1. Rewrite the absolute value inequality as a compound inequality for $|x+8|>5$.

A. $x>-3$ **B.** $-13<x<-3$ **C.** $x<-13$ or $x>-3$ **D.** no solution

First, we break the inequality into two pieces.

- Notice that one of the new inequalities flips the inequality sign and changes the sign of the "5".
- Also note the sign in the given inequality is “>” which, using poor spelling, we can refer to as the “greator than” sign, reminding us to put an “or” between the two new inequalities.

Starting Inequalities:	$x+8>5$	or	$x+8<-5$
Subtract 8:	$-8 \quad -8$		$-8 \quad -8$
Result:	$x \quad >-3$	or	$x \quad <-13$

Answer C

2. Which of the following expresses all of the solutions for the compound inequality below?

$$3(5-z)\geq 3 \text{ or } 5\geq 3-2z$$

A. $z\leq -1$ or $z\geq 4$

B. $-1\leq z\leq 4$

C. no solution

D. all real numbers

Watch the signs carefully as you proceed through this solution.

$3(5-z)\geq 3$	or	$5\geq 3-2z$
$\div 3 \quad \div 3$		$-3 \quad -3$
$5-z \geq 1$	or	$2 \geq -2z$
$-5 \quad -5$		$\div -2 \quad \div -2$
$-z \geq -4$	or	$1 \leq z$
$\div -1 \quad \div -1$		reverse the inequality
$z \leq 4$	or	$z \geq 1$

Interpretation time: We want the set of real numbers that are either less than or equal to 4 or greater than or equal to 1. All real numbers meet one or the other of these conditions.

Answer D

3. Write the equation of the line that passes through the point (-4, -3) and is parallel to the line -6x + 2y = -1.

A. y = -3x + 9 **B.** y = -3x – 15 **C.** y = 3x – 15 **D.** y = 3x +9

Let's find the slope of the original equation; the slope of the parallel line must be equal to it.

Original equation:	$-6x + 2y = \quad -1$
Add $6x$:	$+6x \qquad + 6x$
Result:	$2y = \quad 6x - 1$
Divide by 2:	$\div 2 \qquad \div 2$
Result:	$y = \quad 3x - \frac{1}{2}$

The slope must match the original equation if our line is to be parallel to it. So, $m = 3$. That leaves answers **C** and **D**. Let's use our point, $(-4, -3)$, to see which is correct:

If **C** is correct, then: $-3 = 3(-4) - 15 = -27$ **NOT!**

If **D** is correct, then: $-3 = 3(-4) + 9 = -3$ ✓ **Answer D**

4. Write an equation in point-slope form of the line that passes through the point $(2, -4)$ and has a slope of -4.

Point-slope form is: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is any point on the line.

So, point-slope form for this line would be: $\boldsymbol{y + 4 = -4(x - 2)}$

5. The value of y varies directly with x, and $y = 15$ when $x = 9$. What is the value of x when $y = 20$?

There is a shortcut for direct variations with two variables. Let's use it.

$y = ax$, so we have two equations based on the above: $15 = a \cdot 9$ and $20 = a \cdot x$.

Let's divide the second equation by the first one.

$$\frac{20}{15} = \frac{a \cdot x}{a \cdot 9} \quad \Rightarrow \quad \frac{4}{3} = \frac{x}{9} \quad \Rightarrow \quad \boldsymbol{x = 12}$$

6. What is the value of x in the solution of the following system of linear equations?

$$2x - 4y = 13$$
$$4x - 5y = 8$$

A. -5.5 **B.** 2 **C.** 6.5 **D.** no solution

Let's set this up to eliminate y, which will leave a solution for x.

$2x - 4y = 13$	multiply by (5) $\Rightarrow$	$10x - 20y = 65$
$4x - 5y = 8$	multiply by (-4) $\Rightarrow$	$-16x + 20y = -32$
	Add the equations:	$-6x = 33$
	Divide by -6:	$\div -6 \quad \div -6$
	Add the equations:	$\boldsymbol{x = -5.5}$

Answer A

7. What is the y-coordinate of the solution to the following system of equations?

$$2x + y - z = 5$$
$$x + 3z = 14$$
$$-2x - 3y + 2z = 2$$

A. -2 **B.** 0 **C.** 3 **D.** 5

Let's set this up to eliminate x and z, which will leave a solution for y.

Let's begin by working with the first two equations to eliminate x.

$2x + y - z = 5$	multiply by (-1) $\Rightarrow$	$-2x - y + z = -5$
$x + 3z = 14$	multiply by (2) $\Rightarrow$	$2x + 6z = 28$
	Add the equations:	$-y + 7z = 23$

Let's do the same (eliminate x) with the last two equations. Alternatively, we could use the first and last equation in this step. The final solution for the values of x, y and z would still be the same! Cool, huh?

$x + 3z = 14$	multiply by (2) $\Rightarrow$	$2x + 6z = 28$
$-2x - 3y + 2z = 2$	multiply by (1) $\Rightarrow$	$-2x - 3y + 2z = 2$
	Add the equations:	$-3y + 8z = 30$

(continued on next page)

Now that we are down to 2 equations in 2 unknowns, let's eliminate z.

$-y + 7z = 23$	multiply by (8) $\Rightarrow$	$-8y + 56z = 184$
$-3y + 8z = 30$	multiply by (-7) $\Rightarrow$	$21y - 56z = -210$
	Add the equations:	$13y = -26$
	Divide by 13:	$\div 13 \quad \div 13$
	Add the equations:	$y = -2$

Answer A

8. Solve the following linear system:

$$5x - 2y = 8$$
$$10x - 16 = 4y$$

A. (–4,0) **B.** (2, 9) **C.** infinitely many solutions **D.** no solution

Let's set this up to first eliminate x, which will leave a solution for y. The first step is to get both equations in the same form. Let's begin by getting the second equation in form of the first one. Note that this form is called Standard Form.

