# 2012-2013
# Algebra 2 Semester 1

## Instructional Materials for the WCSD Math Common Finals

The Instructional Materials are for student and teacher use and are aligned to the Math Common Final blueprint for this course. When used as test practice, success on the Instructional Materials does not guarantee success on the district math common final.

Students can use these Instructional Materials to become familiar with the format and language used on the district common finals. Familiarity with standards vocabulary and interaction with the types of problems included in the Instructional Materials can result in less anxiety on the part of the students.

Teachers can use the Instructional Materials in conjunction with the course guides to ensure that instruction and content is aligned with what will be assessed. The Instructional Materials are not representative of the depth or full range of learning that should occur in the classroom.

**Note from Earl.** Many of the solutions in this document use techniques presented in the Algebra Handbook, which is available on the www.mathguy.us website. If you have trouble following any of the techniques used, try looking in the handbook for pages that deal with the issue you are struggling with.

This test seems to require a lot of multiple choice test taking skills and is a bit light on requiring math skills, in my opinion. I would prefer a test that concentrates on math skills and little else, but perhaps that is my own personal shortcoming.

In any case, I solve the problems in this test using the quickest method available in most cases. Occasionally, I also make comments about some of the math involved in an effort to enhance your understanding of what is going on in the problem.

Multiple Choice

*Identify the choice that best completes the statement or answers the question.*

1. The distance a spring will stretch, $S$, varies directly with the force (or weight), $F$, attached to the spring. If a spring stretches 7 inches with 50 pounds attached, how far will it stretch with 40 pounds attached?

A. 280 $in$  B. 5.60 $in$  C. 3.50 $in$  D. 0.14 $in$

The form of the equation is $S = aF$ for direct variation. First we find the value of $a$ by substituting in the given values for $S$ and $F$ that relate to each other.

Starting equation: $S = a \cdot F$

Substitute in values: $7 = a \cdot (50)$

Divide by 50: $\div 50 \qquad \div 50$

Result: $\frac{7}{50} = a$

Revised Starting Equation: $S = \frac{7}{50} \cdot F$

Then, we find the desired value of $S$ when $F = 40$.

Substitute in value of $F$: $S = \frac{7}{50} \cdot (40)$

Multiply to get $S$: $S = \frac{280}{50} = \frac{28}{5} = 5.60$ in. **Answer B**

2. The value of $y$ varies inversely with the product of $x$ and the square of $z$. If $y = 12$ when $x = 6$ and $z = 2$, what is the value of $y$ when $x = 3$ and $z = 4$?

A. 216 B. 42 C. 9 D. 6

The form of the equation is $y = \frac{a}{x \cdot z^2}$ for the joint variation described. First we find the value of $a$ by substituting in the given values for $y, x$ and $z$ that relate to each other.

Starting equation: $y = \frac{a}{x \cdot z^2}$

Substitute in values: $12 = \frac{a}{6 \cdot 2^2}$

Simplify: $12 = \frac{a}{24}$

Multiply by 24: $\cdot 24 \quad \cdot 24$

Result: $288 = a$

Revised Starting Equation: $y = \frac{288}{x \cdot z^2}$

Then, we find the desired value of $y$ when $x = 3$ and $z = 4$.

Substitute in values of $x$ and $z$: $y = \frac{288}{3 \cdot 4^2}$

Multiply to get $y$: $y = \frac{288}{48} = 6$ **Answer D**

3. Which equation of the line passes through $(8, 10)$ and is parallel to the graph of the line $y = \frac{8}{3}x + 7$ .

A. $y = \frac{8}{3}x - \frac{34}{3}$

B. $y = \frac{8}{3}x + \frac{8}{3}$

C. $y = 6x - \frac{34}{3}$

D. $y = 16x + \frac{8}{3}$

The slope must match the original equation if our line is to be parallel to it. So, $m = \frac{8}{3}$. That leaves answers **A** and **B**. Let's use our point, $(8, 10)$, to see which is correct:

If **A** is correct, then: $10 = \frac{8}{3}(8) - \frac{34}{3} = \frac{64}{3} - \frac{34}{3} = \frac{30}{3} = 10$ ✓ **Answer A**

If **B** is correct, then: $10 = \frac{8}{3}(8) + \frac{8}{3} = \frac{64}{3} + \frac{8}{3} = \frac{72}{3} = 24$ **NOT!**

4. Which equation of the line passes through $(29, 8)$ and is perpendicular to the graph of the line $y = \frac{1}{13}x + 17$ .

A. $y = 385x + \frac{1}{13}$

B. $y = \frac{1}{13}x + 385$

C. $y = -13x + 385$

D. $y = -13x - 13$

The slope must be the opposite reciprocal of the slope in the original equation if our line is to be perpendicular to it. So, $m = -13$. That leaves answers **C** and **D**. Let's use our point, $(29, 8)$, to see which is correct:

If C is correct, then: $8 = -13(29) + 385 = -377 + 38 = 8$ ✓ **Answer C**

If D is correct, then: $8 = -13(29) - 13 = -377 - 13 = -390$ **NOT!**

5. Which equation of the line passes through $(4,7)$ and is perpendicular to the graph of the line that passes through the points $(1,3)$ and $(-2,9)$ ?

A. $y = 2x - 1$

B. $y = \frac{1}{2}x + 5$

C. $y = \frac{1}{2}x - 5$

D. $y = -2x + 15$

The slope of the line that passes through the points (1, 3) and (-2, 9) is:

$$m = \frac{9-3}{-2-1} = \frac{6}{-3} = -2$$

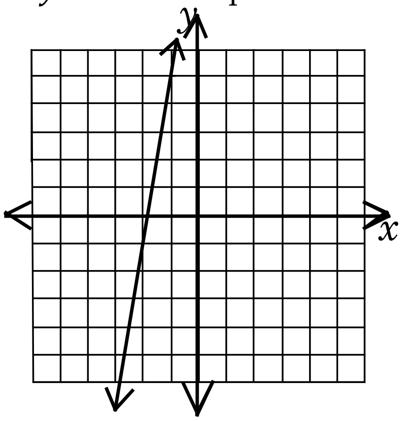
The slope of the desired line must be the opposite reciprocal of this slope if our line is to be perpendicular to it. So, $m = \frac{1}{2}$. That leaves answers **B** and **C**. Let's use our point, $(4, 7)$, to see which is correct:

If B is correct, then: $7 = \frac{1}{2}(4) + 5 = 2 + 5 = 7$ ✓ **Answer B**

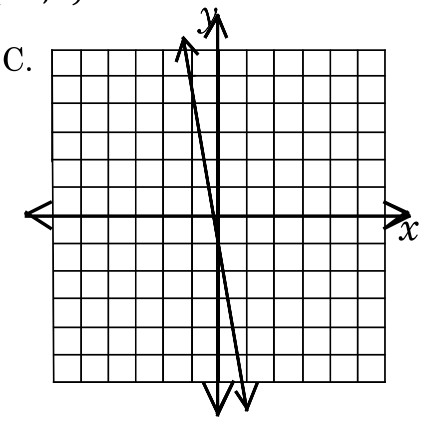
If C is correct, then: $7 = \frac{1}{2}(4) - 5 = 2 - 5 = -3$ **NOT!**

6. Which graph is the graph of the line that is parallel to the line, $x + 6y = 12$ and passes through the point $(-1, 5)$ ?

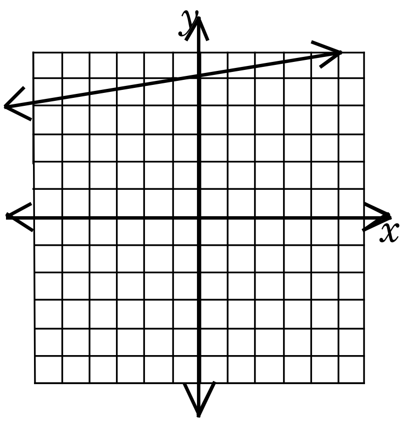
A.



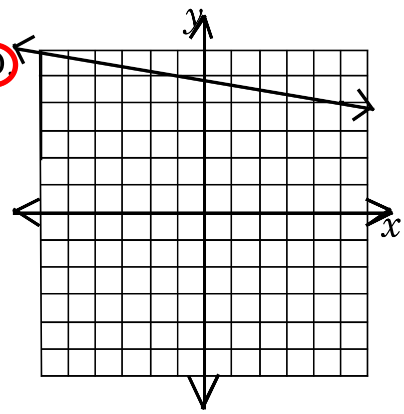
C.



