Algebra
Exponent Formulas

Word Description of Property	Math Description of Property	Limitations on variables	Examples
Product of Powers	$a^p \cdot a^q = a^{(p+q)}$		$x^4 \cdot x^3 = x^7$ $x^5 \cdot x^{-8} = x^{-3}$
Quotient of Powers	$\frac{a^p}{a^q} = a^{(p-q)}$	$a \neq 0$	$\frac{y^5}{y^2} = y^3$
Power of a Power	$(a^p)^q = a^{(p \cdot q)}$		$(z^4)^3 = z^{12}$ $(x^{-3})^{-5} = x^{15}$
Anything to the zero power is 1	$a^0 = 1$	$a \neq 0$	$91^0 = 1$ $(xyz^3)^0 = 1$, if $x, y, z \neq 0$
Negative powers generate the reciprocal of what a positive power generates	$a^{(-p)} = \frac{1}{a^p}$	$a \neq 0$	$x^{(-3)} = \frac{1}{x^3}$ $\left(\frac{1}{x}\right)^{-5} = x^5$
Power of a product	$(a \cdot b)^p = a^p \cdot b^p$		$(3y)^3 = 27y^3$ $[(x+1)z]^4 = (x+1)^4 z^4$
Power of a quotient	$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$	$b \neq 0$	$\left(\frac{x}{4}\right)^3 = \frac{x^3}{64}$
Converting a root to a power	$\sqrt[n]{a} = a^{(1/n)}$	$n \neq 0$	$\sqrt{x} = x^{1/2}$

Table of Exponents and Logarithms

Definition: $b^a = c$ if and only if $\log_b c = a$

$2^0 = 1$	$\log_2 1 = 0$
$2^1 = 2$	$\log_2 2 = 1$
$2^2 = 4$	$\log_2 4 = 2$
$2^3 = 8$	$\log_2 8 = 3$
$2^4 = 16$	$\log_2 16 = 4$
$2^5 = 32$	$\log_2 32 = 5$
$2^6 = 64$	$\log_2 64 = 6$
$2^7 = 128$	$\log_2 128 = 7$
$2^8 = 256$	$\log_2 256 = 8$
$2^9 = 512$	$\log_2 512 = 9$
$2^{10} = 1024$	$\log_2 1024 = 10$
$3^0 = 1$	$\log_3 1 = 0$
$3^1 = 3$	$\log_3 3 = 1$
$3^2 = 9$	$\log_3 9 = 2$
$3^3 = 27$	$\log_3 27 = 3$
$3^4 = 81$	$\log_3 81 = 4$
$3^5 = 243$	$\log_3 243 = 5$
$4^0 = 1$	$\log_4 1 = 0$
$4^1 = 4$	$\log_4 4 = 1$
$4^2 = 16$	$\log_4 16 = 2$
$4^3 = 64$	$\log_4 64 = 3$
$4^4 = 256$	$\log_4 256 = 4$
$5^0 = 1$	$\log_5 1 = 0$
$5^1 = 5$	$\log_5 5 = 1$
$5^2 = 25$	$\log_5 25 = 2$
$5^3 = 125$	$\log_5 125 = 3$
$5^4 = 625$	$\log_5 625 = 4$

$6^0 = 1$	$\log_6 1 = 0$
$6^1 = 6$	$\log_6 6 = 1$
$6^2 = 36$	$\log_6 36 = 2$
$6^3 = 216$	$\log_6 216 = 3$
$7^0 = 1$	$\log_7 1 = 0$
$7^1 = 7$	$\log_7 7 = 1$
$7^2 = 49$	$\log_7 49 = 2$
$7^3 = 343$	$\log_7 343 = 3$
$8^0 = 1$	$\log_8 1 = 0$
$8^1 = 8$	$\log_8 8 = 1$
$8^2 = 64$	$\log_8 64 = 2$
$8^3 = 512$	$\log_8 512 = 3$
$9^0 = 1$	$\log_9 1 = 0$
$9^1 = 9$	$\log_9 9 = 1$
$9^2 = 81$	$\log_9 81 = 2$
$9^3 = 729$	$\log_9 729 = 3$
$10^0 = 1$	$\log_{10} 1 = 0$
$10^1 = 10$	$\log_{10} 10 = 1$
$10^2 = 100$	$\log_{10} 100 = 2$
$10^3 = 1000$	$\log_{10} 1000 = 3$
$11^0 = 1$	$\log_{11} 1 = 0$
$11^1 = 11$	$\log_{11} 11 = 1$
$11^2 = 121$	$\log_{11} 121 = 2$
$11^3 = 1331$	$\log_{11} 1331 = 3$

For now, the student should try to memorize as many of the exponentiations (in magenta) as possible!

For #1 – 11, simplify each expression fully. No decimals or negative exponents. Show your work!