Original equation:	$10x - 16 = 4y$
Add $-4y + 16$:	$-4y + 16 \quad -4y + 16$
Result:	$10x - 4y = 16$

Now let's set it up to eliminate x:

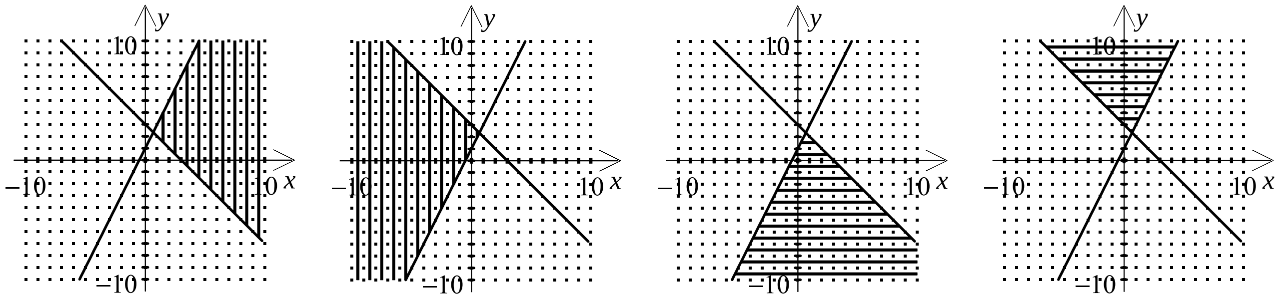
$5x - 2y = 8$	multiply by (2) $\Rightarrow$	$10x - 4y = 16$
$10x - 4y = 16$	multiply by (-1) $\Rightarrow$	$-10x + 4y = -16$
	Add the equations:	$0 = 0$

Since we got a result that is always true, the lines defined by the two equations are the same line. Therefore, there are infinitely many solutions.

Answer C

9. Graph the system of inequalities.

$$y \geq 2x + 1$$
$$x + y \geq 3$$



A **B** **C** **D**

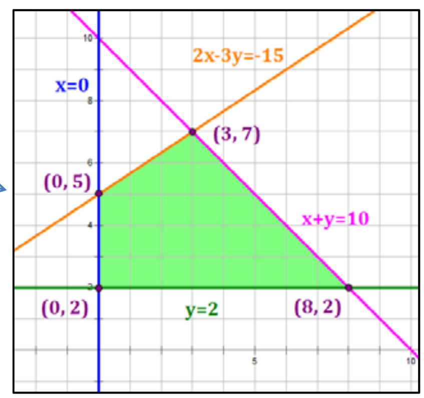
First, notice that the y variable is on the left hand side of both inequalities. This is good; if it were not, we would need to manipulate the offending equation to get y on the left.

Next, note that the $\geq$ signs imply that both shadings will be above the curve and have solid lines. There is only one graph where the shading is above both lines. Answer D

10. Using linear programming procedures, the equation $C = 5x + 9y$ is to be maximized subject to the following constraints.

$$x \geq 0$$
$$y \geq 2$$
$$x + y \leq 10$$
$$2x - 3y \geq -15$$

Graph the region of overlap for all of the inequalities.



The grid may be used to graph the feasible region.

What is the maximum value for the objective function?

A. 88
B. 78
C. 73
D. 53

Linear Programming Theory tells us that the maximum and minimum values of the objective function will be at points of intersection. If you graph carefully, you will be able to read the points of intersection off the graph; if not, you must calculate them.

Then calculate the value of the objective function at each point of intersection:

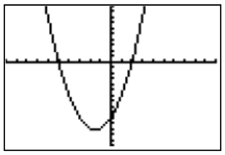
Point: $(0, 2)$ $C = 5(0) + 9(2) = 18$

Point: $(0, 5)$ $C = 5(0) + 9(5) = 45$

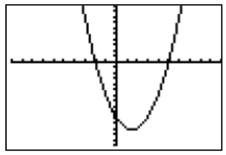
Point: $(8, 2)$ $C = 5(8) + 9(2) = 58$

Point: $(3, 7)$ $C = 5(3) + 9(7) = \mathbf{78}$ ✓ *Maximum value* **Answer B**

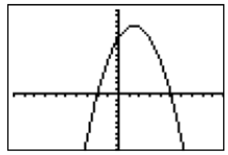
11. **Which graph from a graphing calculator represents the function $y = x^2 + 3x - 10$? (Assume the scale on each graph is one unit per tick mark.)**



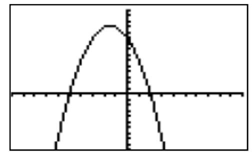
A



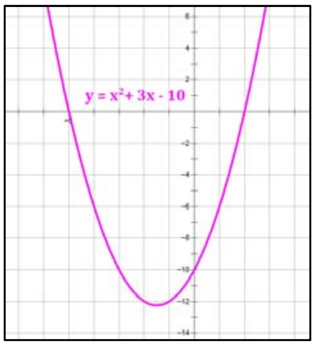
B



C



D



Answer A

Note: This problem can be solved easily without a graphing calculator:

- The lead coefficient is positive, so the curve opens up.
- $x^2 + 3x - 10 = (x+5)(x-2)$, so the roots are $x = \{-5, 2\}$

Voilà! Answer A

12. **Solve the quadratic equation $2x^2 - 15x + 13 = 0$ by factoring.**

A. $x = \frac{1}{2},\ x = 13$

B. $x = 1,\ x = 13$

C. $x = 1,\ x = \frac{13}{2}$

D. no solution

$2x^2 - 15x + 13$

Using the **AC Method**, we seek two numbers that multiply to get:

$2 \cdot 13 = 26$

And add to get -15

After a few tries, we can determine that the values we want are -2 and -13.

$= (2x^2 - 2x - 13x + 13)$

$= (2x^2 - 2x) - (13x - 13)$

$= 2x(x-1) - 13(x-1)$

$= (2x-13)(x-1)$

Then,

$(2x-13) = 0 \quad (x-1) = 0$

So, $x = \left\{\frac{13}{2}, 1\right\}$

Answer C

13. Solve $3(x+4)^2 = 27$

Starting Equation:	$3(x+4)^2 = 27$
Divide by 3:	$\div 3 \quad \div 3$
Result:	$(x+4)^2 = 9$
Take square roots:	$x+4 = \pm 3$
Subtract 4:	$-4 \quad -4$
Result:	$x = -4 \pm 3$
Break into separate solutions:	$x = -4-3 = -7 \qquad x = -4+3 = -1$
Identify solutions:	$\boldsymbol{x = \{-7, -1\}}$

14. Which shows the solutions for $3x^2 - 7x = 1$, using the quadratic formula?

A. $\left\{\frac{-7+\sqrt{61}}{6}, \frac{-7-\sqrt{61}}{6}\right\}$

B. $\left\{\frac{7+\sqrt{61}}{6}, \frac{7-\sqrt{61}}{6}\right\}$

C. $\left\{\frac{-7+\sqrt{35}}{6}, \frac{-7-\sqrt{35}}{6}\right\}$

D. $\left\{\frac{7+\sqrt{35}}{6}, \frac{7-\sqrt{35}}{6}\right\}$

For an equation in general form: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

First, convert the equation to general form.

