For more information on the techniques used in this document, see the "Algebra 2 Chapter 5 Companion," available on www.mathguy.us.

For #1-4, solve.

1) $4y^4 + 4y^3 = 48y^2$

Starting Equation: $4y^4 + 4y^3 \qquad = 48y^2$

Subtract $48y^2$ from both sides: $\qquad -48y^2 \quad -48y^2$

Result: $4y^4 + 4y^3 - 48y^2 = 0$

Factor out $4y^2$: $4y^2\,(y^2 + y - 12) = 0$

Factor the remaining trinomial: $4y^2\,(y-3)\,(y+4) = 0$

Break into separate equations: $y = 0 \qquad y - 3 = 0 \qquad y + 4 = 0$

Identify solutions: $\boldsymbol{y = \{0, 3, -4\}}$

Question: Why didn't we start this problem by dividing by y^2?

Answer: Because it is possible that $y = 0$, and we cannot divide an equation by 0 and get valid results. Also, if 0 is a root (i.e., a zero of the polynomial), we may lose it in the division, meaning that the solution set we find would not include $y = 0$, which it should.

Lesson: don't divide by a term including the variable unless you are very careful.

2) $4x^4 = 9x^2$

Starting Equation: $4x^4 = 9x^2$

Subtract $9x^2$ from both sides: $-9x^2 \quad -9x^2$

Result: $4x^4 - 9x^2 = 0$

Factor out x^2: $x^2\,(4x^2 - 9) = 0$

Factor the difference of squares: $x^2\,(2x+3)\,(2x-3) = 0$

Break into separate equations: $x = 0 \qquad 2x + 3 = 0 \qquad 2x - 3 = 0$

Identify solutions: $\boldsymbol{x = \left\{0,\ -\frac{3}{2},\ \frac{3}{2}\right\}}$

3) $x^4 - 3x^2 - 18 = 0$

This equation has higher powers of x than we are used to for a trinomial, so we may need a trick here. If you see this equation as simply a quadratic in terms of x^2-terms instead of x-terms, then you do not need the trick. If you do not see that, consider this:

Let $u = x^2$. Then, we can re-write our equation as: $u^2 - 3u - 18 = 0$. Then:

Starting Equation: $u^2 - 3u - 18 = 0$

Factor the trinomial: $(u - 6)(u + 3) = 0$

Substitute x^2 back in for u: $(x^2 - 6)(x^2 + 3) = 0$

Break into separate equations: $x^2 - 6 = 0 \quad x^2 + 3 = 0$

Manipulate each equation: $x^2 = 6 \quad x^2 = -3$

Identify solutions: $\boldsymbol{x = \{\pm\sqrt{6},\ \pm i\sqrt{3}\}}$

4) $3x^5 - 9x^3 = 30x$

Starting Equation: $3x^5 - 9x^3 \quad = 30x$

Subtract $30x$ from both sides: $\quad -30x \quad -30x$

Result: $3x^5 - 9x^3 - 30x = 0$

Factor out $3x$: $3x(x^4 - 3x^2 - 10) = 0$

Factor the remaining trinomial: $3x(x^2 - 5)(x^2 + 2) = 0$

Use the "u" method described in #3 above if you need to.

Break into separate equations: $x = 0 \quad x^2 - 5 = 0 \quad x^2 + 2 = 0$

Manipulate each equation: $x = 0 \quad x^2 = 5 \quad x^2 = -2$

Identify solutions: $\boldsymbol{x = \{0,\ \pm\sqrt{5},\ \pm i\sqrt{2}\}}$

5) Given the polynomial $f(x) = x^3 - 5x^2 - 2x + 24$ and a factor $(x+2)$, factor completely.