Questions 1 – 11 require you to simplify an expression with not-so-nice exponents. A key question you will face with each questions is: How do I start? Here's some general advice about that:

- **Decide at the beginning of the problem if you prefer to work with roots, e.g., $\sqrt[3]{64^4}$ or fractional exponents, e.g., $64^{4/3}$. If you prefer one method over the other, convert the problem into that form and continue from there. In the solutions below, I use either form, depending on the problem. You may use the form opposite of what I use if you prefer.**
- **Get rid of negative exponents as soon as possible. Remember to move the term with the negative exponent across the fraction line and change the exponent to positive.**
- **Numbers can be broken into their prime factors for simplification, e.g., $\sqrt[3]{24} = \sqrt[3]{2^3 \cdot 3} = \sqrt[3]{2^3} \cdot \sqrt[3]{3} = 2\sqrt[3]{3}$.**
- **If you have multiple terms with the same base, e.g., $4^{7/3} \cdot 4^{2/3}$, combine them early in the process.**
- **If you have multiple terms with the same root, e.g., $\frac{\sqrt[3]{135}}{\sqrt[3]{5}}$, combine them early in the process.**

1) $-64^{\frac{4}{3}}$ Key point: apply the negative sign last.

$-64^{4/3} = -\left(\sqrt[3]{64}\right)^4 = -4^4 = \mathbf{-256}$

Notice the difference:

$-4^4 = -256$

$(-4)^4 = 256$

2) $\frac{\sqrt[3]{135}}{\sqrt[3]{5}}$ Key point: bring everything under the cube root sign.

$\frac{\sqrt[3]{135}}{\sqrt[3]{5}} = \sqrt[3]{\frac{135}{5}} = \sqrt[3]{27} = \mathbf{3}$

3) $\frac{3}{\sqrt[3]{25}}$ Key point: rationalize the denominator.

$$\frac{3}{\sqrt[3]{25}} = \frac{3}{\sqrt[3]{5^2}}$$

$$= \frac{3}{\sqrt[3]{5^2}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}}$$

$$= \frac{3\sqrt[3]{5}}{\sqrt[3]{5^3}} = \mathbf{\frac{3\sqrt[3]{5}}{5}}$$

We want a denominator with an exponent that is a multiple of 3, so we need one more 5 under the radical in the denominator.

4) $\left(\sqrt[3]{243d^5}\right)^2$ Key point: separate the numbers from the variables.

$$\left(\sqrt[3]{243d^5}\right)^2 = \left(\sqrt[3]{243}\right)^2 \cdot \left(\sqrt[3]{d^5}\right)^2$$

$$= \left(\sqrt[3]{3^5}\right)^2 \cdot \left(\sqrt[3]{d^5}\right)^2$$

Convert any numbers to forms containing exponents.

$$= \left(\sqrt[3]{3^{10}}\right) \cdot \left(\sqrt[3]{d^{10}}\right)$$

Next step: break each term into two terms, one of which has an exponent that is the highest multiple of 3 possible (because of the $\sqrt[3]{\ }$), and the other has the balance of the exponent.

$$= \left(\sqrt[3]{3^9}\right)\left(\sqrt[3]{3}\right) \cdot \left(\sqrt[3]{d^9}\right)\left(\sqrt[3]{d}\right)$$

$$= (3^3)\left(\sqrt[3]{3}\right) \cdot (d^3)\left(\sqrt[3]{d}\right)$$

$$= 27d^3\left(\sqrt[3]{3}\right)\left(\sqrt[3]{d}\right)$$

$$= \mathbf{27d^3\,\sqrt[3]{3d}}$$

5) $(49x^6)^{-\frac{3}{2}}$ Key point: negative exponents move terms over the fraction line.

Be patient and handle this one step at a time.

$$(49x^6)^{-3/2} = \frac{1}{(49x^6)^{3/2}}$$

$$= \frac{1}{(49)^{3/2} \cdot (x^6)^{3/2}}$$

$$= \frac{1}{(7^2)^{3/2} \cdot (x^6)^{3/2}}$$

$$= \frac{1}{(7^3) \cdot (x^9)}$$

When a power is taken to a power, multiply the exponents.

$$= \frac{\mathbf{1}}{\mathbf{343\,x^9}}$$

6) $\sqrt[3]{2} + 3\sqrt[3]{162} - 2\sqrt[3]{128}$ Key point: simplify each term, then see what you can combine.