Original equation:	$3x^2 - 7x = 1$
Subtract 1:	$-1 \quad -1$
Result:	$3x^2 - 7x - 1 = 0$

So, for this problem, we have: $a = 3$, $b = -7$, $c = -1$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{7 \pm \sqrt{61}}{6}$$

Answer B

15. Use the discriminant to determine the number and type of solutions of the equation $7x^2 - 5x = 6$.

A. 1 real solution, 1 complex solution
B. no real solutions, 2 complex solutions
C. 2 real solutions
D. 1 real solution, no complex solutions

The discriminant is the part of the quadratic formula that is under the radical. It is:

$$\Delta = b^2 - 4ac$$

If $\Delta > 0$, there are two real solutions.
If $\Delta = 0$, there is one real solution.
If $\Delta < 0$, there are zero real solutions, and two complex solutions.

First, convert the equation to general form.

Original equation:	$7x^2 - 5x$	$= 6$
Subtract 6:	-6	-6
Result:	$7x^2 - 5x - 6 =$	0

So, for this problem, we have:

$a = 7$, $b = -5$, $c = -6$

$$\Delta = (-5)^2 - 4(7)(-6) = 193$$

Since $\Delta > 0$, there are two real solutions.

Answer C

16. Solve the quadratic equation $2x^2 + 5x = -4$.

A. $\left\{\frac{5+i\sqrt{7}}{4}, \frac{5-i\sqrt{7}}{4}\right\}$

B. $\left\{\frac{-5+i\sqrt{7}}{4}, \frac{-5-i\sqrt{7}}{4}\right\}$

C. $\left\{\frac{-5+\sqrt{57}}{4}, \frac{-5-\sqrt{57}}{4}\right\}$

D. $\left\{\frac{5+\sqrt{57}}{4}, \frac{5-\sqrt{57}}{4}\right\}$

For an equation in general form: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

First, convert the equation to general form.

Original equation:	$2x^2 + 5x$	$= -4$
Add 4:	$+4$	$+4$
Result:	$2x^2 + 5x + 4 =$	0

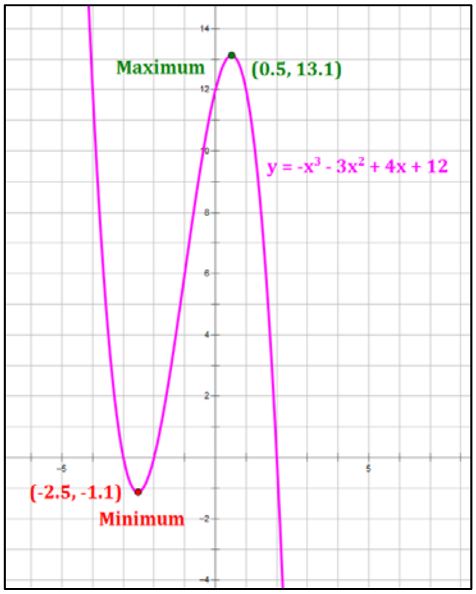
So, for this problem, we have: $a = 2$, $b = 5$, $c = 4$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(4)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{-7}}{4} = \frac{-5 \pm i\sqrt{7}}{4}$$

Answer B

17. Sketch a graph of the polynomial. State when it is increasing and decreasing and state the relative max and min $y = -x^3 - 3x^2 + 4x + 12$.



I assume this will be done on a calculator, so I do not explain all of the math behind the calculations.

Location of relative minimum: $(-2.5, -1.1)$
Value of relative minimum: -1.1

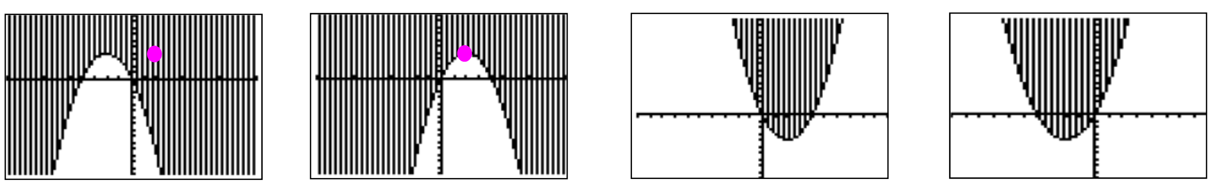
Location of relative maximum: $(0.5, 13.1)$
Value of relative maximum: 13.1

Increasing Interval: $-2.5 < x < 0.5$

Decreasing Intervals: $x < -2.5$ or $x > 0.5$

Note: you are not asked for the x-intercepts, but you could find them by factoring the equation. They are: $x = \{-3, -2, 2\}$.

18. Which of the following graphs from a graphing calculator represents the graph of $y \geq 4x - x^2$**? (Assume the scale on each graph is one unit per tick mark.)**



A **B** **C** **D**

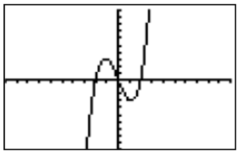
Here's what we can deduce about this curve:

- The shaded area for this graph will be above the curve and have a solid line because of the " $\geq$ " sign in the inequality. Unfortunately, all of the curves above meet this criterion.
- The curve will open down because of the negative sign in front of the x^2 term. This narrows down the answers to **A** and **B**.
- If $x = 2$, then $y \geq 4(2) - 2^2 = 4$. This occurs in **B** but not **A**.

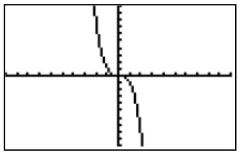
Answer B

19. Which graph represents the polynomial function $f(x) = -x^3 + 4x$? (Assume the scale on each graph is one unit per tick mark.)

