Algebra 2: Chapter 7 Practice Test
"Unofficial" Worked-Out Solutions by Earl Whitney

1) $y = 4^{(x-3)} + 5$ ← *Asymptote also useful in setting the range.*

1st step: Asymptote: $y = 5$

2nd step: Select x-values of points:

Select a point where the exponent is zero;

$$x - 3 = 0$$
$$x = 3$$

Also select a point one <u>higher</u>, since the lead coefficient is positive.

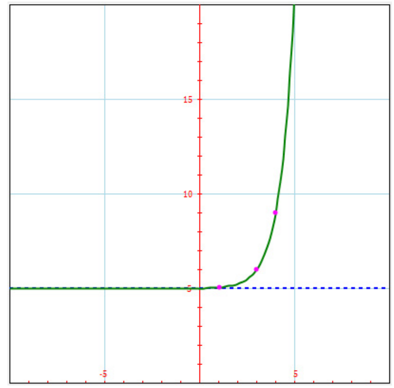
So, our x-values are $x = 3$ and $x = 4$

3rd step: Calculate y-values of points:

$x = 3$: $\quad y = 4^{(3-3)} + 5 = 1 + 5 = 6 \quad$ $Point$: $(3, 6)$

$x = 4$: $\quad y = 4^{(4-3)} + 5 = 4 + 5 = 9 \quad$ $Point$: $(4, 9)$

4th step: Draw the curve based on the asymptote and the two points.



Note: the graph above shows three points, but only two of them are needed if you draw the asymptote.

Domain: all real numbers
Range: $y > 5$

2) $y = -3\left(\frac{2}{5}\right)^{(x+1)} - 2$ ← *Asymptote also useful in setting the range.*

1st step: Asymptote: $y = -2$

2nd step: Select x-values of points:

Select a point where the exponent is zero;

$$x + 1 = 0$$
$$x = -1$$

Also select a point one <u>lower</u>, since the lead coefficient is negative.

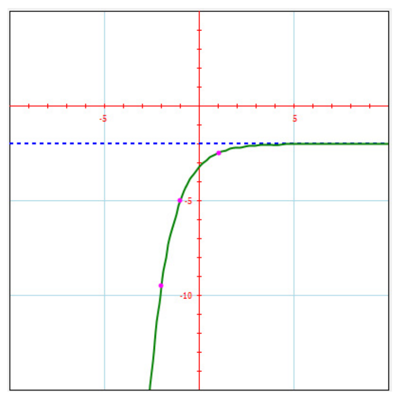
So, our x-values are $x = -2$ and $x = -1$

3rd step: Calculate y-values of points:

$x = -2$: $\quad -3\left(\frac{2}{5}\right)^{(-2+1)} - 2 = -3\left(\frac{5}{2}\right) - 2 = -9.5 \quad$ $Point$: $(-2, -9.5)$

$x = -1$: $\quad -3\left(\frac{2}{5}\right)^{(-1+1)} - 2 = -3(1) - 2 = -5 \quad$ $Point$: $(-1, -5)$

4th step: Draw the curve based on the asymptote and the two points.



Note: the graph above shows three points, but only two of them are needed if you draw the asymptote.

Domain: all real numbers
Range: $y < -2$

3) $y = e^{(x-2)} + 4$

Asymptote also useful in setting the range.

1st step: Asymptote: $y = 4$

2nd step: Select x-values of points:

Select a point where the exponent is zero;

$$x - 2 = 0$$
$$x = 2$$

Also select a point one <u>higher</u>, since the lead coefficient is positive.

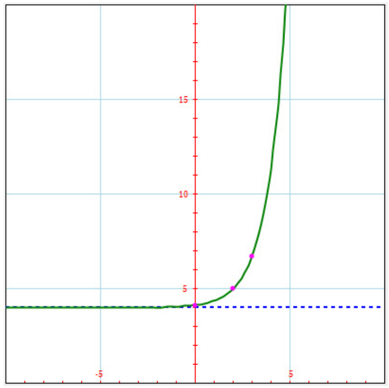
So, our x-values are $x = 2$ and $x = 3$

3rd step: Calculate y-values of points:

$x = 2$: $y = e^{(2-2)} + 4 = 1 + 4 = 5$ $\quad Point$: $(2, 5)$

$x = 3$: $y = e^{(3-2)} + 4 = 2.7 + 4 = 6.7$ $\quad Point$: $(3, 6.7)$

4th step: Draw the curve based on the asymptote and the two points.