This one is hard because of the large numbers involved. If necessary, develop a prime number tree for any numbers you find difficult to reduce to exponential form. See page 10 in the Pre-Algebra Handbook (v 2.1) for instructions on how to create a prime number tree.

$$\sqrt[3]{2} + 3\sqrt[3]{162} - 2\sqrt[3]{128} = \sqrt[3]{2} + 3\sqrt[3]{2 \cdot 3^4} - 2\sqrt[3]{2^7}$$

$$= \sqrt[3]{2} + 3 \cdot \sqrt[3]{3^4} \cdot \sqrt[3]{2} - 2\sqrt[3]{2^7}$$

Next step: break each term into two terms, one of which has an exponent that is the highest multiple of 3 possible (because of the $\sqrt[3]{}$), and the other has the balance of the exponent.

$$= \sqrt[3]{2} + 3 \cdot \sqrt[3]{3^3} \cdot \sqrt[3]{3} \cdot \sqrt[3]{2} - 2\sqrt[3]{2^6} \cdot \sqrt[3]{2}$$

$$= \sqrt[3]{2} + 3 \cdot 3 \cdot \sqrt[3]{6} - 2 \cdot 2^2 \cdot \sqrt[3]{2}$$

$$= \sqrt[3]{2} + 9\sqrt[3]{6} - 8\sqrt[3]{2}$$

$$= \mathbf{-7\sqrt[3]{2} + 9\sqrt[3]{6}}$$

7) $(27x^2)^{\frac{5}{3}}$ Key point: separate the numbers from the variables.

$$(27x^2)^{5/3} = (27)^{5/3} \cdot (x^2)^{5/3}$$

$$= (3^3)^{5/3} \cdot (x^2)^{5/3}$$ Convert any numbers to values with exponents.

$$= 3^5 \cdot x^{10/3}$$ When a power is taken to a power, multiply the exponents.

$$= 3^5 \cdot x^{9/3} \cdot x^{1/3}$$ Break $x^{10/3}$ into two terms.

$$= \mathbf{243\, x^3 \sqrt[3]{x}}$$

8) $\dfrac{3}{\sqrt[3]{4}}$ Key point: rationalize the denominator.

$$\frac{3}{\sqrt[3]{4}} = \frac{3}{\sqrt[3]{2^2}}$$

$$= \frac{3}{\sqrt[3]{2^2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$$

$$= \frac{3\sqrt[3]{2}}{\sqrt[3]{2^3}} = \mathbf{\frac{3\sqrt[3]{2}}{2}}$$

We want a denominator with an exponent that is a multiple of 3, so we need one more 2 under the radical in the denominator.

9) $\left(\sqrt[3]{24x^6}\right)^2$ Key point: separate the numbers from the variables.

$$\left(\sqrt[3]{24x^6}\right)^2 = \left(\sqrt[3]{24}\right)^2 \cdot \left(\sqrt[3]{x^6}\right)^2$$

$$= \left(\sqrt[3]{2^3 \cdot 3}\right)^2 \cdot \left(\sqrt[3]{x^6}\right)^2$$ Convert any numbers to values with exponents.

$$= \left(\sqrt[3]{2^3}\right)^2 \cdot \left(\sqrt[3]{3}\right)^2 \cdot \left(\sqrt[3]{x^6}\right)^2$$ Separate the different numbers (2 and 3).

$$= \left(\sqrt[3]{2^6}\right) \cdot \left(\sqrt[3]{3^2}\right) \cdot \left(\sqrt[3]{x^{12}}\right)$$ Bring the exponents under th radicals.

Next step: we would normally break each term into two terms, one of which has an exponent that is the highest multiple of 3 possible (because of the $\sqrt[3]{\ }$), and the other has the balance of the exponent. However, this is not needed here because each exponent is already either a multiple of 3 or less than 3.

$$= (2^2) \cdot \left(\sqrt[3]{9}\right) \cdot (x^4)$$

$$= \mathbf{4\, x^4 \sqrt[3]{9}}$$

10) $\dfrac{3^{3/4} \cdot 9^{5/4}}{27}$ Key point: After converting 9 to 3^2, work carefully with the exponents of the 3's.

$$\frac{3^{3/4} \cdot 9^{5/4}}{27} = \frac{3^{3/4} \cdot (3^2)^{5/4}}{27}$$ Convert any numbers to values with exponents.

$$= \frac{3^{3/4} \cdot 3^{10/4}}{27}$$ When a power is taken to a power, multiply the exponents.

$$= \frac{3^{13/4}}{27}$$ When multiplying numbers with the same base, add the exponents.

Next step: break the numerator into two terms, one of which has an exponent with a numerator that is the highest multiple of 4 possible (because the exponent has a denominator of 4), and the other has the balance of the exponent.

$$= \frac{\left(3^{12/4}\right) \cdot \left(3^{1/4}\right)}{27}$$

$$= \frac{3^3 \cdot \sqrt[4]{3}}{27}$$

$$= \frac{27\sqrt[4]{3}}{27} = \mathbf{\sqrt[4]{3}}$$

11) $\dfrac{\sqrt[4]{x}\cdot\sqrt[4]{x^7}}{\sqrt[4]{x^{11}}}$ Key point: Combine terms in the numerator; then rationalize the denominator.