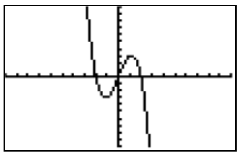
A



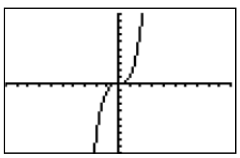
B

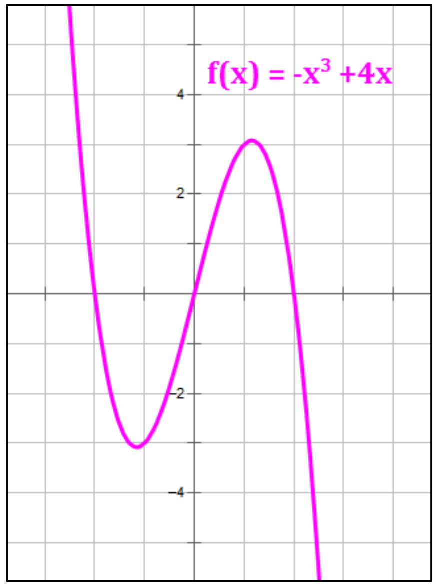


C



D





Answer C

Note: This problem can be solved easily without a graphing calculator:

- The lead coefficient is negative, so the curve goes to ∞ on the left.
- $f(x)$, can be easily factored to get zeros of $\{-2, 0, 2\}$

Voilà. Answer C

20. Solve the polynomial equation $x^4 - 8x^2 + 7 = 0$.

Starting Equation:	$x^4 - 8x^2 + 7 = 0$
Factor the trinomial:	$(x^2 - 7)\,(x^2 - 1) = 0$
Factor the difference of squares:	$(x^2 - 7)\,(x - 1)\,(x + 1) = 0$
Break into separate equations:	$(x^2 - 7) = 0 \quad x - 1 = 0 \quad x + 1 = 0$
Manipulate each equation:	$x^2 = 7 \quad x = 1 \quad x = -1$
Identify solutions:	$\boldsymbol{x = \{\pm\sqrt{7},\ -1,\ 1\}}$

21. Factor the polynomial $27x^3 + 64$.

This is a sum of cubes. Here are the formulas for the sum and difference of cubes. Memorize them!

$$a^3 + b^3 = (a+b)\,(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)\,(a^2 + ab + b^2)$$

Then,

$$27x^3 + 64 = (3x)^3 + 4^3$$ So, we will let $a = 3x$ and $b = 4$.

$$= (3x+4)\,((3x)^2 - 3x \cdot 4 + 4^2)$$
$$= \mathbf{(3x+4)\,(9x^2 - 12x + 16)}$$

22. Which is the set of all <u>real</u> zeros of the polynomial function $f(x) = 3x^3 - 6x^2 + 3x - 6$?

A. $\{2\}$

B. $\{2, 3\}$

C. $\{1, 3\}$

D. $\{-1, 1, 2, 3\}$

Let's factor the polynomial:

$$3x^3 - 6x^2 + 3x - 6 = 0$$
$$3 \cdot [x^3 - 2x^2 + x - 2] = 0$$
$$3 \cdot [(x^3 - 2x^2) + (x - 2)] = 0$$
$$3 \cdot [x^2(x-2) + 1(x-2)] = 0$$
$$3 \cdot [(x^2+1)\,(x-2)] = 0$$

When you have four terms, the first thing to try is to group the terms into pairs. Then, factor each pair. In Algebra 2 problems, this almost always works!

Now, let's look at the factors:

$(x^2 + 1) = 0 \quad \Rightarrow \quad x^2 = -1$ No real zeros

$(x - 2) = 0 \quad \Rightarrow \quad x = 2$

Answer A

23. Which of the following describes the end behavior of the graph of $f(x) = -x^4 + 4x - 7$ as $x \to +\infty$?

A. $f(x) \to +\infty$ **B.** $f(x) \to -\infty$ **C.** $f(x) \to -7$ **D.** $f(x) \to 0$

We need only look at the lead term of the polynomial.

All odd exponents have the same end behavior, and all even exponents have the same end behavior. The chart below provides a summary of end behavior of polynomials based on the sign of the lead coefficient and the degree of the lead term.

End Behavior of Polynomials		
Degree of Lead Term	**Lead Coefficient +**	**Lead Coefficient −**
Even	as $x \to -\infty$, $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to +\infty$	as $x \to -\infty$, $f(x) \to -\infty$ as $x \to +\infty$, $f(x) \to -\infty$
Odd	as $x \to -\infty$, $f(x) \to -\infty$ as $x \to +\infty$, $f(x) \to +\infty$	as $x \to -\infty$, $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to -\infty$

Since our lead coefficient is negative and the degree of the lead term (4) is even, the answer to this question is **Answer B**

24. Which of the following is the remainder when the polynomial $f(x) = 3x^3 - 4x + 8$ is divided by $x - 3$?

A. -59 **B.** -31 **C.** 23 **D.** 77

The easiest approach to this is to use synthetic division. First, note that the root implied by the divisor $(x-3)$ is $x = 3$.

$$
\begin{array}{r|rrrr}
3 & 3 & 0 & -4 & 8 \\
 & & 9 & 27 & 69 \\
\hline
 & 3 & 9 & 23 & 77
\end{array}
$$

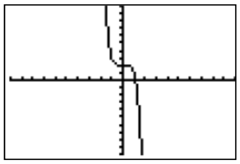
Answer D

For a polynomial fo degree n, synthetic division requires that we have columns for all exponents n and below. That's why we must have a column for x^2 even though there is no x^2 term in the polynomial in this problem.

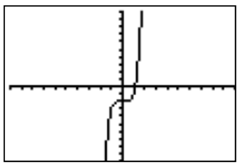
For a full explanation of synthetic division, see pages 122-123 of the Algebra Handbook or use the synthetic division section of the Algebra App, both of which are available at www.mathguy.us.

25. Which best represents the graph of the polynomial function $y = x^5 + 2$ as shown on a graphing calculator? (Assume the scale on each graph is one unit per tick mark.)

