try $x = 2$: $\log_4(3(2)+16) = \log_4(2) + \log_4(2+9)$

$\log_4(22) = \log_4(2) + \log_4(11)$

$\log_4(22) = \log_4(2 \cdot 11)$ ✓

Try $x = -8$: $\log_4(3(-8)+16) = \log_4(-8) + \log_4(-8+9)$

Uh oh! You can't take a log of -8!! ✗

Solution: $x = 2$

Problems 22-24 involve the accumulation of interest. The formulas for this are:

$A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$ and $A = P \cdot e^{rt}$

where, $A =$ the accumulated value at time t

$P =$ the Principal invested at time $t = 0$

$r =$ the interest rate on an annual basis

$n =$ the number of compounding periods in a year (e.g., $n = 4$ for quarterly compounding, $n = 12$ for monthly compounding)

Note that as n increases, $\left(1 + \frac{r}{n}\right)^{nt}$ approaches e^{rt}. That's why the second formula, $A = P \cdot e^{rt}$ is used for continuous compounding.

Shameless self-promotion: the "Algebra (Main) App" allows you to experiment with interest problems, and provides detailed solutions to problems like the ones below. It is available free at http://www.mathguy.us/PCApps.php. *Note: the app is for PCs, not for Macs or phones.*

22) A person invests \$5000 in an account that pays 1.5% interest compounded quarterly. Find the balance after 8 years.

In this problem, $P = 5000$, $r = .015$, $n = 4$, $t = 8$

$$A = 5000 \cdot \left(1 + \frac{0.015}{4}\right)^{4 \cdot 8}$$

$$= 5000 \cdot (1.00375)^{32}$$

$$= \mathbf{\$5,636.22}$$

23) Find the value of \$1500 deposited for 5 years in an account paying 4% annual interest compounded continuously.

In this problem, $P = 1500$, $r = .04$, $t = 5$

Continuous compounding means we must use the $A = P \cdot e^{rt}$ formula

$$A = 1500 \cdot e^{0.04 \cdot 5}$$

$$= 1500 \cdot e^{0.2}$$

$$= \mathbf{\$1,832.10}$$

Note: You can get very close to this answer by using the formula $A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$ with a high value of n, such as $n = 2000$. Try it!

24) If \$2000 is invested at a rate of 3% compounded continuously, what amount of time would be needed to have a balance of \$2500? Use the formula $A = Pe^{rt}$.

In this problem, $A = 2500,\ P = 2000,\ r = .03$

Continuous compounding means we must use the $A = P \cdot e^{rt}$ formula

Starting equation: $2500 = 2000 \cdot e^{0.03\,t}$

Divide by 2000: $1.25 = e^{0.03\,t}$

Take the ln of both sides: $\ln 1.25 = 0.03t$

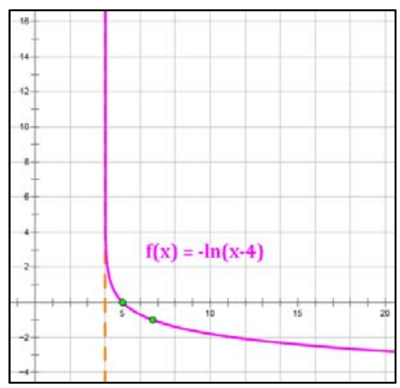
Divide by 0.03: $\frac{\ln 1.25}{0.03} = t$

Simplify $7.438 = t$

Answer: approximately $\mathbf{7.438}$ years

For #25 – 26, graph the function and state the domain and range.

25) $f(x) = -\ln(x-4)$



Domain: $x > 4$
Range: $\mathbb{R}$

1st step: An asymptote occurs where the object of the log is zero:

$x - 4 = 0 \longrightarrow x = 4$

The asymptote is also useful in identifying the domain.

2nd step: Select x-values of points:

Select two points with the following properties:

Select a point so that the object of the log is equal to 1:

$x - 4 = 1 \longrightarrow x = 5$

Select the other point so that the object of the log is equal to the base of the log:

$x - 4 = e \longrightarrow x = e + 4 \sim 6.718$

So, our x-values are $x = 5$ and $x = e + 4 \sim 6.718$

3rd step: Calculate y-values of points:

$x = 5$: $f(x) = -\ln(5-4) = -\ln(1) = 0$ $Point$: $(5, 0)$

$x = 6.718$: $f(x) = -\ln(e+4-4) = -\ln(e) = -1$ $Point$: $(6.718, -1)$

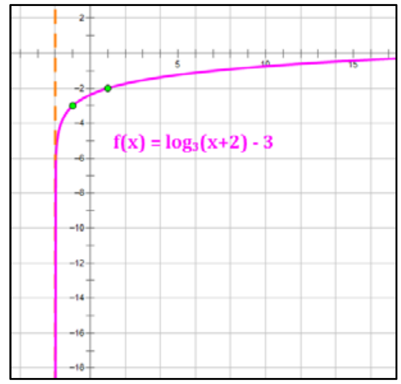
4th step: Draw the curve based on the asymptote and the two points.

26) $y = \log_3(x+2) - 3$

1st step: An asymptote occurs where the object of the log is zero:

$x + 2 = 0 \longrightarrow x = -2$

The asymptote is also useful in identifying the domain.