B.



D.



The slopes of the four graphs are all different. So, let's find the slope of the original equation, and the slope of the parallel equation must be equal to it.

| | |
|---|---|
| Original equation: | $x + 6y = 12$ |
| Subtract $x$: | $-x \qquad -x$ |
| Result: | $6y = -x + 12$ |
| Divide by 6: | $\div 6 \qquad \div 6$ |
| Result: | $y = -\frac{1}{6}x + 2$ |

The resulting equation has a slope of $-\frac{1}{6}$. If you recall the rise-over-run method of defining slope, this translates to moving down 1 unit and right 6 units. A look at the slopes in each of the graphs reveals that Answer **D** is the graph with this slope.

**Answer D**

7. What is the solution for y, in the following system?

$$\begin{cases} 7y = -8x + 18 \\ 3x - 5y = 22 \end{cases}$$

A. $y = -4$ B. $y = -2$ C. $y = 0$ D. $y = 4$

Let's set this up to eliminate $x$, which will leave a solution for $y$. The first step is to get both equations in the same form. Let's begin by getting the first equation in form of the second one. Note that this form is called Standard Form.

Original equation: $7y = -8x + 18$

Add $8x$: $+8x \qquad + 8x$

Result: $8x + 7y = \quad 18$

Now let's set it up to eliminate $x$:

$8x + 7y = 18$ multiply by $(3)$ $\Rightarrow$ $24x + 21y = \quad 54$

$3x - 5y = 22$ multiply by $(-8)$ $\Rightarrow$ $-24x + 40y = -176$

Add the equations: $61y = -122$

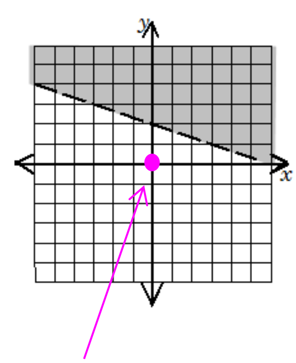
Divide by $61$: $\div 61 \qquad \div 61$

Add the equations: $y = -2$

**Answer B**

8. Which inequality is represented by the shaded area shown in the graph?

A. $3y \geq x + 2$ B. $x - 3y > -18$ C. $x + 3y \leq 2$ D. $x + 3y > 6$




Note that the "$y$" in each of the answers is on the left of the sign. That's a good start. Then note that the line in the graph is dashed and the shaded area is above the line. This means that the inequality must contain the sign "$>$". This eliminates answers **A** and **C**.

Next, we want to test answers **B** and **D**. Let's find a point that satisfies one of the equations and not the other. The simplest point to try is $(0, 0)$. Let's substitute it into the inequalities in **B** and **D**.

**B.** $x - 3y > -18 \Rightarrow 0 - 3 \cdot 0 = 0 > -18$ true

**D.** $x + 3y > 6 \Rightarrow 0 + 3 \cdot 0 = 0 > 6$ false

Since our point, $(0, 0)$ is not in the shaded region, we want the answer with a false result.

**Answer D**

9. Sarah is selling bracelets and earings to make money for summer vacation. The bracelets cost \$2 and earrings cost \$3. She needs to make at least \$500. She knows she will sell at least 50 bracelets. Which set of linear inequalities best represents this situation.

A. $\begin{cases} 2x + 3y > 500 \\ y \geq 50 \end{cases}$ B. $\begin{cases} 2x + 3y < 500 \\ y \geq 50 \end{cases}$ C. $\begin{cases} 2x + 3y \geq 500 \\ x \geq 50 \end{cases}$ D. $\begin{cases} 2x + 3y > 500 \\ x > 50 \end{cases}$

All of the equations contain the term "$2x$". Since bracelets cost \$2 each, we can assume that $x$ represents the number of bracelets sold.

Sarah knows she will sell at least 50 bracelets, so one equation must be $x \geq 50$.

Sarah needs to make at least \$500, so one equation must contain "$\geq 500$".

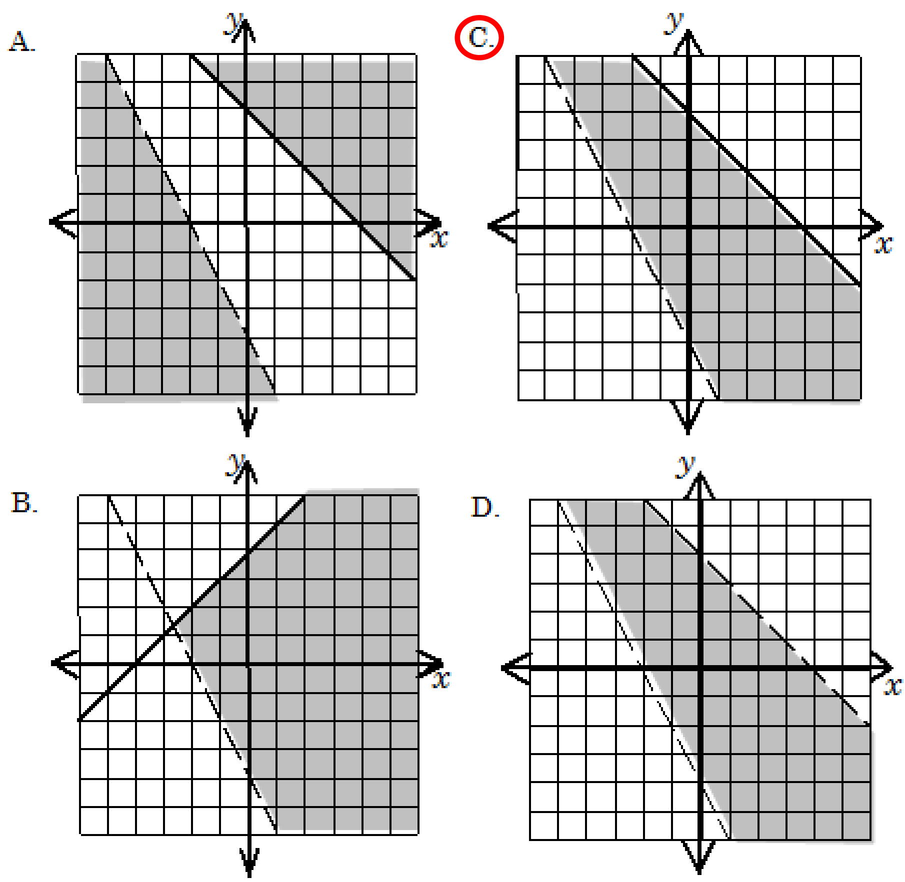
The only answer meeting these conditions is **C**. **Answer C**

10. Which graph shows all solutions to the system of inequalities?

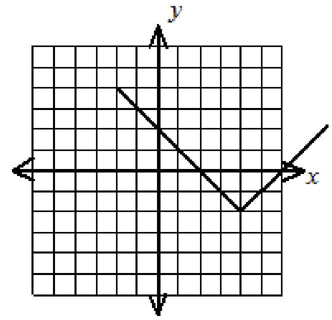
$\begin{cases} y \leq -x + 4 \\ y > -2x - 4 \end{cases}$ $\leq$ implies that the line is solid and shading is below the line.
$>$ implies that the line is dashed and shading is above the line

Based on the information in orange and purple above, we want a solid line with slope $-1$, and shading below it, and a dashed line with slope $-2$, and shading above it.

The only graph with these characteristics is **Answer C**



11. Which equation is obtained after a translation of the graph up 2 and left 6?




A. $f(x) = |x - 2|$

B. $f(x) = |x| - 2$

C. $f(x) = |x + 2|$

D. $f(x) = |x| + 2$

The general equation of an absolute value function is:

$f(x) = a\,|x - h| + k$, where $(h, k)$ is the vertex of the curve.