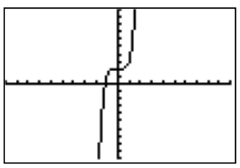
A



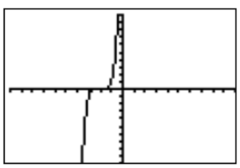
B

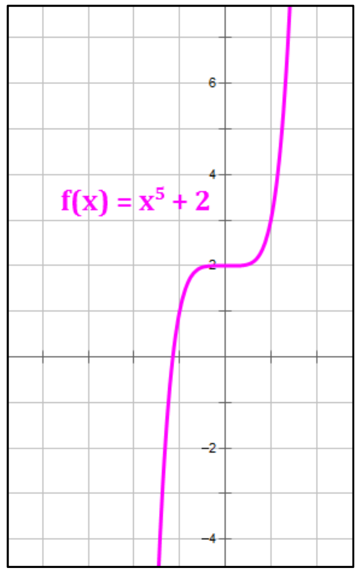


C



D





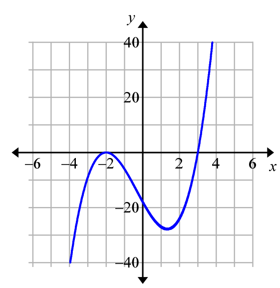
Answer C

Note: This problem can be solved easily without a graphing calculator:

- The lead coefficient is positive, so the curve goes to ∞ on the right.
- $f(0) = 2$, so the curve crosses the y-axis at $y = 2$.

Voilà. Answer C

26. Use the graph of the cubic function $y = s(x)$.



Describe the end behavior of $y = s(x)$ as $x \to \infty$.

Look on the right side of the graph. $s(x) \to \infty$

27. What is the solution set of $|-2x+2| \le 12$? Notice " $\le$ " contains a "less thand" sign. Use "and".

A. $\{x \mid x \ge -5\}$ **B.** $\{x \mid -7 \le x \le 5\}$ **C.** $\{x \mid -5 \le x \le 7\}$ **D.** $\{x \mid -\infty < x < \infty\}$

Starting Inequalities:	$-2x + 2 \le 12$	and	$-2x + 2 \ge -12$
Subtract 2:	$-2 \quad -2$		$-2 \quad -2$
Result:	$-2x \quad \le 10$	and	$-2x \quad \ge -14$
Divide by -2:	$\div -2 \quad \div -2$		$\div -2 \quad \div -2$
Result:	$x \quad \ge -5$	and	$x \quad \le \quad 7$

So, $-5 \le x \le 7$ Answer C

28. What are the solutions of $x^2 - 6x + 10 = 0$?

A. $x = 2$ or $x = 4$

B. $x = -10$ or $x = -4$

C. $x = 3 + i$ or $x = 3 - i$

D. $x = -3 + i$ or $x = -3 - i$

For an equation in general form: $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

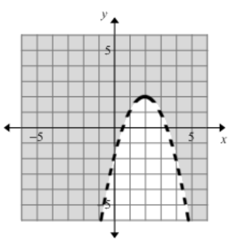
For this problem, we have: $a = 1$, $b = -6$, $c = 10$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$

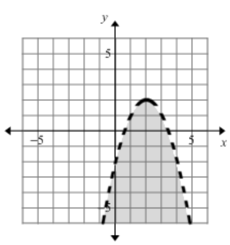
$$= \frac{6 \pm \sqrt{-4}}{2} = \frac{6 \pm 2i}{2} = \mathbf{3 \pm i}$$

Answer C

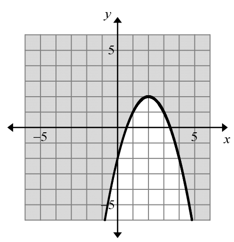
29. Which of the following graphs represents the quadratic inequality $y \le -x^2 + 4x - 2$



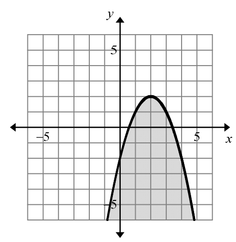
A



B



C



D

The inequality has y on the left and a " $\le$ " sign, so the curve is a solid line and the shaded area is under the curve. That's enough information to select **Answer D**

30. One factor of $x^3 + 2x^2 - 11x - 12$ is $x + 4$. What are the remaining factors?

A. $x + 1$ and $x + 3$

B. $x - 1$ and $x + 3$

C. $x + 1$ and $x - 3$

D. $x - 1$ and $x - 3$

Note that the root implied by the divisor $(x + 4)$ is $x = -4$.

$$\begin{array}{r|rrrr} -4 & 1 & 2 & -11 & -12 \\ & & -4 & 8 & 12 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

The result is: $x^2 - 2x - 3 = 0$

Factor the trinomial: $(\boldsymbol{x - 3})(\boldsymbol{x + 1})$ **Answer C**

31. Which is the product $(8+i)(6+2i)$ in standard form?

A. $50+22i$

B. $48+24i$

C. $46+22i$

D. $48+20i$

$(8+i)\cdot(6+2i)$

F: $8\cdot 6=48$

O: $8\cdot 2i=16i$

I: $i\cdot 6=6i$

L: $i\cdot 2i=2i^2=-2$

Result: $(48-2)+(16i+6i)$

$=\mathbf{46+22i}$ **Answer C**

32. State the end behavior of the graph of $f(x)=-x^3+7x+4$ as $x\to-\infty$.

A. $f(x)\to-\infty$

B. $f(x)\to+\infty$

C. $f(x)\to 4$

D. $f(x)\to 0$

The degree of this equation is "odd", so if the lead coefficient were positive, x and $f(x)$ would tend toward the same limit. However, since the lead coefficient is negative, x and $f(x)$ tend toward opposite limits. **Answer B**

Also, you may want to look at the chart provided in the solution to problem 23.

33. What is x^3-3x^2-6 divided by $x-5$?

A. $x^2+2x+10+\dfrac{44}{x-5}$

B. $x^2-8x+40-\dfrac{206}{x-5}$

C. $x^2-8x-\dfrac{46}{x-5}$

D. $x^2+2x+\dfrac{4}{x-5}$

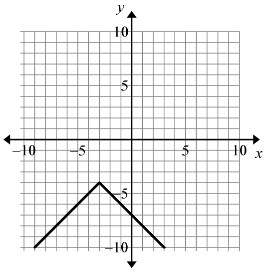
Note that the root implied by the divisor $(x-5)$ is $x=5$.

