Algebra 2 Chapter 6 Practice Test | Name:______________________

For #1-9, graph the function and state the domain and range.

The domain of an exponential function is almost always "all real numbers." In general, there are three exceptions to this rule, i.e., where the domain of a function is not "all real numbers":

- When the function expresses the logarithm of a value that could be negative: e.g., $\log_4(x-3)$
- When the function expresses an even root of a value that could be negative: e.g., $\sqrt{x+6}$
- When the function contains a fraction that could have zero in the denominator: e.g., $\frac{1}{2x-7}$

None of these situations are present in problems 1 to 9, so the domain for all of them is "all real numbers"

The range of an exponential function is determined by its lead coefficient and its constant, as follows:

If you consider the general form of an exponential equation to be: $f(x) = \pm\, a^{bx+c} + d$, then

- If the lead coefficient is positive (+), the range is: $x > d$
- If the lead coefficient is negative (−), the range is: $x < d$

Now, let's do these problems.

Note: An asymptote is a line that a function approaches, but never reaches. For an exponential function, the equation of the asymptote is $y = d$, where d is the constant in the equation.

1. $y = 2^{x+1}$ (note: the constant is 0)

1st step: Asymptote: $y = 0$

Asymptote is also useful in setting the range.

2nd step: Select x-values of points:

Select a point where the exponent is zero;

$x + 1 = 0$

$x = -1$

Also select $x = 0$, to make things easy.

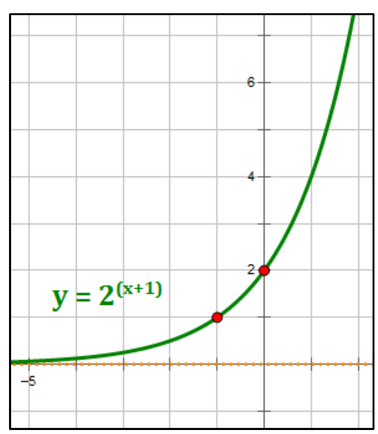
So, our x-values are $x = -1$ and $x = 0$

3rd step: Calculate y-values of points:

$x = -1$: $y = 2^{(-1+1)} = 1$ *Point*: $(-1, 1)$

$x = 0$: $y = 2^{(0+1)} = 2$ *Point*: $(0, 2)$

4th step: Draw the curve based on the asymptote and the two points.



Domain: all real numbers

Range: $y > 0$

2. $y = -4^x$ (note: the constant is 0)

1st step: Asymptote: $y = 0$

Asymptote is also useful in setting the range.

2nd step: Select x-values of points:

Select a point where the exponent is zero;

$x = 0$

We need another point where $x \neq 0$. In this case, let's select $x = 1$, to make things easy.

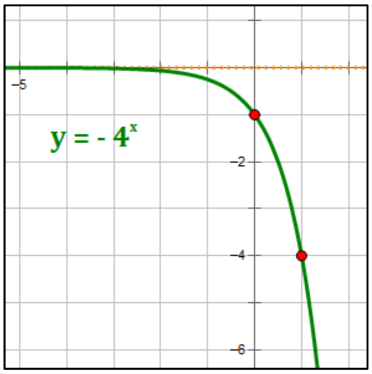
So, our x-values are $x = 0$ and $x = 1$

3rd step: Calculate y-values of points:

$x = 0$: $y = -4^{(0)} = -1$ *Point*: $(0, -1)$

$x = 1$: $y = -4^{(1)} = -4$ *Point*: $(1, -4)$

4th step: Draw the curve based on the asymptote and the two points.



Domain: all real numbers

Range: $y < 0$

3. $y = 3^x + 1$ (note: the constant is 1)

1st step: Asymptote: $y = 1$

Asymptote is also useful in setting the range.

2nd step: Select x-values of points:

Select a point where the exponent is zero;

$x = 0$

We need another point where $x \neq 0$. In this case, let's select $x = 1$, to make things easy.