Note: the graph above shows three points, but only two of them are needed if you draw the asymptote.

Domain: all real numbers
Range: $y > 4$

4) $f(x) = log_2(x + 1) - 6$

1st step: Asymptote occurs where the object of the log is zero:

$$x + 1 = 0 \longrightarrow x = -1$$

Asymptote also useful in setting the domain.

2nd step: Select x-values of points:

Select two points with the following properties:

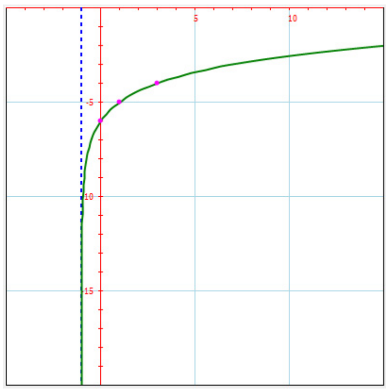
Select a point so that the object of the log is equal to 1:

$$x + 1 = 1 \longrightarrow x = 0$$

Select the other point so that the object of the log is equal to the <u>base</u> of the log:

$$x + 1 = 2 \longrightarrow x = 1$$

So, our x-values are $x = 0$ and $x = 1$



Note: the graph above shows three points, but only two of them are needed if you draw the asymptote.

Domain: $x > -1$
Range: all real numbers

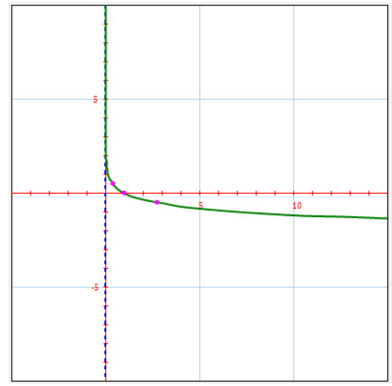
3rd step: Calculate y-values of points:

$x = 0$: $f(x) = log_2(0 + 1) - 6 = 0 - 6 = -6$ $\quad Point$: $(0, -6)$

$x = 1$: $f(x) = log_2(1 + 1) - 6 = 1 - 6 = -5$ $\quad Point$: $(1, -5)$

4th step: Draw the curve based on the asymptote and the two points.

5) $f(x) = -\frac{1}{2} ln(x)$



Note: the graph above shows three points, but only two of them are needed if you draw the asymptote.

1st step: Asymptote occurs where the object of the log is zero:

$x = 0 \longrightarrow x = 0$

Asymptote also useful in setting the domain.

2nd step: Select x-values of points:

Select two points with the following properties:

Select a point so that the object of the log is equal to 1:

$x = 1 \longrightarrow x = 1$

Select the other point so that the object of the log is equal to the base of the log:

$x = e \longrightarrow x = e = 2.7$

So, our x-values are $x = 1$ and $x = e$

Domain: $x > 0$
Range: all real numbers

3rd step: Calculate y-values of points:

$x = 1$	$f(x) = -\frac{1}{2} ln(1) = 0$	$Point: (1, 0)$
$x = e = 2.7$	$f(x) = -\frac{1}{2} ln(e) = -\frac{1}{2}$	$Point: (2.7, -0.5)$

4th step: Draw the curve based on the asymptote and the two points.

6) $A = \$1{,}000 \cdot \left(1 + \frac{.07}{365}\right)^{10 \cdot 365} = \$2{,}013.62$

7) $A = \$1{,}000 \cdot \left(1 + \frac{.08}{52}\right)^{8 \cdot 52} = \$1{,}895.55$

Problems 8 to 9: In evaluating logarithmic expressions, it is often helpful to set the expression equal to x, and then use the "first-last-middle" rule to convert the logarithmic expression to an exponential expression, which is easier to think about. *Note: the "first-last-middle" rule requires that the logarithmic or exponential portion be on the left-hand side of the equation.*

8) In the log expression, $log_3 27 = x$, **first** is "3", **last** is "x" and **middle** is "27." We put these in the exponential expression, from left to right, to get: $3^x = 27$, then solve.