5	1	−3	0	−6
		5	10	50
	1	2	10	44

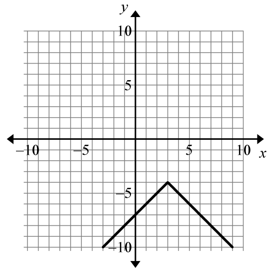
The resulting polynomial (plus remainder) is:

$$\mathbf{x^2+2x+10+\left(\frac{44}{x-5}\right)}$$ **Answer A**

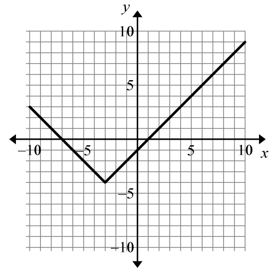
34. Which is the graph of $y = -|x+3| - 4$?



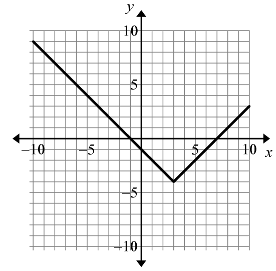
A



B



C



D

The general equation of an absolute value function is:

$f(x) = a\,|x - h| + k$, where (h, k) is the vertex of the curve.

So, for the equation above, we have $(h, k) = (-3, -4)$. Further, the equation has a negative lead coefficient, so it opens down. That's enough information to select **Answer A.**

35. Write the expression $\dfrac{7+3i}{3+9i}$ as a complex number in standard form.

A. $\dfrac{1}{12} - \dfrac{3}{4}i$

B. $\dfrac{8}{15} - \dfrac{3}{5}i$

C. $\dfrac{8}{15} + \dfrac{4}{5}i$

D. $\dfrac{1}{12} + i$

Multiply by the conjugate of the complex number in the denominator.

$$\frac{7+3i}{3+9i} = \frac{7+3i}{3+9i} \cdot \frac{3-9i}{3-9i}$$

$$= \frac{7 \cdot 3 + 7 \cdot (-9i) + (3i) \cdot 3 + (3i) \cdot (-9i)}{3 \cdot 3 + 3 \cdot (-9i) + (9i) \cdot 3 + (9i) \cdot (-9i)}$$

$$= \frac{21 - 63i + 9i - 27i^2}{9 - 27i + 27i - 81i^2}$$

$$= \frac{21 + 27 - 54i}{9 + 0 + 81}$$

$$= \frac{48 - 54i}{90} = \frac{8}{15} - \frac{3}{5}i$$

Answer B

36. What is the best representation of the solution of the inequality $2x^2 + x - 36 < 0$?

A. $-5 < x < 4$ **B.** $x < -5$ *or* $x > 4$ **C.** $-4 < x < \frac{9}{2}$ **D** $-\frac{9}{2} < x < 4$

Write the corresponding equation: $2x^2 + 1x - 36 = 0$

Using the **AC Method**, we seek two numbers that multiply to get:
$2 \cdot (-36)$
$= -72$

And add to get $+1$

After a few tries, we can determine that the values we want are -8 and $+9$.

Split the middle term: $2x^2 - 8x + 9x - 36 = 0$

Group terms: $(2x^2 - 8x) + (9x - 36) = 0$

Factor each term: $2x(x - 4) + 9(x - 4) = 0$

Factor the trinomial: $(2x + 9)(x - 4) = 0$

Break into separate equations: $2x + 9 = 0$ $x - 4 = 0$

Solutions for x in the equation: $x = \left\{-\frac{9}{2},\ 4\right\}$

Then, set up a table of intervals based on the solutions for x and test each interval to determine the sign of the function in that interval:

Interval	$x < -\frac{9}{2}$	$-\frac{9}{2} < x < 4$	$x > 4$
Terms of $f(x)$	$(2x + 9)(x - 4)$	$(2x + 9)(x - 4)$	$(2x + 9)(x - 4)$
Signs of terms	$- \cdot -$	$+ \cdot -$	$+ \cdot +$
Sign of $f(x)$	$+$	$-$	$+$

Based on the results in the table, $2x^2 + x - 36 < 0$ when $-\frac{9}{2} < x < 4$.

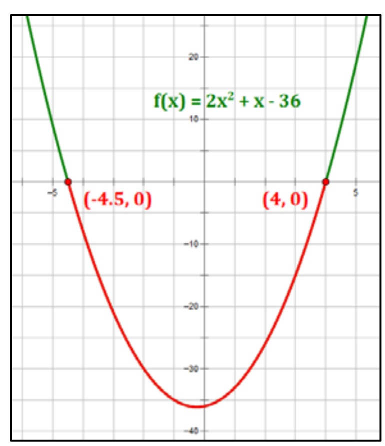
So, the solution set is: $-\frac{9}{2} < x < 4$ **Answer D**

ALTERNATIVE (CALCULATOR) APPROACH

If you are allowed to use a calculator, graph the function and isolate where it is below the x-axis (for a function of form $f(x) < 0$).

In the graph at right, the portion of the curve that is below the x-axis is shown in red. Clearly, the associated x-interval is: $-\frac{9}{2} < x < 4$.

If the problem had been of the form $f(x) > 0$, you would want the intervals in green, i.e., the intervals above the x-axis.



37. Solve the System: $\begin{cases} y = 3x^2 + 4x - 9 \\ y = 2x^2 - 2x + 7 \end{cases}$

A. (2, 11), (8, 151) B. (2, 11), (-8, 151) C. (-2, 11), (-8, 151) D. (-2, 11), (8, -151)

Both equations are of the form: $y = \cdots$, so set them equal to each other.