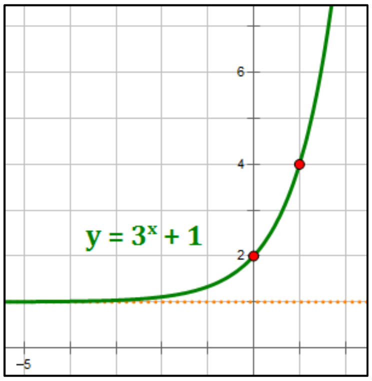
So, our x-values are $x = 0$ and $x = 1$

3rd step: Calculate y-values of points:

$x = 0$: $y = 3^{(0)} + 1 = 2$ *Point*: $(0, 2)$

$x = 1$: $y = 3^{(1)} + 1 = 4$ *Point*: $(1, 4)$

4th step: Draw the curve based on the asymptote and the two points.



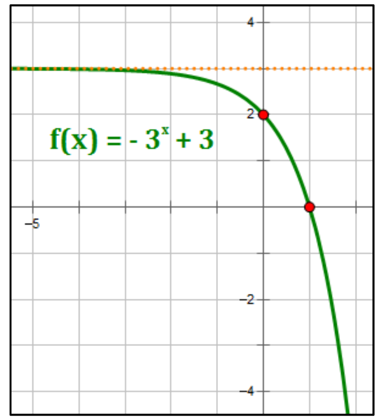
Domain: all real numbers

Range: $y > 1$

4. $f(x) = -3^x + 3$ (note: the constant is 3)

1[st] step: Asymptote: $y = 3$

Asymptote is also useful in setting the range.



Domain: all real numbers

Range: $y < 3$

2[nd] step: Select x-values of points:

Select a point where the exponent is zero;

$x = 0$

We need another point where $x \neq 0$. In this case, let's select $x = 1$, to make things easy.

So, our x-values are $x = 0$ and $x = 1$

3[rd] step: Calculate y-values of points:

$x = 0$: $f(x) = -3^{(0)} + 3 = 2$ $Point$: $(0, 2)$

$x = 1$: $f(x) = -3^{(1)} + 3 = 0$ $Point$: $(1, 0)$

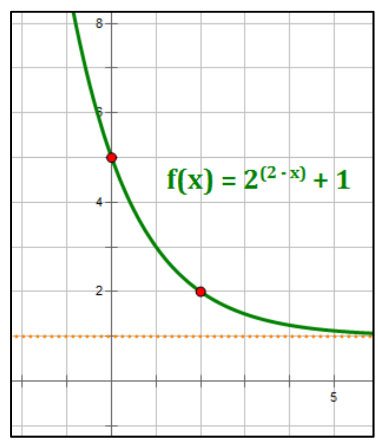
4[th] step: Draw the curve based on the asymptote and the two points.

5. $f(x) = \left(\frac{1}{2}\right)^{x-2} + 1$ (note: the constant is 1)

Uh oh! We hate fractions! So convert this equation to: $f(x) = 2^{(2-x)} + 1$ by inverting the base and negating the exponent. This is the same function, but in a form that is easier to work with.

1[st] step: Asymptote: $y = 1$

Asymptote is also useful in setting the range.



Domain: all real numbers

Range: $y > 1$

2[nd] step: Select x-values of points:

Select a point where the exponent is zero;

$2 - x = 0$

$x = 2$

Also select $x = 0$, to make things easy.

So, our x-values are $x = 2$ and $x = 0$

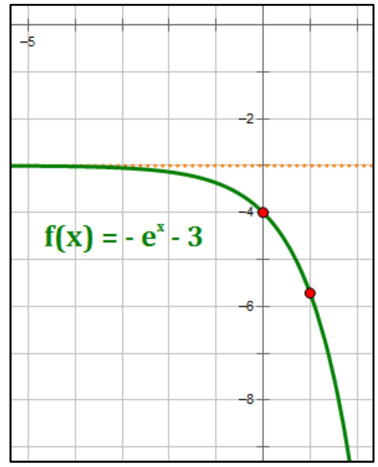
3[rd] step: Calculate y-values of points:

$x = 2$: $f(x) = 2^{(2-2)} + 1 = 2$ $Point$: $(2, 2)$

$x = 0$: $f(x) = 2^{(2-0)} + 1 = 5$ $Point$: $(0, 5)$

4[th] step: Draw the curve based on the asymptote and the two points.