The value of $a$ in all of the answers is 1, so the general equation simplifies to:

$f(x) = |x - h| + k$

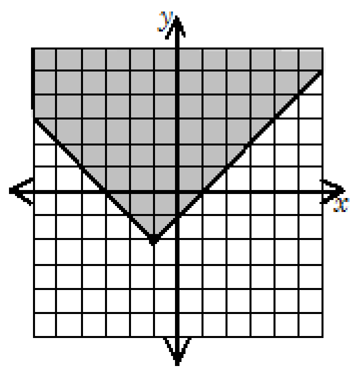
The vertex is currently at $(4, -2)$. A translation of $\langle -6, +2 \rangle$ will put the vertex at:

$(4, -2) + \langle -6, +2 \rangle = (4 - 6, -2 + 2) = (-2, 0) = (h, k)$

So, $h = -2$ and $k = 0$, which gives an equation of

$\boldsymbol{f(x) = |x - (-2)| + 0 = |x + 2|}$ **Answer C**

12. Which inequality is represented by the shaded area shown in the graph?




A. $y < |x + 1| - 2$

B. $y < |x - 1| - 2$

C. $y \leq |x + 1| - 2$

D. $y \geq |x + 1| - 2$

The vertex is at $(-1, -2)$, which means the right side of the inequality is: $|x - (-1)| + (-2) = |x + 1| - 2$

The shaded area is above the curve, meaning the inequality sign must be ">" or "≥".

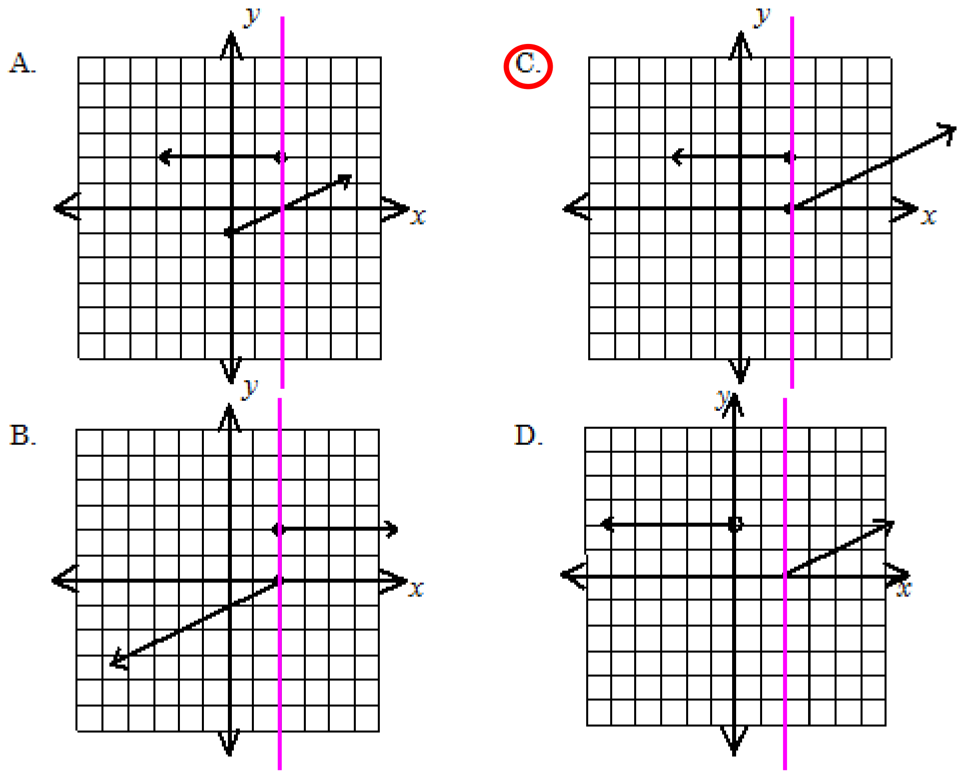
The line is solid, meaning there must be an equal sign in the inequality sign ("≥"). So, the resulting inequality is:

$\boldsymbol{f(x) \geq |x + 1| - 2}$ **Answer D**

13. What graph represents the piecewise function?

$$f(x) = \begin{cases} \frac{1}{2}x - 1, & x \geq 2 \\ 2, & x < 2 \end{cases}$$

The magenta line indicates the break point in the graph. One piece of the function is to its left and the other is to its right. Since the pieces in the above function definition "meet" at $x = 2$, this leaves out answers **A** and **D**, which do not meet at $x = 2$. So, the answer is narrowed down to **B** or **C**.



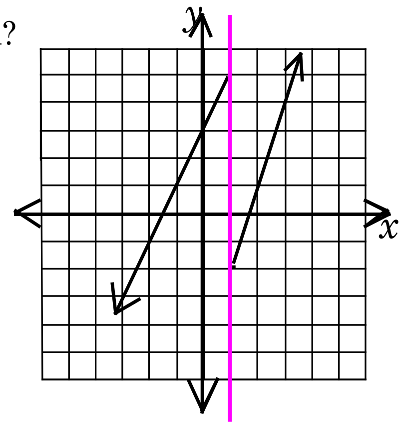

**First Equation:**

$x \geq 2$ is the right side of the graph. The line $f(x) = \frac{1}{2}x - 1$ has slope $\frac{1}{2}$. Since the slope is positive, the line extends up and to the right. This works for answer **C**, but not for answer **D**. So **C** must be the right answer. Let's proceed further, just for fun (and to make sure you understand more of the math involved in piecewise functions).

**Second Equation:**

$x < 2$ is the left side of the graph. The line $f(x) = 2$ is a flat line at $y = 2$. This works for answer **C**, but we need to check something. Since the function values are for $x$-values less than 2, there should be an open circle at the point $(2,2)$ for the flat line. If you look closely, you can see the open circle at $(2,2)$, so we can conclude with confidence that the answer is **C**. **Answer C**

14. Which piecewise function is represented by the graph?




A. $f(x) = \begin{cases} 2x - 3, & x < 1 \\ 3x - 5, & x > 1 \end{cases}$

B. $f(x) = \begin{cases} 2x + 3, & x \leq 1 \\ 3x - 5, & x > 1 \end{cases}$

C. $f(x) = \begin{cases} 2x + 3, & x \geq 1 \\ 3x - 5, & x > 1 \end{cases}$

D. $f(x) = \begin{cases} 2x - 3, & x < 1 \\ 3x - 5, & x \geq 1 \end{cases}$

The magenta line indicates the break point in the graph. One piece of the function is to its left and the other is to its right. Note that the slopes of the lines are the same across all of the answers, so let's concern ourselves with the $y$-intercepts and any open circles.

**Left Line:**

The $y$-intercept of the left line is $+3$, which narrows our answers to **B** and **C**. In addition, the endpoint at $x = 1$ is closed, indicating the interval for the line is $x \leq 1$. This pretty much sums up that the answer must be **B**. Let's look at the second line anyway.

**Right Line:**

The endpoint at $x = 1$ is open, indicating the interval for the line is $x > 1$. This matches answer **B**. so, now we can be confident in our answer. **Answer B**

15. Solve the system of equations, state the solution for $b$.

$$\begin{cases} -3a = 36 \\ 10a + 3c = 9 \\ 2b + 5c = 23 \end{cases}$$

A. $b = 96$

B. $b = 43$

C. $b = -12$

D. $b = -96$

Solving the first equation, we find that $a = -12$.

Putting this in the second equation gives:

$$10(-12) + 3c = 9$$

$$-120 + 3c = 9$$

From which we can solve to get $c = 43$.

Putting this in the third equation gives:

$$2b + 5(43) = 23$$

$$2b + 215 = 23$$

From which we can solve to get $\boldsymbol{b = -96}$. **Answer D**

16. Solve the system of equations, state the solution for $z$.

$$\begin{cases} 2x - y - 3z = 5 \\ x + 2y - 5z = -11 \\ -x - 3y = 10 \end{cases}$$

A. $z = -4$ B. $z = 1$ C. $z = 2$ D. $z = 4$

Let's set this up to eliminate $x$ and $y$, which will leave a solution for $z$.

Let's begin by working with the first two equations to eliminate $x$.

| | | |
|---|---|---|
| $2x - y - 3z = 5$ | multiply by $(-1)$ $\Rightarrow$ | $-2x + y + 3z = -5$ |
| $x + 2y - 5z = -11$ | multiply by $(2)$ $\Rightarrow$ | $2x + 4y - 10z = -22$ |
| | Add the equations: | $5y - 7z = -27$ |

Let's do the same (eliminate $x$) with the last two equations. Alternatively, we could use the first and last equation in this step. The final solution for the values of $x$, $y$ and $z$ would still be the same! Cool, huh?