$$\frac{\sqrt[4]{x}\cdot\sqrt[4]{x^7}}{\sqrt[4]{x^{11}}}=\frac{\sqrt[4]{x\cdot x^7}}{\sqrt[4]{x^{11}}}$$

$$=\frac{\sqrt[4]{x^8}}{\sqrt[4]{x^{11}}}\cdot\frac{\sqrt[4]{x}}{\sqrt[4]{x}}$$ We want a denominator with an exponent that is a multiple of 3.

$$=\frac{x^2\cdot\sqrt[4]{x}}{\sqrt[4]{x^{12}}}$$ Simplify along the way where possible.

$$=\frac{x^2\cdot\sqrt[4]{x}}{x^3}$$

$$=\frac{\sqrt[4]{x}}{x}$$

12) Are the following functions inverses? Explain your reasoning.

$f(x)=24x^3$ and $g(x)=\dfrac{\sqrt[3]{x}}{3}$

Two functions $f(x)$ and $g(x)$ are inverses if and only if:

- $f(g(x))=x$ for every x in the domain of g, and
- $g(f(x))=x$ for every x in the domain of f.

For this problem, try:

$$f(g(x))=24\left(\frac{\sqrt[3]{x}}{3}\right)^3=24\left(\frac{\sqrt[3]{x^3}}{3^3}\right)=24\,\frac{x}{27}=\frac{24x}{27}\neq x$$

Therefore, the functions are not inverses.

Note: if **either** $(g(x))\neq x$ **or** $f(g(x))\neq x$, the functions are **not** inverses. You do not need to show that both are not equal to x in order to prove two functions are not inverses.

13) For $f(x) = 4x^2$, find f^{-1} if $x \geq 0$. Then graph both $f(x)$ and the inverse on the same coordinate system.

Original equation: $y = 4x^2$

Note: If $x \geq 0$ for $f(x)$, then $y \geq 0$ for $f^{-1}(x)$.

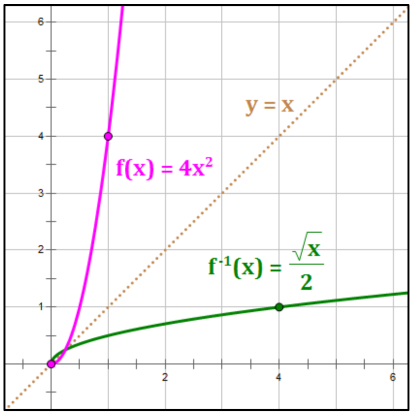
Switch variables: $x = 4y^2$

Divide by 4: $\frac{x}{4} = y^2$

Take square roots: $\pm\sqrt{\frac{x}{4}} = y$

Recall that $y \geq 0$: $\sqrt{\frac{x}{4}} = y$

Simplify: $\frac{\sqrt{x}}{2} = f^{-1}(x)$



Identify some points for graphing:

$f(0) = 4(0)^2 = 0$ $f(0) = 0$ $f^{-1}(0) = 0$

$f(1) = 4(1)^2 = 4$ $f(1) = 4$ $f^{-1}(4) = 1$

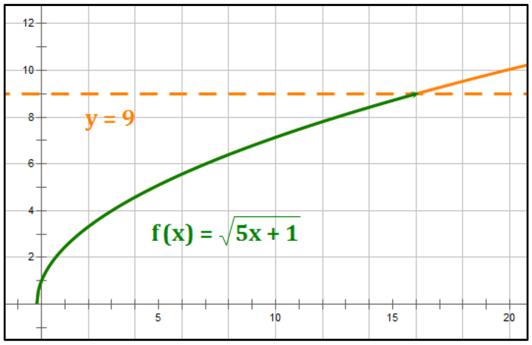
Notice the symmetry.

If (a, b) is a point of $f(x)$, then (b, a) is a point of $f^{-1}(x)$.

Solve and graph the solution to each inequality on the number line:

14) $\sqrt{5x+1} < 9$

This graph not required, but is informative!



First get the function's domain:

Radicand must be ≥ 0: $5x + 1 \geq 0$

Subtract 1: $5x \geq -1$

Divide by 5: $x \geq -\frac{1}{5}$ or $-\frac{1}{5} \leq x$

Solve the inequality:

Original inequality: $\sqrt{5x+1} < 9$

Square both sides: $5x + 1 < 81$

Subtract 1: $5x < 80$

Divide by 5: $x < 16$

Number line representation of solution (required)

Combine the results: $-\frac{1}{5} \leq x < 16$

15) $2\sqrt{3x-8}+2 \geq 4$

First get the function's domain:

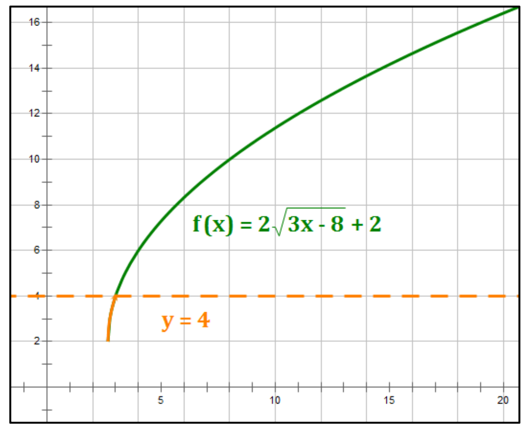
Radicand must be ≥ 0:	$3x-8 \geq 0$
Add 8:	$3x \geq 8$
Divide by 3:	$x \geq \frac{8}{3}$

Solve the inequality:

Original inequality:	$2\sqrt{3x-8}+2 \geq 4$
Subtract 2:	$2\sqrt{3x-8} \geq 2$
Divide by 2:	$\sqrt{3x-8} \geq 1$
Square both sides:	$3x-8 \geq 1$
Add 8:	$3x \geq 9$
Divide by 3:	$x \geq 3$

Combine the results: $x \geq 3$

This graph not required, but is informative!



Number line representation of solution (required)

For #16-21, solve the equations. Check for extraneous solutions. If needed, write answers in simplified radical form.

16) $\sqrt[3]{5x-1}+6=10$

Original equation:	$\sqrt[3]{5x-1}+6=10$
Subtract 6:	$\sqrt[3]{5x-1}=4$
Cube both sides:	$5x-1=64$
Add 1:	$5x=65$
Divide by 5:	$x=13$

Check the results: $\sqrt[3]{5(13)-1}+6=\sqrt[3]{64}+6=4+6=10$ ✓

Solution: $x = 13$

17) $9 = \sqrt{4x-3}$

Original equation:	$9 = \sqrt{4x-3}$
Square both sides:	$81 = 4x - 3$
Add 3:	$84 = 4x$
Divide by 4:	$21 = x$

Check the results: $\sqrt{4(21)-3} = \sqrt{81} = 9$ ✓

Solution: $x = 21$

18) $\sqrt{6x+1} = \sqrt{2x+13}$

Original equation:	$\sqrt{6x+1} = \sqrt{2x+13}$
Square both sides:	$6x + 1 = 2x + 13$
Subtract $2x$:	$4x + 1 = 13$
Subtract 1:	$4x = 12$
Divide by 4:	$x = 3$

Check the results: $\sqrt{6(3)+1} = \sqrt{2(3)+13}$

$\sqrt{19} = \sqrt{19}$ ✓

Solution: $x = 3$

19) $-5\sqrt[3]{x-1} = 40$

Original equation:	$-5\sqrt[3]{x-1} = 40$
Divide by -5:	$\sqrt[3]{x-1} = -8$
Cube both sides:	$x - 1 = -512$
Add 1:	$x = -511$

Check the results: $-5\sqrt[3]{(-511)-1}$

$= -5\sqrt[3]{-512}$

$= -5(-8) = 40$ ✓

Solution: $x = -511$

A Note about Square Rooting

Square roots are always positive. What?!? you say! What about this?

$$x^2 = 4$$

$$\sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2$$

It looks like square rooting causes both positive and negative roots!

As Johnny Carson used to say, "wrong, moose-breath!"

The above development implies that $\sqrt{x^2} = x$, which is not correct! At this stage in your math career, you need to learn a subtle, but very important point:

$$\sqrt{x^2} = |x|$$

Now, let's re-do the above problem:

$$x^2 = 4$$

$$\sqrt{x^2} = \sqrt{4}$$

$$|x| = 2$$

$$x = \pm 2$$

Same answer, but done without skipping the step involving the absolute value. So, remember: **Square roots are always positive.** That's why, in problem 17, $\sqrt{81} = 9$ and not -9.

20) $x^{2/3} = 16$

Original equation: $x^{2/3} = 16$

Take both sides to the $\frac{3}{2}$ power: $\left(x^{2/3}\right)^{3/2} = 16^{3/2}$

Simplify: $x = \left(\sqrt{16}\right)^3$

Simplify: $x = 4^3$

Simplify: $x = 64$

Check the results: $64^{2/3} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$ ✓

Solution: $x = 64$

21) $(x-3)^3 - 12 = 52$

Original equation: $(x-3)^3 - 12 = 52$

Add 12: $(x-3)^3 = 64$

Take cube roots: $x - 3 = \sqrt[3]{64}$

Simplify: $x - 3 = 4$

Add 3: $x = 7$

Check the results: $(7-3)^3 - 12 = 64 - 12 = 52$ ✓

Solution: $x = 7$

#22 – 24: For each radical function $y = a\sqrt[n]{x-h}+k$, describe the transformation from the parent function $y=\sqrt[n]{x}$, identify the domain and range, sketch the graph, and identify the end behavior.