6. $f(x) = -e^x - 3$ (note: the constant is -3)



Domain: all real numbers
Range: $y < -3$

1st step: Asymptote: $y = -3$ *Asymptote is also useful in setting the range.*

2nd step: Select x-values of points:

Select a point where the exponent is zero;

$x = 0$

We need another point where $x \neq 0$. In this case, let's select $x = 1$, to make things easy.

So, our x-values are $x = 0$ and $x = 1$

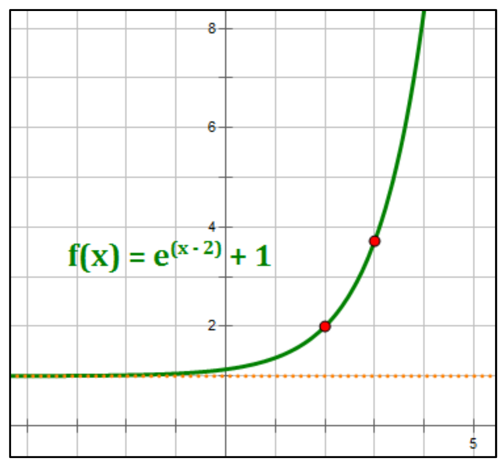
3rd step: Calculate y-values of points:

$x = 0$: $f(x) = -e^{(0)} - 3 = -1 - 3 = -4$ *Point*: $(0, -4)$

$x = 1$: $f(x) = -e^{(1)} - 3 \sim -2.7 - 3 = -5.7$ *Point*: $(1, -5.7)$

4th step: Draw the curve based on the asymptote and the two points.

7. $f(x) = e^{x-2} + 1$ (note: the constant is 1)



Domain: all real numbers
Range: $y > 1$

1st step: Asymptote: $y = 1$ *Asymptote is also useful in setting the range.*

2nd step: Select x-values of points:

Select a point where the exponent is zero;

$x - 2 = 0$

$x = 2$

Also select $x = 3$, to make things easy.
Note, picking $x = 0$ would make things difficult!

So, our x-values are $x = 2$ and $x = 3$

3rd step: Calculate y-values of points:

$x = 2$: $f(x) = e^{(2-2)} + 1 = 1 + 1 = 2$ *Point*: $(2, 2)$

$x = 3$: $f(x) = e^{(3-2)} + 1 \sim 2.7 + 1 = 3.7$ *Point*: $(3, 3.7)$

4th step: Draw the curve based on the asymptote and the two points.

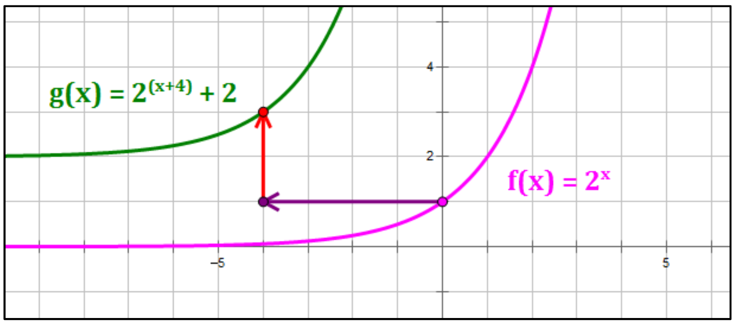
8. Write an equation for a function that shifts left 4 and up 2 from the parent function, $f(x) = 2^x$.