*Note: it's not necessary to multiply each equation by 1 in this case, but I show it this way so the problem can be used as a general example of how to solve 3 equations in 3 unknowns.*

| | | |
|---|---|---|
| $x + 2y - 5z = -11$ | multiply by $(1)$ $\Rightarrow$ | $x + 2y - 5z = -11$ |
| $-x - 3y = 10$ | multiply by $(1)$ $\Rightarrow$ | $-x - 3y = 10$ |
| | Add the equations: | $-y - 5z = -1$ |

Now that we are down to 2 equations in 2 unknowns, let's eliminate $y$.

| | | |
|---|---|---|
| $5y - 7z = -27$ | multiply by $(1)$ $\Rightarrow$ | $5y - 7z = -27$ |
| $-y - 5z = -1$ | multiply by $(5)$ $\Rightarrow$ | $-5y - 25z = -5$ |
| | Add the equations: | $-32z = -32$ |
| | Divide by $-32$: | $\div -32 \quad \div -32$ |
| | Add the equations: | $\mathbf{z = 1}$ |

**Answer B**

17. Given the function, $f(x) = -(x-4)^2 - 3$, state whether the parabola opens up or down and the maximum or minimum value.

A. Opens down, Maximum value is $-3$

B. Opens down, Maximum value is $4$

C. Opens up, Minimum value is $4$

D. Opens up, Minimum value is $-3$

Vertex form:

$$f(x) = a(x-h)^2 + k$$

where $(h, k)$ is the vertex of the parabola.

If the equation is in vertex form and the lead coefficient is negative, the parabola opens down and has a maximum value equal to $k$ (the constant at the end of the equation).

This parabola opens down and has a maximum value of $-3$. **Answer A**

18. Given the function, $f(x) = x^2 + 2x + 7$, state whether the parabola opens up or down and the maximum or minimum value.

A. Opens up, Minimum value is $7$

B. Opens up, Minimum value is $6$

C. Opens down, Maximum value is $7$

D. Opens down, Maximum value is $6$

General form:

$$f(x) = ax^2 + bx + c$$

If the equation is in general form and the lead coefficient is positive, the parabola opens up. It has a minimum at the vertex, which occurs at $= -\frac{b}{2a}$.

The $\boldsymbol{a}$ and the $\boldsymbol{b}$ are defined the same way as in the quadratic formula. You may be able to identify $-\frac{b}{2a}$ as the average of the two roots in the quadratic formula.

For the equation in this problem, we find the vertex at $x = -\frac{b}{2a} = -\frac{2}{2} = -1$.

The minimum value, then, is the $y$-value of the vertex.

$$f(-1) = (-1)^2 + 2(-1) + 7 = 6$$

Result: This parabola opens up and has a minimum value of 6. **Answer B**

You will need the quadratic formula for the next couple of questions:

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ provides solutions to the equation: $ax^2 + bx + c = 0$

19. What are the solutions to the quadratic equation, $3x^2 + 7x + 11 = 5x + 7$ ?

A. $x = \frac{-2 \pm 2i\sqrt{11}}{3}$ B. $x = \frac{-1 \pm 2i\sqrt{11}}{3}$ C. $x = \frac{-1 \pm i\sqrt{11}}{3}$ D. $x = \frac{\pm i\sqrt{11}}{3}$

First, combine all terms on one side of the equation.

$$\begin{array}{ll} \text{Original Equation:} & 3x^2 + 7x + 11 = 5x + 7 \\ \text{Subtract } 5x + 7\text{:} & \quad -5x - 7 \quad -5x - 7 \\ \hline \text{Result:} & 3x^2 + 2x + 4 = 0 \end{array}$$

Next, use the quadratic formula with $a = 3$, $b = 2$, $c = 4$

$$x = \frac{-(2) \pm \sqrt{2^2 - 4(3)(4)}}{2(3)} = \frac{-2 \pm \sqrt{-44}}{6} = \frac{-2 \pm 2i\sqrt{11}}{6} = \mathbf{\frac{-1 \pm i\sqrt{11}}{3}}$$

**Answer C**

20. What are the solutions to the quadratic equation, $y^2 + 2y = 9 + 5y$ ?

A. $y = \frac{3 \pm 3i\sqrt{3}}{2}$ B. $y = \frac{-3 \pm 3\sqrt{5}}{2}$ C. $y = \frac{3 \pm 3i\sqrt{5}}{2}$ D. $y = \frac{3 \pm 3\sqrt{5}}{2}$

First, combine all terms on one side of the equation.

$$\begin{array}{ll} \text{Original Equation:} & y^2 + 2y = 9 + 5y \\ \text{Subtract } 5y + 9\text{:} & \quad -5y - 9 \quad -9 - 5y \\ \hline \text{Result:} & y^2 - 3y - 9 = 0 \end{array}$$

Next, use the quadratic formula with $a = 1$, $b = -3$, $c = -9$

$$y = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)} = \frac{3 \pm \sqrt{45}}{2} = \mathbf{\frac{3 \pm 3\sqrt{5}}{2}}$$

**Answer D**

21. Solve: $x^2 + 6x + 5 < 0$

A. $x > -5$ *and* $x > -1$ B. $-5 < x < -1$ C. $x < -5$ *or* $x > -1$ D. $x > -5$ *or* $x > -1$

Write the corresponding equation: $x^2 + 6x + 5 = 0$

Factor the trinomial: $(x+1)\,(x+5) = 0$

Break into separate equations: $x + 1 = 0$ $x + 5 = 0$

Solutions for $x$ in the equation: $x = \{-1, -5\}$

Then, set up a table of intervals based on the solutions for $x$ and test each interval to determine the sign of the function in that interval:

| Interval | $x < -5$ | $-5 < x < -1$ | $x > -1$ |
|---|---|---|---|
| Terms of $f(x)$ | $(x+1)\,(x+5)$ | $(x+1)\,(x+5)$ | $(x+1)\,(x+5)$ |
| Signs of terms | $- \cdot -$ | $- \cdot +$ | $+ \cdot +$ |
| Sign of $f(x)$ | $+$ | $-$ | $+$ |

Based on the results in the table, $x^2 + 6x + 5 < 0$ when $-5 < x < -1$.

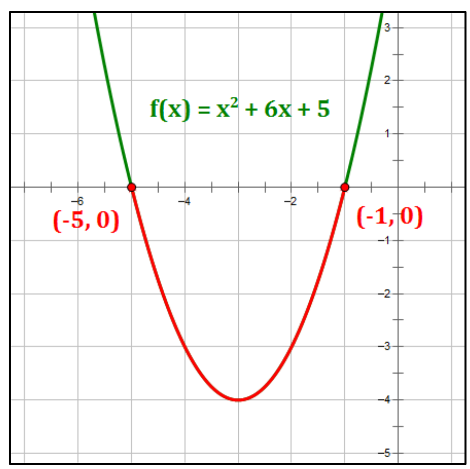
So, the solution set is: $\mathbf{-5 < x < -1}$ **Answer B**

**ALTERNATIVE (CALCULATOR) APPROACH**

If you are allowed to use a calculator, graph the function and isolate where it is below the x-axis (for a function of form $\boldsymbol{f(x) < 0}$).

In the graph at right, the portion of the curve that is below the x-axis is shown in red. Clearly, the associated $\boldsymbol{x}$-interval is: $\mathbf{-5 < x < -1}$.

If the problem had been of the form $\boldsymbol{f(x) > 0}$, you would want the intervals in green, i.e., the intervals above the x-axis.



22. Solve: $x^2 - 49 \geq 0$

A. $x \geq -7$ *and* $x \geq 7$ B. $x \geq -7$ *or* $x \geq 7$ C. $x \leq -7$ *or* $x \geq 7$ D. $-7 \leq x \leq 7$

Write the corresponding equation: $x^2 - 49 = 0$

Factor the trinomial: $(x+7)\,(x-7) = 0$

Break into separate equations: $x + 7 = 0 \quad x - 7 = 0$

Solutions for $x$ in the equation: $x = \{-7, 7\}$

Then, set up a table of intervals based on the solutions for $x$ and test each interval to determine the sign of the function in that interval:

| Interval | $x < -7$ | $-7 < x < 7$ | $x > 7$ |
|---|---|---|---|
| Terms of $f(x)$ | $(x+7)\,(x-7)$ | $(x+7)\,(x-7)$ | $(x+7)\,(x-7)$ |
| Signs of terms | $- \cdot -$ | $+ \cdot -$ | $+ \cdot +$ |
| Sign of $f(x)$ | $+$ | $-$ | $+$ |

Based on the results in the table, $x^2 - 49 \geq 0$ when $x < -7$ or $x > 7$. Finally, add equal signs to the inequality signs because the original inequality sign $(\geq)$ contains one.