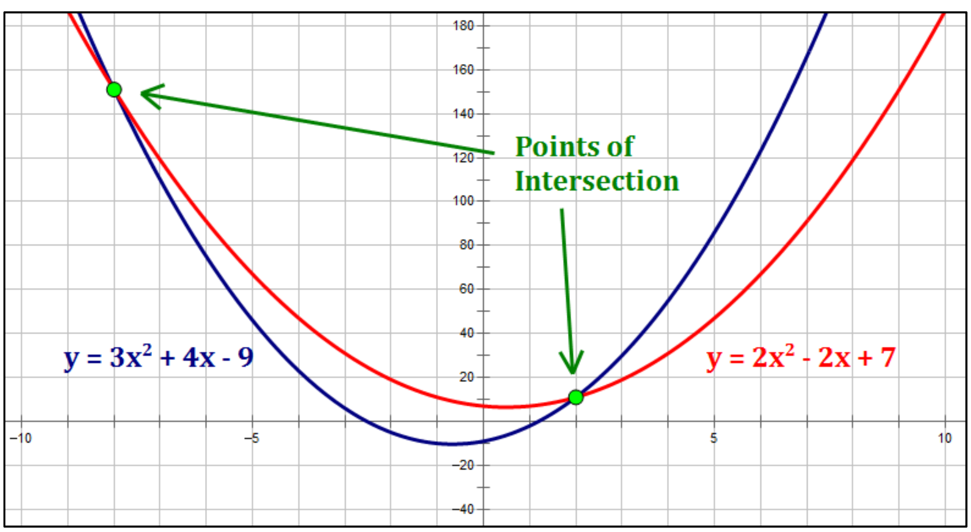
Original equations:	$3x^2 + 4x - 9 = 2x^2 - 2x + 7$
Subtract $2x^2 - 2x + 7$:	$\underline{-2x^2 + 2x - 7 \quad - 2x^2 + 2x - 7}$
Result:	$x^2 + 6x - 16 = 0$
Factor the trinomial:	$(x + 8)(x - 2) = 0$
Break into separate equations:	$(x + 8) = 0 \qquad (x - 2) = 0$
Solutions for x:	$x = \{-8, 2\}$

Now, we have to find the y-values that go with these x-values. You can use either equation for this. I typically go with the simpler equation, in this case: $y = 2x^2 - 2x + 7$.

$x = -8$: $y = 2 \cdot (-8)^2 - 2 \cdot (-8) + 7 = 128 + 16 + 7 = 151$

$x = 2$: $y = 2 \cdot (2)^2 - 2 \cdot (2) + 7 = 8 - 4 + 7 = 11$

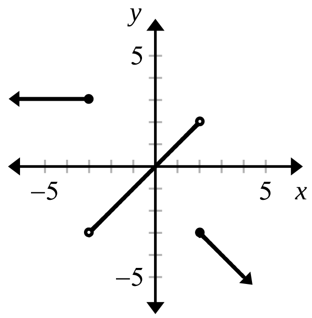
So, the two solutions are: $\mathbf{(-8, 151), (2, 11)}$ **Answer B**



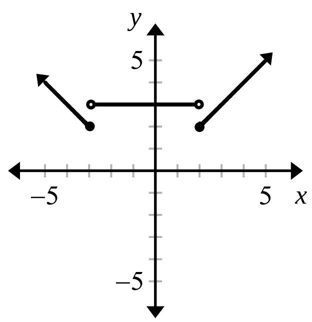
38. Which graph represents the piecewise function below?

$$f(x)=\begin{cases} x, & \text{if } x \geq 2 \\ 3, & \text{if } -3 < x < 2 \\ -x-1, & \text{if } x \leq -3 \end{cases}$$

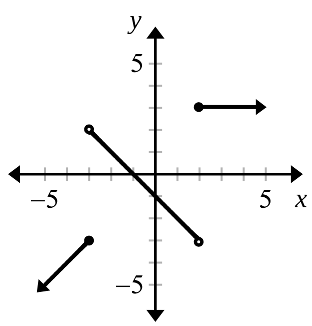
A.



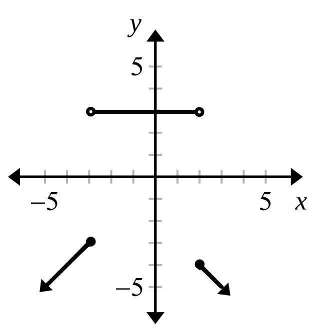
B.



C.



D.



Don't get freaked out by this. Let's just take it one piece at a time and see what happens.

- The first thing I notice is that the middle interval has a constant value over the whole interval.

  $f(x) = 3, \text{ if } -3 < x < 2$

  Cool! That narrows our answers down to **B** and **D** because they have flat lines in the middle.

- Then, notice that the line on the right side of answer **B** has a positive slope, whereas the line on the right side of answer **D** has a negative slope. Looking back up at $f(x)$ reveals that for $x \geq 2$, the slope is 1, which is positive.

Therefore, our answer must be **Answer B**.

39. $\begin{cases} y = -5x \\ 2x^2 + y = 3 \end{cases}$

A. $(3, 15), \left(-\frac{1}{2}, \frac{5}{2}\right)$ B. $(-3, 15), \left(-\frac{1}{2}, \frac{5}{2}\right)$ C. $(3, -15), \left(\frac{1}{2}, -\frac{5}{2}\right)$ D. $(3, -15), \left(-\frac{1}{2}, \frac{5}{2}\right)$

This system is tailor made for substitution because one of the equations contains an unfettered variable (i.e., a variable that is not multiplied by or added to anything).

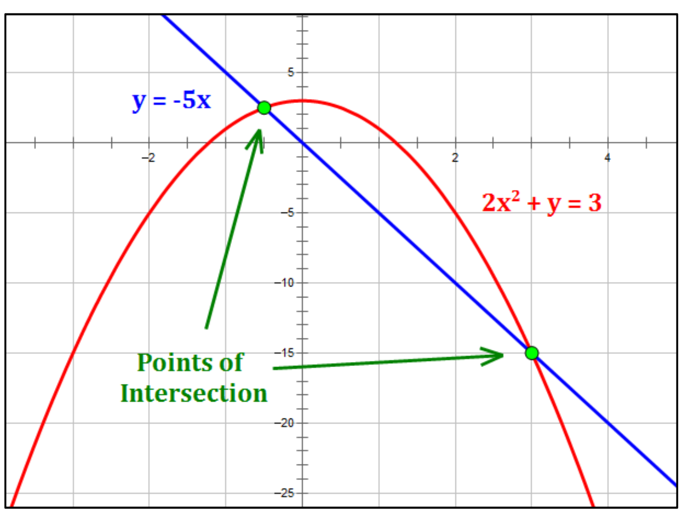
Original equation:	$2x^2 + y = 3$
Substitute $-5x$ for y:	$2x^2 + (-5x) = 3$
Clean up:	$2x^2 - 5x = 3$
Subtract 3:	$-3 \quad -3$
Result:	$2x^2 - 5x - 3 = 0$
Factor the trinomial:	$(2x+1)(x-3) = 0$
Break into separate equations:	$(2x+1) = 0 \qquad (x-3) = 0$
Solutions for x:	$x = \left\{-\frac{1}{2},\ 3\right\}$

Now, we have to find the y-values that go with these x-values. You can use either equation for this. I typically go with the simpler equation, in this case: $y = -5x$.