Look at each piece separately:

Shift the graph four units to the left: add 4 to x $\Rightarrow$ $(x+4)$

Shift the graph two units up: add 2 as a constant $\Rightarrow$ $+2$ at the end

$g(x) = 2^{(x+4)} + 2$



For $g(x)$:
Domain: all real numbers
Range: $y > 2$

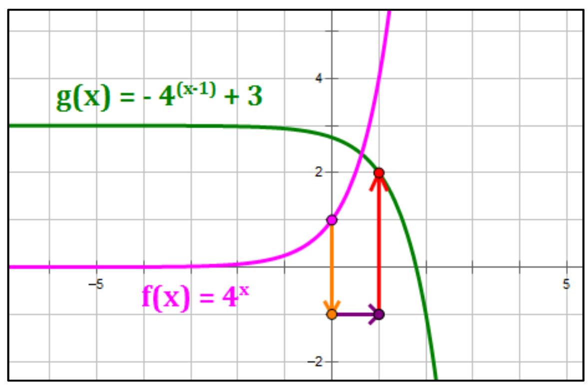
9. Describe in words how the graph of $f(x) = -4^{x-1} + 3$ would be transformed from the parent function $f(x) = 4^x$.

Look at each piece separately:

$-$ sign: **Reflect the graph over the x-axis**

$(x-1)$ exponent: **Shift the graph one unit to the right**

$(+3)$ constant: **Shift the graph three units up**



For $g(x)$:
Domain: all real numbers
Range: $y < 3$

For #10-21, simplify. No decimals or negative exponents. Show your work!

10. $\dfrac{3^2 \cdot 9^4}{3^5}$

$$\frac{3^2 \cdot 9^4}{3^5} = \frac{3^2 \cdot (3^2)^4}{3^5} = \frac{3^2 \cdot 3^8}{3^5} = 3^{2+8-5} = 3^5 = 243$$

11. $\dfrac{5^3 \cdot 25^2}{(5^{-1})^3}$

$$\frac{5^3 \cdot 25^2}{(5^{-1})^3} = \frac{5^3 \cdot (5^2)^2}{5^{-3}} = \frac{5^3 \cdot 5^4}{5^{-3}} = 5^3 \cdot 5^4 \cdot 5^3 = 5^{3+4+3} = \mathbf{5^{10}} = \mathbf{9,765,625}$$

12. $\dfrac{(x^{-3}y^4)^3}{x^2y^3}$

$$\frac{(x^{-3}y^4)^3}{x^2 \cdot y^3} = \frac{x^{-9} \cdot y^{12}}{x^2 \cdot y^3} = \frac{y^{12}}{x^9 \cdot x^2 \cdot y^3} = \left(\frac{1}{x^9 \cdot x^2}\right) \cdot \left(\frac{y^{12}}{y^3}\right) = \left(\frac{1}{x^{11}}\right) \cdot \left(\frac{y^9}{1}\right) = \frac{\boldsymbol{y^9}}{\boldsymbol{x^{11}}}$$

13. $\dfrac{18x^7y^{-2}z}{3x^{-1}yz}$

$$\frac{18 \cdot x^7 \cdot y^{-2} \cdot z}{3 \cdot x^{-1} \cdot y \cdot z} = \frac{18 \cdot x^7 \cdot x \cdot z}{3 \cdot y \cdot y^2 \cdot z} = \left(\frac{18}{3}\right) \cdot \left(\frac{x^8}{1}\right) \cdot \left(\frac{1}{y^3}\right) \cdot \left(\frac{z}{z}\right) = 6 \cdot \frac{x^8}{y^3} \cdot 1 = \frac{\mathbf{6}\,\boldsymbol{x^8}}{\boldsymbol{y^3}}$$

14. $\dfrac{\sqrt[4]{a^3} \cdot \sqrt[4]{a^2}}{a^2}$

$$\frac{\sqrt[4]{a^3} \cdot \sqrt[4]{a^2}}{a^2} = \frac{\sqrt[4]{a^3 \cdot a^2}}{a^2} = \frac{\sqrt[4]{a^5}}{a^2} = \frac{\sqrt[4]{a^4} \cdot \sqrt[4]{a}}{a^2} = \frac{a \cdot \sqrt[4]{a}}{a^2} = \frac{\boldsymbol{\sqrt[4]{a}}}{\boldsymbol{a}}$$