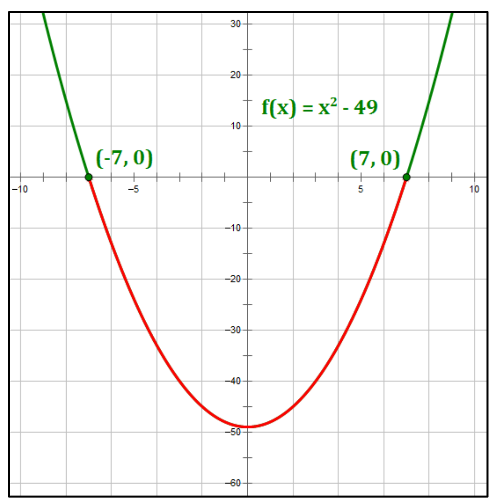
So, the final solution set is: $x \leq -7$ or $x \geq 7$ **Answer C**

**ALTERNATIVE (CALCULATOR) APPROACH**

If you are allowed to use a calculator, graph the function and isolate where it is above the x-axis (for a function of form $\boldsymbol{f(x) > 0}$).

In the graph at right, the portion of the curve that is above the x-axis is shown in green. Clearly, the associated $\boldsymbol{x}$-intervals are: $\boldsymbol{x \leq -7}$ **or** $\boldsymbol{x \geq 7}$. Note: since the original inequality uses the " $\geq$ " sign, you must include the equal sign in your inequalities.

If the problem had been of the form $\boldsymbol{f(x) < 0}$, you would want the interval in red, i.e., the interval below the x-axis.



23. Simplify: $\sqrt{-25}\cdot\sqrt{-36}$

A. $30i$

B. $-30$

C. $30$

D. $900$

$\sqrt{-25}\cdot\sqrt{-36} = 5i\cdot 6i = 30i^2 = 30\cdot(-1) = -30$

**Answer B**

24. Simplify: $(11+i)+(3-15i)$

A. $14-14i$

B. $-4+4i$

C. $12-12i$

D. $14+16i$

For addition of complex numbers, I like to line things up vertically.

$$\begin{array}{r} 11 + \quad i \\ 3 - 15i \\ \hline \mathbf{14 - 14}\boldsymbol{i} \end{array}$$

**Answer A**

25. Simplify: $3i(6-5i)-4(2+3i)$

A. $23+6i$

B. $-8-9i$

C. $8-9i$

D. $7+6i$

$3i(6-5i)-4(2+3i)$

$= (18i - 15i^2) - (8+12i)$

$= (15+18i) + (-8-12i)$

$$\begin{array}{r} 15 + 18i \\ -8 - 12i \\ \hline \mathbf{7 + 6}\boldsymbol{i} \end{array}$$

**Answer D**

26. Simplify: $(3-7i)^2$

A. $-40+0i$

B. $-40-42i$

C. $58+0i$

D. $58-42i$

$(3-7i)\cdot(3-7i)$

$F$: $3\cdot 3 = 9$

$O$: $3\cdot(-7i) = -21i$

$I$: $-7i\cdot 3 = -21i$

$L$: $-7i\cdot(-7i) = 49i^2 = -49$

Result: $(9-49)+(-21i-21i)$

$= \mathbf{-40-42}\boldsymbol{i}$ **Answer B**

27. Simplify: $(4-5i)(4+5i)$

A. $-9$
B. $41$
C. $16+25i$
D. $16-25i$

$(4-5i)\cdot(4+5i)$

$F$: $4\cdot 4=16$

$O$: $4\cdot 5i=20i$

$I$: $-5i\cdot 4=-20i$

$L$: $-5i\cdot 5i=-25i^2=25$

Result: $(16+25)+(20i-20i)$

$=\mathbf{41}$ **Answer B**

28. Simplify: $\frac{6+2i}{2-i}$

A. $2-3i$
B. $2+3i$
C. $2+2i$
D. $3+2i$

$$=\frac{6+2i}{2-i}\cdot\frac{2+i}{2+i}$$

Multiply by the conjugate of the complex number in the denominator.

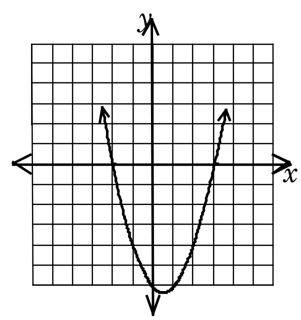
$$=\frac{6\cdot 2+6\cdot i+(2i)\cdot 2+(2i)\cdot i}{2\cdot 2+2\cdot i+(-i)\cdot 2+(-i)\cdot i}$$

$$=\frac{12+6i+4i+2i^2}{4+2i-2i-i^2}$$

$$=\frac{12-2+10i}{4+0+1}$$

$$=\frac{10+10i}{5}=\mathbf{2+2i}$$ **Answer C**

29. Which function is represented by the graph?



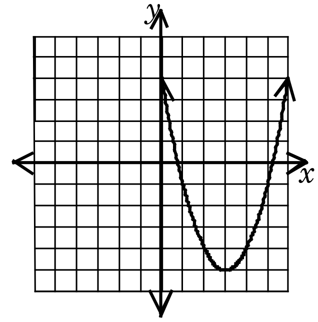

A. $f(x)=(x-2)(x+3)$
B. $f(x)=(x-2)(x-3)$
C. $f(x)=(x+2)(x-3)$
D. $f(x)=(x+2)(x+3)$

Intercept form for a quadratic equation is: $f(x)=a(x-r_1)(x-r_2)$

Note that the two roots are $x=-2,3$ and that the lead coefficient is $a=1$, in all of the answers given. The equation, then, must be: $f(x)=1\cdot(x-(-2))(x-3)$.

Therefore: $\boldsymbol{f(x)=(x+2)(x-3)}$ **Answer C**

30. Which equation is represented by the graph?



A. $y = (x-3)^2 - 5$
B. $y = 2(x-3)^2 - 5$
C. $y = 2(x+3)^2 - 5$
D. $y = -(x+3)^2 - 5$

Note that the vertex is $(3, -5)$

Vertex form for a quadratic equation is: $f(x) = a(x-h)^2 + k$, where $(h, k)$ is the vertex.

The curve opens up if $a > 0$ and opens down if $a < 0$.

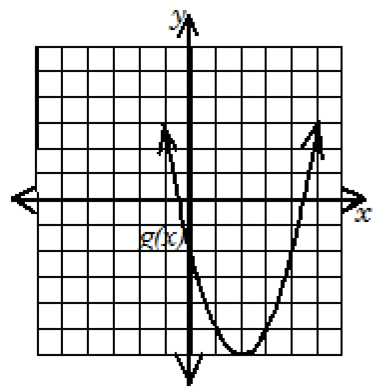
The equation, then, must be of the form: $f(x) = a(x-3)^2 - 5$, which narrows down our answers to **A** and **B**. To determine which is the correct answer, we need to find the value of $a$. To do that, let's pick a point on the curve and test the equations in **A** and **B**.

Let's pick an easy point from the curve, say $(1, -1)$, and see how **A** and **B** perform.

If **A** is correct, then: $-1 = (1-3)^2 - 5 = 4 - 5 = -1$ ✓ **Answer A**

If **B** is correct, then: $-1 = 2 \cdot (1-3)^2 - 5 = 8 - 5 = 3$ **NOT!**

31. Which description explains how the graph of $f(x) = x^2 - 4x + 4$ is related to the graph of $g(x) = x^2 - 4x - 2$ shown here?




A. $f(x)$ is vertically stretched to make $g(x)$
B. $f(x)$ is translated down 6 units to make $g(x)$
C. $f(x)$ is translated 6 units to the left to make $g(x)$
D. $f(x)$ is compressed vertically to make $g(x)$

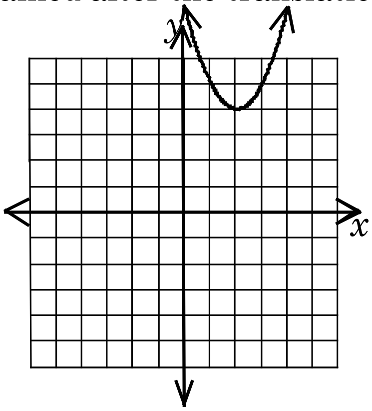
Just to say it, I would ignore the illustration on this one. It makes this problem more confusing. Concentrate on $f(x)$ and $g(x)$, and the answers.