First, divide out $(x+2)$ from $f(x)$.

$$
\begin{array}{r|l}
 & x^2 - 7x + 12 \\
\hline
x+2 & x^3 - 5x^2 - 2x + 24 \\
 & x^3 + 2x^2 \\
\hline
 & -7x^2 - 2x + 24 \\
 & -7x^2 - 14x \\
\hline
 & 12x + 24 \\
 & 12x + 24 \\
\hline
 & 0
\end{array}
$$

Then, factor the resulting trinomial.

$$x^2 - 7x + 12 = (x-3)(x-4)$$

So, the fully factored polynomial is:

$$\boldsymbol{f(x) = (x+2)\,(x-3)\,(x-4)}$$

6) What is the remaining term in the division:
$(4x^3 - 6x^2 + 8x - 2) \div (2x - 1)$?

$$
\begin{array}{r|l}
 & 2x^2 - 2x + 3 \\
\hline
2x-1 & 4x^3 - 6x^2 + 8x - 2 \\
 & 4x^3 - 2x^2 \\
\hline
 & -4x^2 + 8x - 2 \\
 & -4x^2 + 2x \\
\hline
 & 6x - 2 \\
 & 6x - 3 \\
\hline
 & 1
\end{array}
$$

$$\boldsymbol{\frac{1}{2x-1}}$$

7) Solve $x^3 + 3x^2 - 28x \le 0$

Let's call the polynomial "$f(x)$"

Write the corresponding equation: $x^3 + 3x^2 - 28x = 0$

Factor out x: $x\,(x^2 + 3x - 28) = 0$

Factor the remaining trinomial: $x\,(x+7)\,(x-4) = 0$

Break into separate equations: $x = 0$ $\quad x + 7 = 0$ $\quad x - 4 = 0$

Solutions for x in the equation: $x = \{0, -7, 4\}$

(continued on the next page)

Then, set up a table of intervals based on the solutions for x and test each interval to determine the sign of the function in that interval:

Interval	$x < -7$	$-7 < x < 0$	$0 < x < 4$	$x > 4$
Terms of $f(x)$	$x\ (x+7)\ (x-4)$	$x\ (x+7)\ (x-4)$	$x\ (x+7)\ (x-4)$	$x\ (x+7)\ (x-4)$
Signs of terms	$- \cdot - \cdot -$	$- \cdot + \cdot -$	$+ \cdot + \cdot -$	$+ \cdot + \cdot +$
Sign of $f(x)$	$-$	$+$	$-$	$+$

Based on the results in the table, $x^3 + 3x^2 - 28x \le 0$ when $x < -7$ or $0 < x < 4$. Then, because the sign in the inequality is "≤", which includes the equal sign, we must add to this set the solutions for x in the equation $(x = \{0, -7, 4\})$. So, the final solution set is:

$$\boldsymbol{x \le -7 \text{ or } 0 \le x \le 4}$$

8) Solve $x^3 + 5x^2 - 4x - 20 > 0$

Let's call the polynomial "$f(x)$"

Write the corresponding equation: $x^3 + 5x^2 - 4x - 20 = 0$

Group the terms in pairs: $(x^3 + 5x^2) - (4x + 20) = 0$

Be careful with your signs when grouping.

Factor out greatest common factors: $x^2\ (x+5) - 4\ (x+5) = 0$

Collect terms: $(x^2 - 4)\ (x+5) = 0$

Factor the difference of squares: $(x-2)\ (x+2)\ (x+5) = 0$

Break into separate equations: $x - 2 = 0$ $\quad x + 2 = 0$ $\quad x + 5 = 0$

Solutions for x in the equation: $x = \{2, -2, -5\}$

Then, set up a table of intervals based on the solutions for x and test each interval to determine the sign of the function in that interval:

Interval	$x < -5$	$-5 < x < -2$	$-2 < x < 2$	$x > 2$
Terms of $f(x)$	$(x-2)\ (x+2)\ (x+5)$	$(x-2)\ (x+2)\ (x+5)$	$(x-2)\ (x+2)\ (x+5)$	$(x-2)\ (x+2)\ (x+5)$
Signs of terms	$- \cdot - \cdot -$	$- \cdot - \cdot +$	$- \cdot + \cdot +$	$+ \cdot + \cdot +$
Sign of $f(x)$	$-$	$+$	$-$	$+$