15. $\sqrt[3]{-64a^3} = \sqrt[3]{-64} \cdot \sqrt[3]{a^3}$

$$= \sqrt[3]{-1} \cdot \sqrt[3]{64} \cdot \sqrt[3]{a^3}$$

$$= -1 \cdot 4 \cdot a$$

$$= \boldsymbol{-4a}$$

16. $\sqrt[4]{1250x^{24}y^{31}} = \sqrt[4]{1250} \cdot \sqrt[4]{x^{24}} \cdot \sqrt[4]{y^{31}}$

$= \sqrt[4]{5^4 \cdot 2} \cdot \sqrt[4]{x^{24}} \cdot \sqrt[4]{y^{28}} \cdot \sqrt[4]{y^3}$

$= \sqrt[4]{5^4} \cdot \sqrt[4]{2} \cdot x^6 \cdot y^7 \cdot \sqrt[4]{y^3}$

$= 5\,x^6y^7 \cdot \sqrt[4]{2} \cdot \sqrt[4]{y^3}$

$= \mathbf{5\,x^6\,y^7\,\sqrt[4]{2y^3}}$

17. $\sqrt[5]{480x^{12}y^{15}z^8} = \sqrt[5]{480} \cdot \sqrt[5]{x^{12}} \cdot \sqrt[5]{y^{15}} \cdot \sqrt[5]{z^8}$

$= \sqrt[5]{2^5 \cdot 3 \cdot 5} \cdot \sqrt[5]{x^{10}} \cdot \sqrt[5]{x^2} \cdot \sqrt[5]{y^{15}} \cdot \sqrt[5]{z^5} \cdot \sqrt[5]{z^3}$

$= \sqrt[5]{2^5} \cdot \sqrt[5]{3 \cdot 5} \cdot x^2 \cdot \sqrt[5]{x^2} \cdot y^3 \cdot z \cdot \sqrt[5]{z^3}$

$= 2\,x^2y^3z \cdot \sqrt[5]{15} \cdot \sqrt[5]{x^2} \cdot \sqrt[5]{z^3}$

$= \mathbf{2\,x^2\,y^3\,z\,\sqrt[5]{15\,x^2\,z^3}}$

18. $\dfrac{\sqrt[5]{x^7} \cdot \sqrt[5]{x^6}}{x^3}$

$$\frac{\sqrt[5]{x^7} \cdot \sqrt[5]{x^6}}{x^3} = \frac{\sqrt[5]{x^7 \cdot x^6}}{x^3} = \frac{\sqrt[5]{x^{13}}}{x^3} = \frac{\sqrt[5]{x^{10}} \cdot \sqrt[5]{x^3}}{x^3} = \frac{x^2 \cdot \sqrt[5]{x^3}}{x^3} = \frac{\sqrt[5]{x^3}}{x}$$

19. $\sqrt[4]{240x^{16}y^{27}} = \sqrt[4]{240} \cdot \sqrt[4]{x^{16}} \cdot \sqrt[4]{y^{27}}$

$= \sqrt[4]{2^4 \cdot 3 \cdot 5} \cdot \sqrt[4]{x^{16}} \cdot \sqrt[4]{y^{24}} \cdot \sqrt[4]{y^3}$

$= \sqrt[4]{2^4} \cdot \sqrt[4]{3 \cdot 5} \cdot x^4 \cdot y^6 \cdot \sqrt[4]{y^3}$

$= 2\,x^4y^6 \cdot \sqrt[4]{15} \cdot \sqrt[4]{y^3}$

$= \mathbf{2\,x^4\,y^6\,\sqrt[4]{15y^3}}$

20. $\dfrac{\sqrt[3]{k^2} \cdot \sqrt[3]{k^5}}{k^4}$

$$\frac{\sqrt[3]{k^2} \cdot \sqrt[3]{k^5}}{k^4} = \frac{\sqrt[3]{k^2 \cdot k^5}}{k^4} = \frac{\sqrt[3]{k^7}}{k^4} = \frac{\sqrt[3]{k^6} \cdot \sqrt[3]{k}}{k^4} = \frac{k^2 \cdot \sqrt[3]{k}}{k^4} = \frac{\sqrt[3]{k}}{k^2}$$