The only difference between $f(x)$ and $g(x)$ is the constant term, which is 6 smaller in $g(x)$ than it is in $f(x)$. If you add or subtract a constant from any function, all you do is move it up or down the amount of the constant (note: this is called a vertical translation).

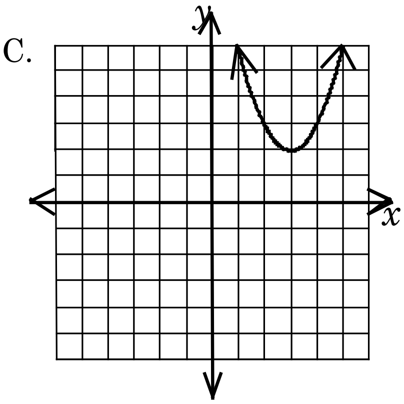
Since $g(x) = f(x) - 6$, we can conclude that to get $g(x)$ from $f(x)$, we need to translate down 6 units. **Answer B**

32. Translate $y = x^2 - 4x + 6$ five (5) units to the left. What is the graph obtained after the translation?

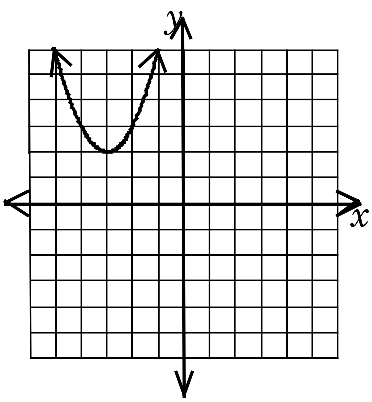
A.



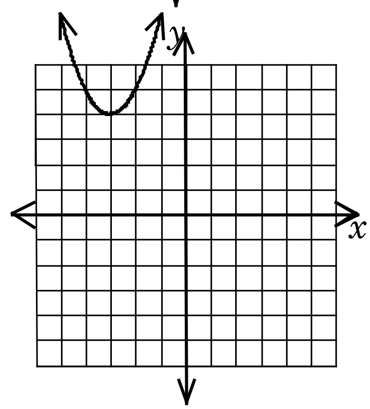

C.




B.




D.




Notice that the four graphs have different vertices, so finding the vertex of our equation and moving it 5 units to the left will produce the desired solution.

The $x$-value of the vertex of a quadratic equation in general form is $= -\frac{b}{2a}$.

For the equation identified above, $a = 1$ and $b = -4$, so this is $x = -\frac{-4}{2(1)} = 2$.

Next, let's find the $y$-value of the vertex.

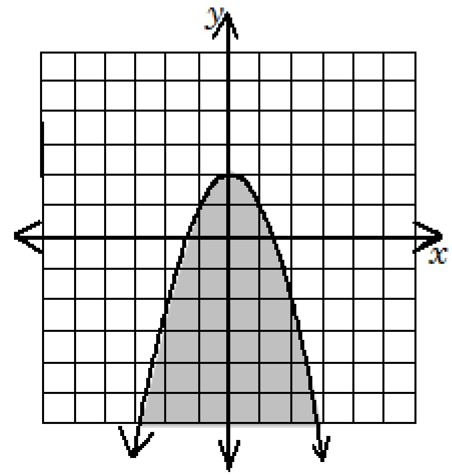
$f(2) = 2^2 - 4 \cdot 2 + 6 = 2$

The vertex of the starting equation, then, is $(2, 2)$, and we must shift it 5 units left. So, the vertex of the translated curve is $(2 - 5, 2) = (-3, 2)$.

The only answer above with a vertex of $(\mathbf{-3, 2})$ is **Answer B**

33. Which inequality is represented by the shaded area shown in the graph?

A. $y \geq x^2 + 2$ B. $y \leq (x-2)^2$ C. $y > -(x-2)^2$ D. $y \leq -x^2 + 2$




Let's look at three things:

- Shaded area is below the curve.
- The curve line is solid, not dashed or dotted.

Together, these require a " $\leq$ " sign.

- The curve opens down, so it must have a negative lead coefficient.

The only answer with a negative lead coefficient and a " $\leq$ " sign is **Answer D**

34. What is the end behavior for the function, $f(x) = (x^2 + 4x - 3)(3x^5 + 6x^3)$ ?

A. as $x \to -\infty, f(x) \to -\infty$ and as $x \to +\infty,\ f(x) \to +\infty$

B. as $x \to -\infty, f(x) \to +\infty$ and as $x \to +\infty,\ f(x) \to +\infty$

C. as $x \to -\infty, f(x) \to -\infty$ and as $x \to +\infty,\ f(x) \to -\infty$

D. as $x \to -\infty, f(x) \to +\infty$ and as $x \to +\infty,\ f(x) \to -\infty$

We need only look at the lead term of the resulting polynomial.

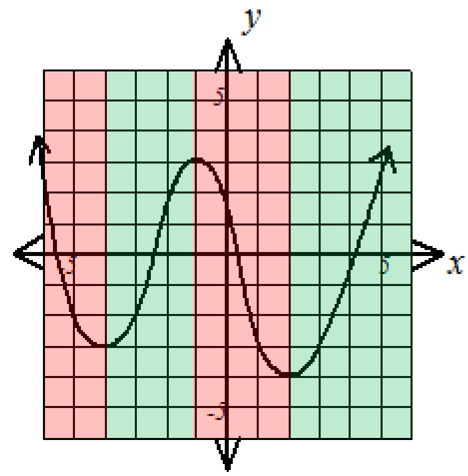
$$f(x) = (x^2 + 4x - 3)(3x^5 + 6x^3) = 3x^7 + \cdots$$

All odd exponents have the same end behavior, and all even exponents have the same end behavior. The chart below provides a summary of end behavior of polynomials based on the sign of the lead coefficient and the degree of the lead term.

| End Behavior of Polynomials | | |
|---|---|---|
| **Degree of Lead Term** | **Lead Coefficient +** | **Lead Coefficient −** |
| **Even** | as $x \to -\infty,\ f(x) \to +\infty$<br>as $x \to +\infty,\ f(x) \to +\infty$ | as $x \to -\infty,\ f(x) \to -\infty$<br>as $x \to +\infty,\ f(x) \to -\infty$ |
| **Odd** | as $x \to -\infty,\ f(x) \to -\infty$<br>as $x \to +\infty,\ f(x) \to +\infty$ | as $x \to -\infty,\ f(x) \to +\infty$<br>as $x \to +\infty,\ f(x) \to -\infty$ |

Since our lead coefficient is positive and the degree of the lead term is odd, the answer to this question is **Answer A**

35. State where the function is increasing and decre




A. Increasing: $x < -\infty, -3 < x < -1$
Decreasing: $x < -\infty, -1 < x < 2$

B. Increasing: $-4 < x < -1,\ x > +\infty$
Decreasing: $x < -\infty, -1 < x < 2$

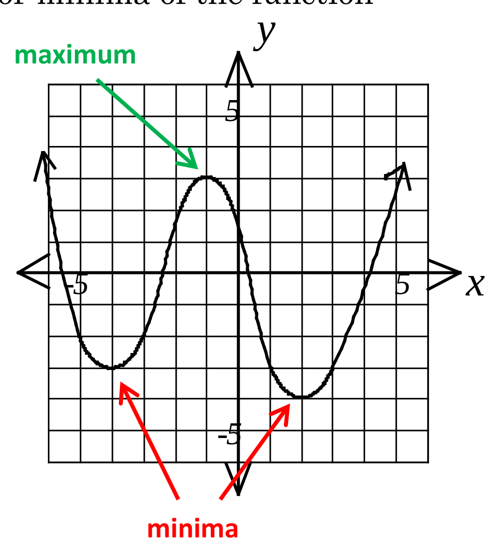
C. Increasing: $-4 < x < -1,\ x > 2$
Decreasing: $x < -4, -1 < x < 2$

D. Increasing: everywhere
Decreasing: $x > +\infty$

Any function is increasing where the curve is going up as you cross from left to right; any function is decreasing where the curve is going down as you cross from left to right.

The intervals where this function is increasing are marked by green shading, and the intervals where it is decreasing are marked by red shading. **Answer C**

36. What are the values of the relative maxima and/or minima of the function graphed?




A. relative maxima, unknown
relative minima, $-4$ and $2$

B. relative maxima, $3$
relative minima, $-3$ and $-4$

C. relative maxima, $-3$
relative minima, $3$ and $4$

D. relative maxima, $4$ and $5$
relative minima, $-1$ and $2$

Relative minima are the ***y*-values** on a curve at points where it changes from decreasing to increasing.