$log_3 27 = x$ converts to: $3^x = 27 \longrightarrow x = 3$

9) $\log 100 = x$ converts to: $10^x = 100$ $\longrightarrow$ $x = 2$

10) $3 \cdot log_2 4$

First, calculate: $log_2 4 = x$ which converts to: $2^x = 4$ $\longrightarrow$ $x = 2$

But, we want 3 times this: $3 \cdot 2 = 6$

11) $\ln e = x$ (remember that: ln is shorthand for: log_e)
$log_e\, e = x$ converts to: $e^x = e$ $\longrightarrow$ $x = 1$

Problems 12 to 13: Exponentiation and taking logarithms are inverse operations, so when they both exist, *with the same base*, they cancel each other out.

12) $e^{\ln 5} = 5$

13) $8^{log_8 11} = 11$

14) Find the inverse of: $y = \ln(x + 3)$

Step 1: switch variables	$x = \ln(y + 3)$
Step 2: take e to the power of both sides	$e^x = e^{\ln(y+3)}$
Step 3: simplify	$e^x = y + 3$
Step 4: subtract 3	$e^x - 3 = y$
Step 5 (optional): switch sides	$y = e^x - 3$

15) $y = 2.9^x$ is an example of growth because $2.9 > 1$.
If the base were less than 1, (example: $y = 0.87^x$), it would be an example of decay.

16) $x = log_7 49$
Re-write this as: $log_7 49 = x$ and use the "first-last-middle" rule.
$log_7 49 = x$ converts to: $7^x = 49$ $\longrightarrow$ $x = 2$

17) $0 = log_8 x$

Step 1: take 8 to the power of both sides $8^0 = 8^{(log_8 x)}$

Step 2: simplify $1 = x$

Step 3 (optional): switch sides $x = 1$

18) $log_x 64 = 2$

This form is set up to use the "first-last-middle" rule, as follows:

$log_x 64 = 2$ converts to: $x^2 = 64 \longrightarrow x = 8$

Note: $x \neq -8$ because -8 is not an acceptable base for a logarithm.

19) $\log(2x) = 2$

Step 1: take 10 to the power of both sides $10^{\log(2x)} = 10^2$

Step 2: simplify $2x = 100$

Step 3: divide by 2 $x = 50$

20) $\ln(2x - 3) = \ln 21$

Step 1: objects of the two ln's must be equal $2x - 3 = 21$

Step 2: add 3 $2x = 24$

Step 3: divide by 2 $x = 12$

21) $log_4(x + 2) + log_4(x - 1) = log_4 10$

Step 1: combine logs on the left side $log_4[(x + 2)(x - 1)] = log_4 10$

Step 2: objects of the two log_4's must be equal $(x + 2)(x - 1) = 10$

Step 3: multiply through on the left $x^2 + x - 2 = 10$

Step 4: subtract 10 $x^2 + x - 12 = 0$

Step 5: factor $(x - 3)(x + 4) = 0$

Step 6: solve for x $x = 3, -4$

Step 7: test value: $\boldsymbol{x = 3}$ **passes** because $log_4(3 + 2) + log_4(3 - 1) = log_4 10$

test value: $\boldsymbol{x = -4}$ **fails** because you cannot take a log of a negative number.

$log_4(-4 + 2)$ and $log_4(-4 - 1)$ are both undefined!