21. $\sqrt[3]{216x^{13}y^6z^{17}} = \sqrt[3]{216} \cdot \sqrt[3]{x^{13}} \cdot \sqrt[3]{y^6} \cdot \sqrt[3]{z^{17}}$

$= \sqrt[3]{216} \cdot \sqrt[3]{x^{12}} \cdot \sqrt[3]{x} \cdot \sqrt[3]{y^6} \cdot \sqrt[3]{z^{15}} \cdot \sqrt[3]{z^2}$

$= 6 \cdot x^4 \cdot \sqrt[3]{x} \cdot y^2 \cdot z^5 \cdot \sqrt[3]{z^2}$

$= 6x^4y^2z^5 \cdot \sqrt[3]{x} \cdot \sqrt[3]{z^2}$

$= \mathbf{6\ x^4\ y^2\ z^5\ \sqrt[3]{xz^2}}$

For #22-26, solve for x.

22. $3^{2x+1} = 3^{4x-5}$

When the bases are the same, you can set the exponents equal.

$2x + 1 = 4x - 5$

$6 = 2x$

$\mathbf{3 = x}$

23. $2^{x+3} = 32$

$2^{x+3} = 32$

$2^{x+3} = 2^5$

Once the bases are the same, you can set the exponents equal.

$x + 3 = 5$

$\mathbf{x = 2}$

24. $3^{x-2} = 81$

$3^{x-2} = 81$

$3^{x-2} = 3^4$

Once the bases are the same, you can set the exponents equal.

$x - 2 = 4$

$\mathbf{x = 6}$

25. $4^{3x} = 64^{2x-7}$

Method 1:

$4^{3x} = 64^{2x-7}$

$4^{3(x)} = 64^{2x-7}$

$64^{x} = 64^{2x-7}$

Once the bases are the same, you can set the exponents equal.

$x = 2x - 7$

$\mathbf{7 = x}$

Method 2:

$4^{3x} = 64^{2x-7}$

$4^{3x} = (4^3)^{2x-7}$

$4^{3x} = 4^{3(2x-7)}$

Once the bases are the same, you can set the exponents equal.

$3x = 6x - 21$

$21 = 3x$

$\mathbf{7 = x}$

26. $2^{4x+4} = 16^{3x-1}$

Method 1:

$2^{4x+4} = 16^{3x-1}$

$2^{4(x+1)} = 16^{3x-1}$

$16^{(x+1)} = 16^{3x-1}$

Once the bases are the same, you can set the exponents equal.

$x + 1 = 3x - 1$

$2 = 2x$

$\mathbf{1 = x}$

Method 2:

$2^{4x+4} = 16^{3x-1}$

$2^{4x+4} = (2^4)^{3x-1}$

$2^{4x+4} = 2^{4(3x-1)}$

Once the bases are the same, you can set the exponents equal.

$4x + 4 = 12x - 4$

$8 = 8x$

$\mathbf{1 = x}$

Given $f(x) = 5x - 7, \; g(x) = 8x, \; and \; h(x) = 4x + 1$:

27. Find $g(f(x))$

$g\big(f(x)\big) = g(5x - 7) = 8(5x - 7) = \mathbf{40x - 56}$

28. Find $f(h(x))$

$f\big(h(x)\big) = f(4x + 1) = 5(4x + 1) - 7 = 20x + 5 - 7 = \mathbf{20x - 2}$

29. Find $f(x) - h(x)$

$f(x) - h(x) = (5x - 7) - (4x + 1) = (5x - 4x) + (-7 - 1) = \mathbf{x - 8}$

30. Find $f(x) * g(x)$

$f(x) \cdot g(x) = (5x - 7) \cdot (8x) = \mathbf{40x^2 - 56x}$