22) $y = -\sqrt{x-4}-2$

Transformation:

$a = -1 < 0$ **Reflection over the x-axis**

$|a| = 1$ **No vertical compression or vertical stretch**

$h = 4$ **Translate 4 units right**

$k = -2$ **Translate 2 units down**

Domain:

$n = 2$ is even, so the radicand must be ≥ 0: $x - 4 \geq 0$

Add 4: $x \geq 4$

Domain: $x \geq 4$

Range:

$n = 2$ is even, so the function is limited to half the Cartesian Plane

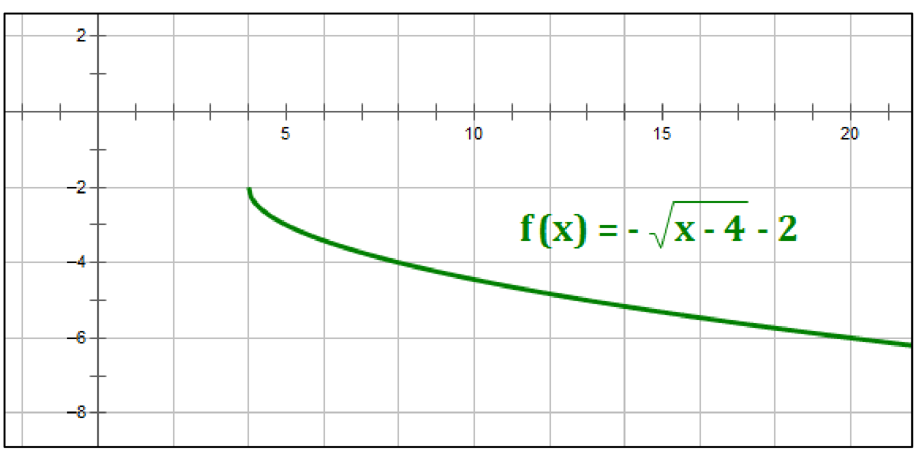
$a < 0$, so the half plane used is the portion below or equal to the constant k

Range: $y \leq -2$

End Behavior:

With the range being $y \leq -2$, the right side of the curve goes to $-\infty$.

End Behavior: As $x \to +\infty$, $f(x) \to -\infty$



23) $y = \frac{1}{3}\sqrt[3]{x} + 5$

Transformation:

$a = \frac{1}{3} > 0$ **No reflection over the x-axis**

$|a| = \frac{1}{3} < 1$ **Vertical compression**

$h = 0$ **No translation left or right**

$k = +5$ **Translate 5 units up**

Domain:

$n = 3$ is odd, so the domain is all real numbers

Domain: $\mathbb{R}$

Range:

$n = 3$ is odd, so the range is all real numbers

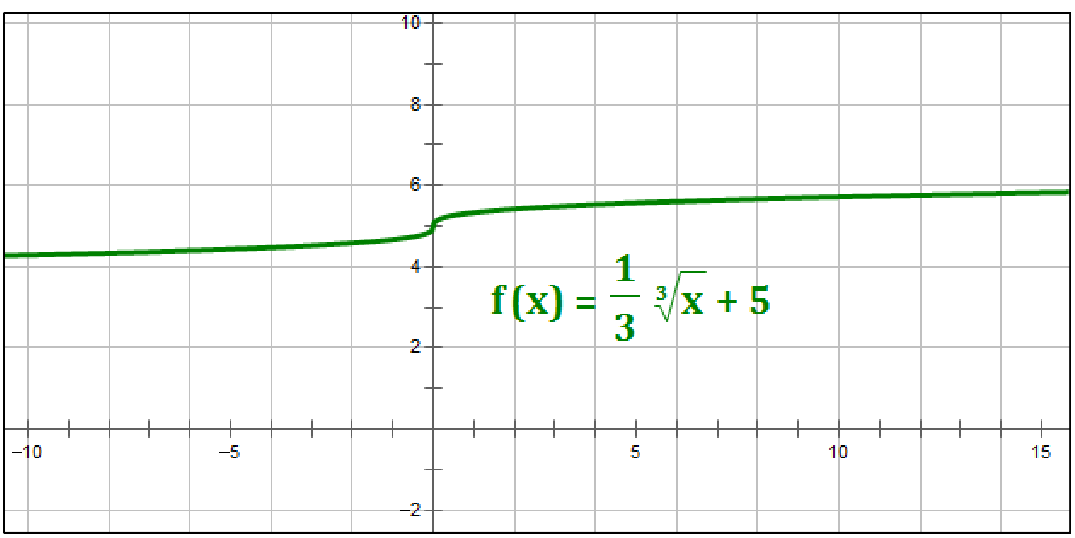
Range: $\mathbb{R}$

End Behavior:

$a > 0$, so the curve moves from lower left to top right

End Behavior: As $x \to +\infty$, $f(x) \to +\infty$

As $x \to -\infty$, $f(x) \to -\infty$



24) $y = 3\sqrt{x+2} + 4$

Transformation:

$a = 3 > 0$ **No reflection over the x-axis**

$|a| = 3 > 1$ **Vertical stretch**

$h = -2$ **Translate 2 units left**

$k = 4$ **Translate 4 units up**

Domain:

$n = 2$ is even, so the radicand must be ≥ 0: $x + 2 \geq 0$

Subtract 2: $x \geq -2$

Domain: $x \geq -2$

Range:

$n = 2$ is even, so the function is limited to half the Cartesian Plane

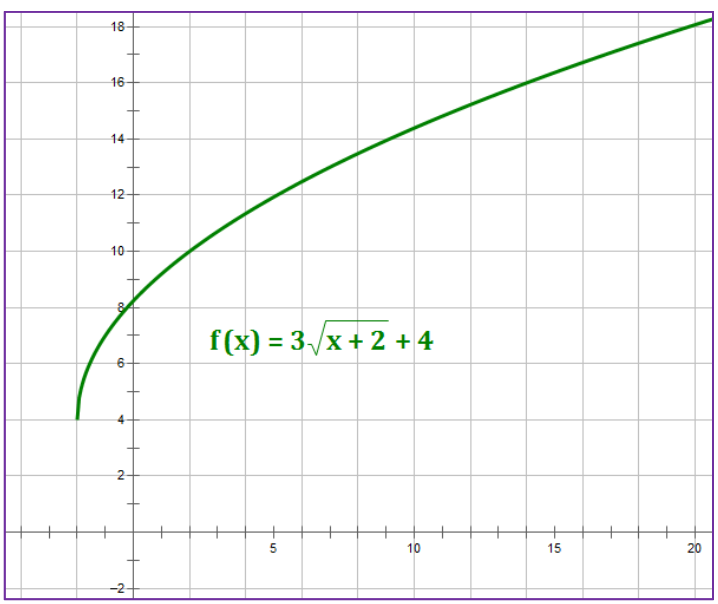
$a > 0$, so the half plane used is the portion above or equal to the constant k

Range: $y \geq 4$

End Behavior:

With the range being $y \geq 4$, the right side of the curve goes to $+\infty$.

End Behavior: As $x \to +\infty$, $f(x) \to +\infty$



25) Are $f(x)$ and $g(x)$ inverse functions? Explain.

$$f(x) = 4x^2,\ x \geq 0 \quad \text{and} \quad g(x) = \frac{\sqrt{x}}{2}.$$

Two functions $f(x)$ and $g(x)$ are inverses if and only if:

- $f(g(x)) = x$ for every x in the domain of g, and
- $g(f(x)) = x$ for every x in the domain of f.

For this problem, try:

$$f(g(x)) = 4\left(\frac{\sqrt{x}}{2}\right)^2 = 4\left(\frac{x}{4}\right) = \boldsymbol{x}$$

$$g(f(x)) = \frac{\sqrt{4x^2}}{2} = \frac{2\,|x|}{2} = |\boldsymbol{x}| = \boldsymbol{x} \quad \text{as long as } x \geq 0$$

Since the domain of $g(x)$ is $x \geq 0$, this condition is met.

Therefore, the functions are inverses.

26) Find the inverse, f^{-1}, of $f(x) = x^2 + 3$ if $x \geq 0$. Then graph both $f(x)$ and the inverse on the same coordinate system.

Original equation: $y = x^2 + 3$ Note: If $x \geq 0$ for $f(x)$, then $y \geq 0$ for $f^{-1}(x)$.

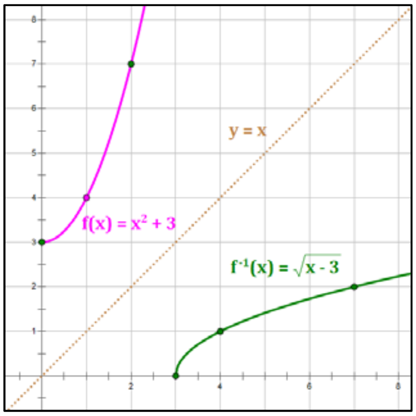
Switch variables: $x = y^2 + 3$

Subtract 3: $x - 3 = y^2$

Take square roots: $\pm\sqrt{x-3} = y$

Recall that $y \geq 0$: $\sqrt{x-3} = y$

Simplify: $\boldsymbol{\sqrt{x-3} = f^{-1}(x)}$



Identify some points for graphing:

$f(0) = (0)^2 + 3 = 3$	$f(0) = 3$	$f^{-1}(3) = 0$
$f(1) = (1)^2 + 3 = 4$	$f(1) = 4$	$f^{-1}(4) = 1$
$f(2) = (2)^2 + 3 = 7$	$f(2) = 7$	$f^{-1}(7) = 2$

Notice the symmetry.