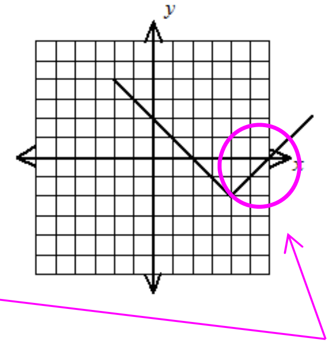
$x = -\frac{1}{2}$: $\quad y = -5 \cdot \left(-\frac{1}{2}\right) = \frac{5}{2}$

$x = 3$: $\quad y = -5 \cdot (3) = -15$

So, the two solutions are: $\left(-\frac{1}{2}, \frac{5}{2}\right), (3, -15)$ **Answer D**



40. Translate the graph up 4 units and left 3 units. What is the function for the graph obtained after the translation?



The general equation of an absolute value function is:

$$f(x) = a\,|x - h| + k,$$

where (h, k) is the vertex of the curve.

The value of a is the slope of the curve to the right of the vertex. In this case, it appears that $a = 1$. So, we can simplify the general equation for this problem to:

$$f(x) = |x - h| + k$$

The vertex is currently at $(4, -2)$. A translation of $\langle -3, +4\rangle$ will put the vertex at:

$$(4, -2) + \langle -3, +4\rangle \;=\; (4 - 3, -2 + 4) \;=\; (1, 2) = (h, k)$$

So, $h = 1$ and $k = 2$, which gives a translated equation of: $\boldsymbol{f(x) = |x - 1| + 2}$

41. Translate $y = x^2 - 4x + 6$ five (5) units to the left and up two (2) units. What is the graph obtained after the translation?

The vertex form of a quadratic function is $y = a(x - h)^2 + k$, where (h, k) is the vertex of the curve. We want our answer in this form because it is the easiest one to use when making a translation.

Note that the lead coefficient in our starting equation is 1. So, $a = 1$.

Let's find the vertex, (h, k) of this curve. Recall that the x-value of the vertex of a quadratic equation in general form is $h = -\frac{b}{2a}$.

Then, $h = -\frac{-4}{2(1)} = 2$; $k = f(h) = (2)^2 - 4(2) + 6 = 2$

So, the vertex of the starting equation is $(2,\ 2)$. A translation of $\langle -5, +2\rangle$ will put the vertex at:

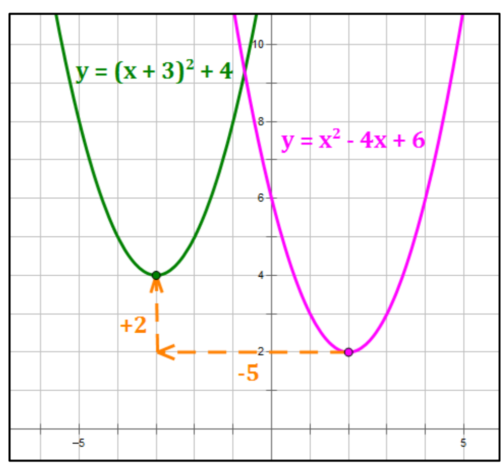
$$(2,\ 2) + \langle -5, +2\rangle = (-3, 4) = (h, k)$$

Then, in the translated equation, $h = -3$ and $k = 4$, which gives an equation of

$$f(x) = 1 \cdot (x - (-3))^2 + 4$$

$$= \boldsymbol{(x + 3)^2 + 4} \quad \text{(vertex form)}$$

$$= \boldsymbol{x^2 + 6x + 13} \quad \text{(general form)}$$



42. Solve: $x^3 + 12x^2 + 20x \le 0$ (Write the answer algebraically.)

Write the corresponding equation: $x^3 + 12x^2 + 20x = 0$

Factor out x: $x\,(x^2 + 12x + 20) = 0$

Factor the trinomial: $x\,(x+2)\,(x+10) = 0$

Break into separate equations: $x = 0 \quad x + 2 = 0 \quad x + 10 = 0$

Solutions for x in the equation: $x = \{0, -2, -10\}$

Then, set up a table of intervals based on the solutions for x and test each interval to determine the sign of the function in that interval:

Interval	$x < -10$	$-10 < x < -2$	$-2 < x < 0$	$x > 0$
Terms of $f(x)$	$x\,(x+2)\,(x+10)$	$x\,(x+2)\,(x+10)$	$x\,(x+2)\,(x+10)$	$x\,(x+2)\,(x+10)$
Signs of terms	$- \cdot - \cdot -$	$- \cdot - \cdot +$	$- \cdot + \cdot +$	$+ \cdot + \cdot +$
Sign of $f(x)$	$-$	$+$	$-$	$+$

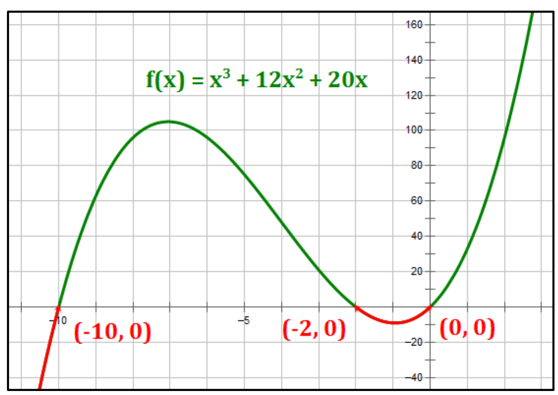
Based on the results in the table, $x^3 + 12x^2 + 20x < 0$ when $x < -10$ or $-2 < x < 0$. Finally, add equal signs to the inequality signs because the original inequality sign $(\le)$ contains one.

So, the solution set is: $\boldsymbol{x \le -10}$ or $\boldsymbol{-2 \le x \le 0}$

ALTERNATIVE (CALCULATOR) APPROACH

If you are allowed to use a calculator, graph the function and isolate where it is below the x-axis (for a function of form $f(x) < 0$).

In the graph at right, the portions of the curve that are below the x-axis are shown in red. Clearly, the associated x-intervals are: $x \le -10$ or $-2 \le x \le 0$. Note: since the original inequality uses the " $\ge$ " sign, you must include the equal sign in your inequalities.



If the problem had been of the form $f(x) > 0$, you would want the intervals in green, i.e., the intervals above the x-axis.