Domain: $x > -2$

Range: $\mathbb{R}$

2nd step: Select x-values of points:

Select two points with the following properties:

Select a point so that the object of the log is equal to 1:

$x + 2 = 1 \longrightarrow x = -1$

Select the other point so that the object of the log is equal to the base of the log:

$x + 2 = 3 \longrightarrow x = 1$

So, our x-values are $x = -1$ and $x = 1$

3rd step: Calculate y-values of points:

$x = -1$: $f(x) = \log_3(-1+2) - 3 = \log_3(1) - 3 = -3$ $\quad Point: (-1, -3)$

$x = 1$: $f(x) = \log_3(1+2) - 3 = \log_3(3) - 3 = -2$ $\quad Point: (1, -2)$

4th step: Draw the curve based on the asymptote and the two points.

27) **Simplfy:** $7^{\log_7 4}$

7 to a power and $\log_7$ are inverse operations, so,

$7^{\log_7 4} = \mathbf{4}$

exponents

28) **Condense:** $5\log_2 3 - 3\log_2 x + \log_2 y$

"−" indicates term goes in denominator

"+" indicates term goes In numerator

$$5\log_2 3 - 3\log_2 x + \log_2 y = \log_2 \frac{3^5 y}{x^3} = \mathbf{\log_2 \frac{243\, y}{x^3}}$$

29) **Expand:** $\ln\left(\frac{3x^4}{yz^5}\right)$

Steps to laying this out:

Step 1: write ln of all of the items in parentheses in the original problem:

$\ln 3 \quad \ln x \quad \ln y \quad \ln z$

Step 2: add the exponents from the original problem as coefficients of each log:

$\ln 3 \quad 4\ \ln x \quad \ln y \quad 5\ \ln z$

Step 3: add the signs ("+" for items in the numerator; "–" for items in the denominator):

$\ln 3 + 4\ \ln x - \ln y - 5\ \ln z$

$$\ln\left(\frac{3x^4}{yz^5}\right) = \ln 3 + 4\ \ln x - \ln y - 5\ \ln z$$

30) **Simplify:** $\log_4 64 - \log_3 81 + \ln(e^3)$

$\log_4 64 - \log_3 81 + \ln(e^3)$

$= 3 - 4 + 3 = \mathbf{2}$

Note: use the first-last-middle approach on each term of this expression if it helps.

For #31-33, solve the equation. Check for extraneous solutions. Round to the nearest hundredth when necessary.

31) $5\log_4(x-3) + 7 = 22$

Original equation:	$5\log_4(x-3) + 7 = 22$
Subtract 7:	$5\log_4(x-3) = 15$
Divide by 5:	$\log_4(x-3) = 3$
Take 4 to the power of both sides:	$4^{\log_4(x-3)} = 4^3$
Simplify:	$x - 3 = 64$
Add 3:	$x = 67$

Check the solutions in the original equation:

$5\log_4(x-3) + 7 = 22$

Try $x = 67$: $5 \cdot \log_4(67-3) + 7 = 22$

$5 \cdot 3 + 7 = 22$ ✓

Solution: $x = 67$

32) $e^{0.04t} + 6 = 6.43$

Original equation: $e^{0.04t} + 6 = 6.43$

Subtract 6: $e^{0.04t} = 0.43$

Take the $\ln$ of both sides: $0.04t = \ln 0.43$

Divide by 0.04: $t = \frac{\ln 0.43}{0.04}$

Simplify: $t = -21.099$

Check the solutions in the original equation:

$e^{0.04t} + 6 = 6.43$

Try $t = -21.099$: $e^{0.04\cdot(-21.099)} + 6 = 6.43$

$e^{-0.84396} + 6 = 6.43$ ✓

Solution: $t = -21.099$

33) $\log_5(3x + 21) = \log_5 x + \log_5(x + 7)$

Original equation: $\log_5(3x + 21) = \log_5 x + \log_5(x + 7)$

Condense terms: $\log_5(3x + 21) = \log_5[x(x + 7)]$

Extract the objects of the logarithms: $3x + 21 = x(x + 7)$

Simplify: $3x + 21 = x^2 + 7x$

Subtract $(3x + 21)$: $0 = x^2 + 4x - 21$

Factor the trinomial: $0 = (x - 3)(x + 7)$

Separate the factors: $(x - 3) = 0$ and $(x + 7) = 0$

Solve the two equations: $x = \{3, -7\}$

Check the solutions in the original equation:

$\log_5(3x + 21) = \log_5 x + \log_5(x + 7)$

Try $x = 3$: $\log_5(3\cdot 3 + 21) = \log_5 3 + \log_5(3 + 7)$

$\log_5(30) = \log_5(3) + \log_4(10)$

$\log_5(30) = \log_5(3\cdot 10)$ ✓

Try $x = -7$: $\log_5(3(-7) + 21) = \log_5(-7) + \log_5(-7 + 7)$

Uh oh! You can't take a log of -7!! ✗

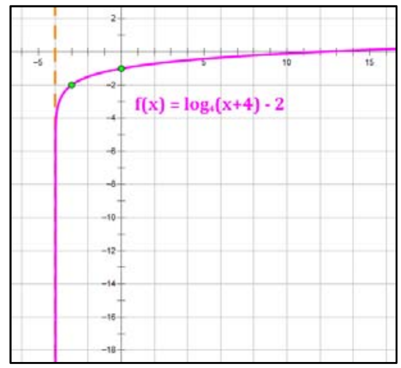
Solution: $x = 3$

34) State the Domain and Range of the function $y = \log_4(x+4) - 2$

$y = \log_4(x+4) - 2$

The Domain is determined by the object of the logarithm. This also determines the location of the vertical asymptote.

$x + 4 = 0$ ⟶ $x = -4$ is the asymptote.



Therefore,

Domain: $x > -4$

Range: $\mathbb{R}$ (as it almost always is with logarithmic functions)