Step 8: final solution: $x = 3$

22) $5^{(x-1)} = 8$

Step 1: take the log of both sides $\log 5^{(x-1)} = \log 8$

Step 2: simplify $(x-1) \cdot \log 5 = \log 8$

Step 3: divide by $\log 5$ $x - 1 = \frac{\log 8}{\log 5}$

Step 4: add 1 $x = \frac{\log 8}{\log 5} + 1$

Step 2: simplify $x = 2.29$

23) $3 \cdot e^{(2x-1)} = 18$

Step 1: divide by 3 $e^{(2x-1)} = 6$

Step 2: take the ln of both sides $\ln e^{(2x-1)} = \ln 6$

Step 3: simplify $2x - 1 = \ln 6$

Step 4: add 1 $2x = \ln 6 + 1$

Step 5: divide by 2 $x = \frac{\ln 6 + 1}{2}$

Step 6: simplify $x = 1.40$

24) $3 \cdot \ln x + 7 = 5$

Step 1: subtract 7 $3 \cdot \ln x = -2$

Step 2: divide by 3 $\ln x = -\frac{2}{3}$

Step 3: take e to the power of both sides $e^{\ln x} = e^{-\frac{2}{3}}$

Step 4: simplify $x = e^{-\frac{2}{3}}$

Step 5: calculate $x = 0.51$

exponents

25) $\textcircled{2} \cdot log_7(2x) - log_7 3y + \textcircled{4} log_7 z = log_7\left(\frac{(2x)^2 z^4}{3y}\right) = log_7\left(\frac{4x^2 z^4}{3y}\right)$

"-" indicates term goes in denominator

"+" indicates term goes In numerator

exponents

26) $④ \cdot \ln x + ③ \cdot \ln y^3 - \ln z^4 \quad = \quad \ln\left(\frac{x^4 y^9}{z^4}\right)$

"+" indicates term goes In numerator

"-" indicates term goes in denominator

> Note: in Problem 26, y already has an exponent of 3, so the coefficient in front of $\ln y^3$ (also,3) is multiplied by that exponent to get the exponent of y in the condensed logarithmic form $(3 \cdot 3 = 9)$.

27) $\log_2\left(\frac{6a^3b^2}{9c^4}\right) \quad = \quad log_2 6 + 3log_2 a + 2log_2 b - log_2 9 - 4log_2 c$

Steps to laying this out:

Step 1: write log_2 of all of the items in parentheses in the original problem:

$$log_2 6 \quad log_2 a \quad log_2 b \quad log_2 9 \quad log_2 c$$

Step 2: add the exponents from the original problem as coefficients of each log:

$$log_2 6 \quad \mathbf{3}log_2 a \quad \mathbf{2}log_2 b \quad log_2 9 \quad \mathbf{4}log_2 c$$

Step 3: add the signs ("+" for items in the numerator; "−" for items in the denominator):

$$log_2 6 + 3log_2 a + 2log_2 b - log_2 9 - 4log_2 c$$

28) $\ln\left(\frac{12x^4}{5y^6}\right) \quad = \quad \ln 12 + 4 \ln x - \ln 5 - 6 \ln y$

Note: try laying this out using the above steps.

29) $log_2 9 = \frac{\log 9}{\log 2} = 3.17$ OR $log_2 9 = \frac{\ln 9}{\ln 2} = 3.17$

30) $log_3 5 = \frac{\log 5}{\log 3} = 1.46$ OR $log_3 5 = \frac{\ln 5}{\ln 3} = 1.46$

31) $7.5 = \frac{2}{3} \cdot \log\left(\frac{E}{10^{11.8}}\right)$

Step 1: multiply by $\frac{3}{2}$	$11.25 = \log\left(\frac{E}{10^{11.8}}\right)$
Step 2: expand	$11.25 = \log \mathrm{E} - \log 10^{11.8}$
Step 3: simplify	$11.25 = \log \mathrm{E} - 11.8$
Step 4: add 11.8	$23.05 = \log \mathrm{E}$
Step 5: take 10 to the power of both sides	$10^{23.05} = 10^{\log \mathrm{E}}$
Step 6: simplify	$10^{23.05} = E$
Step 7: break out $10^{23.05}$	$10^{.05} \cdot 10^{23} = E$
Step 8: calculate $10^{.05}$	$1.12 \cdot 10^{23} = E$
Step 9 (optional): switch sides	$E = 1.12 \cdot 10^{23}$ (units unknown)

32) $\log_3 243 + \ln(e^{10}) - \log_5 625$

Note the following:

$3^5 = 243$

$5^4 = 625$

Then, $\log_3 243 + \ln(e^{10}) - \log_5 625$

$= \quad 5 \quad + \quad 10 \quad - \quad 4 \quad = \quad 11$ Answer **C**