Relative maxima are the ***y*-values** on a curve at points where it changes from increasing to decreasing.

The values are marked on the graph. **Answer B**

Minima is the plural form of minimum.
Maxima is the plural form of maximum.
Together, they make up the extrema of a curve, which is the plural form of extreme.

37. Factor: $2x^3 - 7x^2 - 4x$

A. $x(2x+1)(x-4)$

B. $x(2x-2)(x+2)$

C. $(2x^2+1)(x-4)$

D. $x(x-8)(x+1)$

$2x^3 - 7x^2 - 4x$

$= x(2x^2 - 7x - 4)$

$= x(2x^2 - 8x + 1x - 4)$

$= x[(2x^2 - 8x) + (x - 4)]$

$= x[2x(x-4) + 1(x-4)]$

$= x(2x+1)(x-4)$

**Answer A**

Using the **AC Method**, we seek two numbers that multiply to get:

$2 \cdot (-4) = -8$

And add to get $-7$

After a few tries, we can determine that the values we want are $-8$ and $+1$.

To learn more about the **AC Method**, you may want to review page 69 of the Algebra Handbook, available at www.mathguy.us.

38. Factor: $x^4 - 13x^2 + 36$

A. $(x^2-9)(x^2+4)$

B. $(x-3)(x+3)(x+4)(x-4)$

C. $(x-3)(x+3)(x+2)^2$

D. $(x-3)(x+3)(x+2)(x-2)$

Memorize the difference of squares formula!

$a^2 - b^2 = (a+b)\,(a-b)$

Then,

$x^4 - 13x^2 + 36$

$= (x^2-4)(x^2-9)$

$= (x+2)(x-2)(x+3)(x-3)$

**Answer D**

39. Factor: $64x^3 - 27$

A. $(4x-3)(4x^2+12x+9)$

B. $(4x-3)(4x^2+12x-9)$

C. $(4x-3)(16x^2+12x+9)$

D. $(4x-3)(16x^2-12x-9)$

This is a difference of cubes. Here are the formulas for the sum and difference of cubes. Memorize them!

$a^3 + b^3 = (a+b)\,(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)\,(a^2 + ab + b^2)$

Then,

$64x^3 - 27 = (4x)^3 - 3^3$ Let: $a = 4x,\ b = 3.$

$= (4x-3)\,(16x^2 + 4x \cdot 3 + 3^2)$

$= \mathbf{(4x+3)\,(16x^2 + 12x + 9)}$

**Answer C**

Note: This problem could be solved simply by checking the lead coefficients and knowing the patterns in the formulas for sums and differences of cubes.

40. Solve: $3m^3 - 2m^2 - 5m = 0$

A. $m = -1, m = 0, m = \frac{5}{3}$
B. $m = -3, m = 0, m = 5$
C. $m = -\frac{5}{3}, m = 0, m = 1$
D. $m = -5, m = 0, m = 3$

Starting Equation: $3m^3 - 2m^2 - 5m = 0$

Factor out $m$: $m\,(3m^2 - 2m - 5) = 0$

Factor the remaining trinomial: $m\,(3m - 5)\,(m + 1) = 0$

Use the **AC Method** if you need to.

Break into separate equations: $m = 0$ $3m - 5 = 0$ $m + 1 = 0$

Identify solutions: $\boldsymbol{m = \left\{0, \frac{5}{3}, -1\right\}}$ **Answer A**

41. Solve: $x^4 - 3x^2 = 10$

A. $x = 5, x = -2$
B. $x = \pm 5, x = \pm 2i$
C. $x = \pm\sqrt{5}, x = \pm\sqrt{2}$
D. $x = \pm\sqrt{5}, x = \pm i\sqrt{2}$

Starting Equation: $x^4 - 3x^2 = 10$

Subtract 10: $-10 \quad -10$

Result: $x^4 - 3x^2 - 10 = 0$

Factor the trinomial: $(x^2 - 5)\,(x^2 + 2) = 0$

Break into separate equations: $x^2 - 5 = 0$ $x^2 + 2 = 0$

Manipulate each equation: $x^2 = 5$ $x^2 = -2$

Identify solutions: $\boldsymbol{x = \{\pm\sqrt{5}, \pm i\sqrt{2}\}}$ **Answer D**

42. Solve: $x^4 - 36 = 0$

A. $x = \pm\sqrt{6}, x = \pm 6i$
B. $x = \pm\sqrt{6}, x = \pm i\sqrt{6}$
C. $x = \pm 6, x = \pm 6i$
D. $x = \pm 6, x = \pm i\sqrt{6}$

Starting Equation: $x^4 - 36 = 0$

Factor the trinomial: $(x^2 - 6)\,(x^2 + 6) = 0$

Break into separate equations: $x^2 - 6 = 0$ $x^2 + 6 = 0$

Manipulate each equation: $x^2 = 6$ $x^2 = -6$

Identify solutions: $\boldsymbol{x = \{\pm\sqrt{6}, \pm i\sqrt{6}\}}$ **Answer B**

43. Solve: $x^3 - x^2 - 6x > 0$

A. $-2 < x < 0$ or $x > 3$ B. $x < -2$ or $x > 3$

C. $-2 < x < 0$ or $x > 0$ D. $x < -2$ or $0 < x < 3$

Write the corresponding equation: $x^3 - x^2 - 6x = 0$

Factor out $x$: $x(x^2 - x - 6) = 0$

Factor the trinomial: $x\,(x-3)\,(x+2) = 0$

Break into separate equations: $x = 0$ $x - 3 = 0$ $x + 2 = 0$

Solutions for $x$ in the equation: $x = \{0, 3, -2\}$

Then, set up a table of intervals based on the solutions for $x$ and test each interval to determine the sign of the function in that interval:

| Interval | $x < -2$ | $-2 < x < 0$ | $0 < x < 3$ | $x > 3$ |
|---|---|---|---|---|
| Terms of $f(x)$ | $x\,(x-3)\,(x+2)$ | $x\,(x-3)\,(x+2)$ | $x\,(x-3)\,(x+2)$ | $x\,(x-3)\,(x+2)$ |
| Signs of terms | $- \cdot - \cdot -$ | $- \cdot - \cdot +$ | $+ \cdot - \cdot +$ | $+ \cdot + \cdot +$ |
| Sign of $f(x)$ | $-$ | $+$ | $-$ | $+$ |

Based on the results in the table, $x^3 - x^2 - 6x > 0$ when $-2 < x < 0$ or $x > 3$.

So, the solution set is: $\mathbf{-2 < x < 0}$ or $\mathbf{x > 3}$ **Answer A**

44. Solve: $x^3 + 3x^2 - 4x - 12 \leq 0$

A. $x < -3$ or $-2 < x < 2$ B. $-3 < x < -2$ or $x > 2$

C. $-3 \leq x \leq -2$ or $x \geq 2$ D. $x \leq -3$ or $-2 \leq x \leq 2$

Write the corresponding equation: $x^3 + 3x^2 - 4x - 12 = 0$

Group the four terms in pairs: $(x^3 + 3x^2) - (4x + 12) = 0$

Factor each pair: $x^2(x+3) - 4(x+3) = 0$

Factor the trinomial: $(x^2 - 4)\,(x+3) = 0$

Factor the difference of squares: $(x-2)\,(x+2)\,(x+3) = 0$

Break into separate equations: $x - 2 = 0$ $x + 2 = 0$ $x + 3 = 0$

Solutions for $x$ in the equation: $x = \{2, -2, -3\}$

(continued on next page)

Then, set up a table of intervals based on the solutions for $x$ and test each interval to determine the sign of the function in that interval:

| Interval | $x < -3$ | $-3 < x < -2$ | $-2 < x < 2$ | $x > 2$ |
|---|---|---|---|---|
| Terms of $f(x)$ | $(x-2)\ (x+2)\ (x+3)$ | $(x-2)\ (x+2)\ (x+3)$ | $(x-2)\ (x+2)\ (x+3)$ | $(x-2)\ (x+2)\ (x+3)$ |
| Signs of terms | $- \cdot - \cdot -$ | $- \cdot - \cdot +$ | $- \cdot + \cdot +$ | $+ \cdot + \cdot +$ |
| Sign of $f(x)$ | $-$ | $+$ | $-$ | $+$ |

Based on the results in the table, $x^3 + 3x^2 - 4x - 12 \leq 0$ when $x < -3$ or $-2 < x < 2$. Finally, add equal signs to the inequality signs because the original inequality sign ($\geq$) contains one.