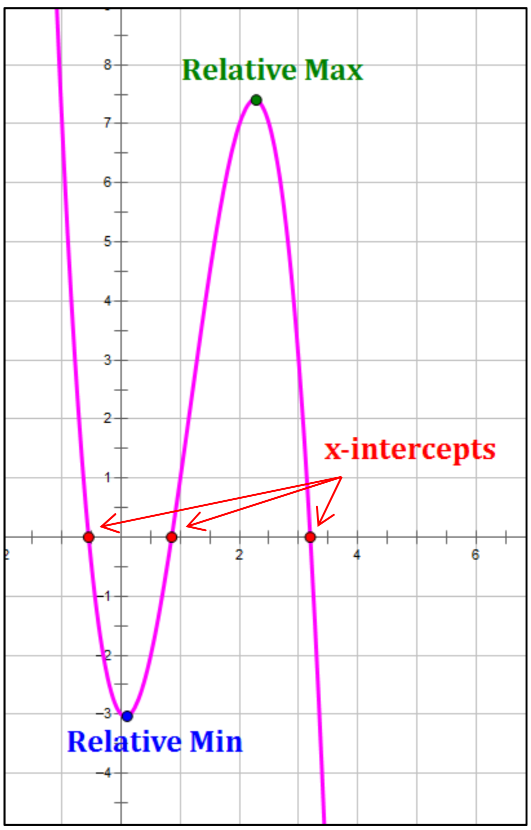
Based on the results in the table, $x^3 + 5x^2 - 4x - 20 > 0$ when $-5 < x < -2$ or $x > 2$. We do not need to add anything to this set because the sign in the original inequality is ">", which does not include an equal sign. So, the final solution set is:

$$\boldsymbol{-5 < x < -2 \text{ or } x > 2}$$

For #9–10, graph the polynomial functions. State the x-intercepts, relative max and min, end behavior, and where it is increasing and decreasing.

9) $f(x) = -2x^3 + 7x^2 - x - 3$

You will need to do this on a graphing calculator. Answers are shown in **green** below. Notes are in red.



x-intercepts: $\boldsymbol{x = \{-0.55, 0.85, 3.20\}}$

relative max: $\mathbf{(2.3, 7.4)}$

relative min: $\mathbf{(0.1, -3.0)}$

end behavior: $: as\ x \to \infty, f(x) \to -\infty$
$as\ x \to -\infty, f(x) \to +\infty$

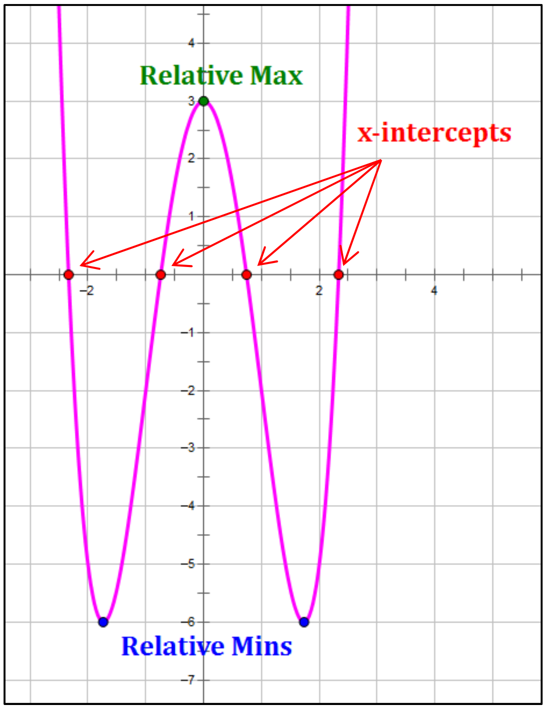
increasing: $\mathbf{0.1 < x < 2.3}$ (from min to max)

decreasing: $\boldsymbol{x < 0.1}$ **or** $\boldsymbol{x > 2.3}$ (everywhere else)

Note: at $x = 0.1$ and at $x = 2.3$, the function is flat, and is neither increasing nor decreasing.