If (a, b) is a point of $f(x)$, then (b, a) is a point of $f^{-1}(x)$.

For #27 – 30, solve the equations. Check for extraneous solutions. If needed, write answers in simplified radical form.

27) $-5x^6 = -320$

Original equation: $-5x^6 = -320$

Divide by -5: $x^6 = 64$

Take sixth roots: $\sqrt[6]{x^6} = \sqrt[6]{64}$

Simplify: $|x| = 2$

Solve for x: $x = \pm 2$

Check the results: $-5(2)^6 = -5(64) = -320$ ✓

$-5(-2)^6 = -5(64) = -320$ ✓

Solution: $x = \pm 2$

28) $x - 8 = \sqrt{18x}$

Original equation: $x - 8 = \sqrt{18x}$

Square both sides: $x^2 - 16x + 64 = 18x$

Subtract $18x$: $x^2 - 34x + 64 = 0$

Factor the trinomial: $(x-2)(x-32) = 0$

Solve for x: $x \in \{2, 32\}$

Check the results: $2 - 8 = \sqrt{18 \cdot 2}$ $\quad$ $32 - 8 = \sqrt{18 \cdot 32}$

$-6 = 6$ ✗ $\quad$ $24 = 24$ ✓

Solution: $x = 32$

29) $\sqrt{8x+3} = -2$

Easy peezy. Remember that **square roots are always positive**. See the explanation in the box on page 9 above. So, no square root is equal to -2. Therefore, **this equation has no solution.**

30) $\sqrt{5x+9} - 10 = 12$

Original equation: $\sqrt{5x+9} - 10 = 12$

Add 10: $\sqrt{5x+9} = 22$

Square both sides: $5x + 9 = 484$

Subtract 9: $5x = 475$

Divide by 5: $x = 95$

Check the results: $\sqrt{5(95)+9} - 10 = \sqrt{484} - 10 = 22 - 10 = 12$ ✓

Solution: $x = 95$

For #31 – 32, solve and graph the solution to each inequality on the number line:

31) $1 + \sqrt{2x+2} > 5$

First get the function's domain:

Radicand must be ≥ 0: $2x + 2 \geq 0$

Subtract 2: $2x \geq -2$

Divide by 2: $x \geq -1$

Solve the inequality:

Original inequality: $1 + \sqrt{2x+2} > 5$

Subtract 1: $\sqrt{2x+2} > 4$

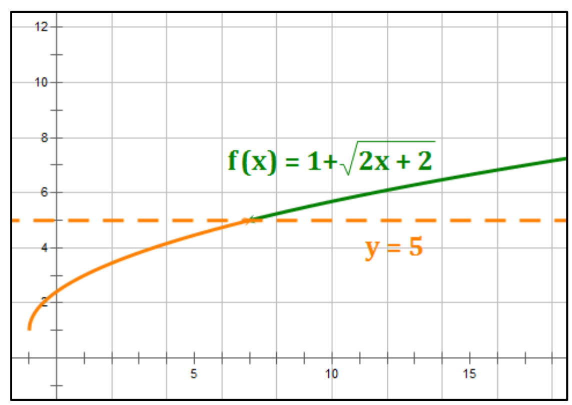
Square both sides: $2x + 2 > 16$

Subract 2: $2x > 14$

Divide by 2: $x > 7$

Combine the results: $x > 7$

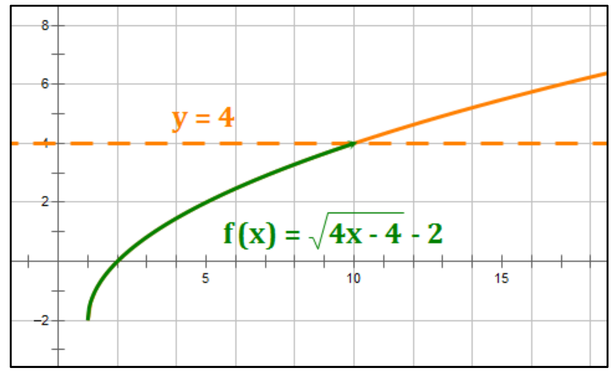
This graph not required, but is informative!



Number line representation of solution (required)

32) $\sqrt{4x-4}-2 \le 4$

This graph not required, but is informative!



First get the function's domain:

Radicand must be ≥ 0: $4x-4 \ge 0$

Add 4: $4x \ge 4$

Divide by 4: $x \ge 1$ or $1 \le x$

Solve the inequality:

Original inequality: $\sqrt{4x-4}-2 \le 4$

Add 2: $\sqrt{4x-4} \le 6$

Square both sides: $4x-4 \le 36$

Add 4: $4x \le 40$

Divide by 4: $x \le 10$

Combine the results: $\mathbf{1 \le x \le 10}$

Number line representation of solution (required)