So, the solution set is: $\mathbf{x \leq -3}$ **or** $\mathbf{-2 \leq x \leq 2}$ **Answer D**

45. What is the remainder in the division $(6x^3 - x^2 + 4x - 9) \div (2x - 3)$ ?

A. $-15$

B. $-3$

C. $3$

D. $15$

The easiest approach to this is to use synthetic division. First, note that the root implied by the divisor $(2x - 3)$ is $x = \frac{3}{2} = 1.5$.

$$\begin{array}{r|rrrr} 1.5 & 6 & -1 & 4 & -9 \\ & & 9 & 12 & 24 \\ \hline & 6 & 8 & 16 & 15 \end{array}$$

**Answer D**

46. Use synthetic or long division to find the quotient of $(2x^2 - 33x + 16) \div (x - 16)$ .

A. $2x - 33 + \frac{16}{x-16}$

B. $2x - 1$

C. $2x - 1 + \frac{-32}{x-16}$

D. $2x + 1 + \frac{32}{x-16}$

The easiest approach to this is to use synthetic division. First, note that the root implied by the divisor $(x - 16)$ is $x = 16$.

$$\begin{array}{r|rrr} 16 & 2 & -33 & 16 \\ & & 32 & -16 \\ \hline & 2 & -1 & 0 \end{array}$$

The result is $\mathbf{2x - 1}$ **Answer B**

For a full explanation of synthetic division, see pages 122-123 of the Algebra Handbook or use the synthetic division section of the Algebra App, both of which are available at www.mathguy.us.

47. Given $x = 5$, use synthetic division to find the two remaining real solutions of $3x^3 - 35x^2 + 128x - 140 = 0$

A. $x = 2\,,\; x = \frac{14}{3}$

B. $x = 6\,,\; x = \frac{14}{3}$

C. $x = -2\,,\; x = \frac{14}{3}$

D. no other solutions

Note that the root is given to us directly: $x = 5$.

$$\begin{array}{r|rrrr} 5 & 3 & -35 & 128 & -140 \\ & & 15 & -100 & 140 \\ \hline & 3 & -20 & 28 & 0 \end{array}$$

The result is: $3x^2 - 20x + 28 = 0$

Split the middle term: $3x^2 - 6x - 14x + 28 = 0$

The split was determined using the **AC Method.**

Pair terms: $(3x^2 - 6x) - (14x - 28) = 0$

Factor pairs: $3x(x-2) - 14(x-2) = 0$

Collect terms: $(3x - 14)(x - 2) = 0$

Break into separate equations: $3x - 14 = 0 \quad x - 2 = 0$

Solutions for $x$ in the equation: $\boldsymbol{x = \left\{\frac{14}{3},\ 2\right\}}$ **Answer A**

48. Given $(x - 9)$ is a factor of the polynomial, $f(x) = 16x^3 - 144x^2 - 81x + 729$, what are the remaining factors?

A. $(4x - 9)(4x + 9)$

B. $(16x^2 + 81)$

C. $(4x - 9)$

D. $(4x - 9)(4x - 9)$

Note that the root implied by the divisor $(x - 9)$ is $x = 9$.

$$\begin{array}{r|rrrr} 9 & 16 & -144 & -81 & 729 \\ & & 144 & 0 & -729 \\ \hline & 16 & 0 & -81 & 0 \end{array}$$

The result is: $16x^2 - 81 = 0$

Factor the difference of squares: $\boldsymbol{(4x + 9)(4x - 9)}$ **Answer A**

49 Solve the system: $\begin{cases} y = -12x \\ 4x^2 + y = 16 \end{cases}$

A. $(-4, 48)$, $(1, -12)$ B. $(4, -48)$, $(1, -12)$

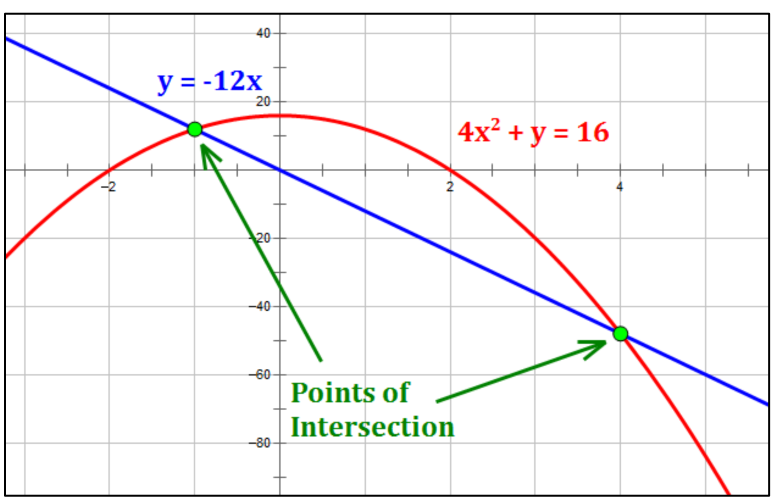
C. $(4, -48)$, $(1, 12)$ D. $(4, -48)$, $(-1, 12)$

This system is tailor made for substitution because one of the equations contains an unfettered variable (i.e., a variable that is not multiplied by or added to anything).

| | |
|---|---|
| Original equation: | $4x^2 + y = 16$ |
| Substitute $-12x$ for $y$: | $4x^2 + (-12x) = 16$ |
| Clean up: | $4x^2 - 12x = 16$ |
| Subtract 16: | $-16 \quad -16$ |
| Result: | $4x^2 - 12x - 16 = 0$ |
| Factor out 4: | $4(x^2 - 3x - 4) = 0$ |
| Factor the trinomial: | $4(x-4)(x+1) = 0$ |
| Break into separate equations: | $(x-4) = 0 \qquad (x+1) = 0$ |
| Solutions for $x$: | $x = \{4, -1\}$ |

At this point, check the answers. There is only one answer with this pair of $x$ values. If this were not the case, we would have to calculate $y$-values for each $x$-value, like we do in Problem 50.

**Answer D**



50. Solve the system: $\begin{cases} y = 8x^2 + 9x + 2 \\ y = 3x^2 + 2x \end{cases}$

A. $(-1, 1)\,,\ \left(\frac{-2}{5}\,, \frac{-24}{5}\right)$ B. $(-1, -1)\,,\ \left(\frac{-2}{5}\,, \frac{-24}{5}\right)$

C. $(-1, 1)\,,\ \left(\frac{-2}{5}\,, \frac{-8}{25}\right)$ D. $(-1, -1)\,,\ \left(\frac{-2}{5}\,, \frac{-8}{25}\right)$

Both equations are of the form: $y = \cdots$, so set them equal to each other.

| | |
|---|---|
| Original equations: | $8x^2 + 9x + 2 = 3x^2 + 2x$ |
| Subtract $3x^2 + 2x$: | $-3x^2 - 2x \qquad - 3x^2 - 2x$ |
| Result: | $5x^2 + 7x + 2 = 0$ |
| Factor the trinomial: | $(5x + 2)(x + 1) = 0$ |
| Break into separate equations: | $(5x + 2) = 0 \qquad (x + 1) = 0$ |
| Solutions for $x$: | $x = \left\{-\frac{2}{5}, -1\right\}$ |

Note: you could have seen that all four answers given above have these two $x$-values as solutions. However, you cannot be sure this will be the case on a real test or final. So, make sure you can do the math described above.

Now, we have to find the $y$-values that go with these $x$-values. You can use either equation for this. I typically go with the simpler equation, in this case: $y = 3x^2 + 2x$.

$$x = -\frac{2}{5}: \quad y = 3 \cdot \left(-\frac{2}{5}\right)^2 + 2 \cdot \left(-\frac{2}{5}\right) = 3 \cdot \left(\frac{4}{25}\right) - \frac{4}{5} = \frac{12}{25} - \frac{20}{25} = -\frac{8}{25}$$

$$x = -1: \quad y = 3 \cdot (-1)^2 + 2 \cdot (-1) = 3 \cdot 1 - 2 = 1$$

So, the two solutions are: $(-1, 1), \left(-\frac{2}{5}, -\frac{8}{25}\right)$ **Answer C**