10) $f(x) = x^4 - 6x^2 + 3$

You will need to do this on a graphing calculator. Answers are shown in **green** below. Notes are in red.



x-intercepts: $x = \{-2.33, -0.74, 0.74, 2.33\}$

relative max: $(0, 3)$

relative min: $(-1.7, -6.0)$ **and** $(1.7, -6.0)$

end behavior: : $as\ x \to \infty, f(x) \to +\infty$
$as\ x \to -\infty, f(x) \to +\infty$

increasing: $-1.7 < x < 0$ **or** $x > 1.7$ (from the left min to the max, and to the right of the right min)

decreasing: $x < -1.7$ **or** $0 < x < 1.7$ (everywhere else)

Note: at $x = -1.7$, at $x = 0$, and at $x = 1.7$, the function is flat, and is neither increasing nor decreasing.

For #11 –17, perform the indicated operation:

11) $(5x^3 - x + 3) + (x^3 - 9x^2 + 4x)$

$$\begin{array}{r} 5x^3 \qquad\quad - x + 3 \\ + \ x^3 - 9x^2 + 4x \\ \hline 6x^3 - 9x^2 + 3x + 3 \end{array}$$

I like to set these up vertically. It's easier on the eyes.

12) $(x^3 + 4x^2 - 5x) - (4x^3 + x^2 - 7)$

$$\begin{array}{r} x^3 + 4x^2 - 5x \qquad \\ + \ -4x^3 - x^2 \qquad + 7 \\ \hline -3x^3 + 3x^2 - 5x + 7 \end{array}$$

When subtracting, I prefer to change all of the signs of the expression being subtracted and then add. (I like adding better than subtracting.)

13) $(\boldsymbol{x}-\mathbf{1})(\mathbf{2}\boldsymbol{x}+\mathbf{3})^{\mathbf{2}}$

$$(x-1)\,(2x+3)^2 = (x-1)\,(2x+3)\,(2x+3)$$

I would start by FOIL-ing the two $(2x+3)$ terms.

$(2x+3)\,(2x+3)$

F $2x \cdot 2x = 4x^2$
O $2x \cdot 3 = 6x$
I $3 \cdot 2x = 6x$
(O + I: $12x$)
L $3 \cdot 3 = 9$

$(2x+3)\,(2x+3) = 4x^2 + 12x + 9$

Multiply the result by $(x-1)$.

$$\begin{array}{r} 4x^2 + 12x + 9 \\ x - 1 \\ \hline -4x^2 - 12x - 9 \\ 4x^3 + 12x^2 + 9x \qquad \\ \hline \mathbf{4x^3 + 8x^2 - 3x - 9} \end{array}$$

14) $(\mathbf{4}\boldsymbol{x}^{\mathbf{4}}-\mathbf{7}\boldsymbol{x}^{\mathbf{3}}+\mathbf{15}\boldsymbol{x}-\mathbf{7})-(-\mathbf{4}\boldsymbol{x}^{\mathbf{2}}-\mathbf{10}\boldsymbol{x})$

$$\begin{array}{rrrrr} 4x^4 - 7x^3 & & + 15x - 7 \\ + & 4x^2 & + 10x & \\ \hline \mathbf{4x^4 - 7x^3} & \mathbf{+ 4x^2} & \mathbf{+ 25x - 7} \end{array}$$

When lining things up vertically to add or subtract, make sure you leave room for "missing terms." For example, in Problem 14, there is no x^2 term in the minuend (the top expression), so we leave an open space for it.

15) $(\boldsymbol{x}-\mathbf{6})(\mathbf{5}\boldsymbol{x}^{\mathbf{2}}+\boldsymbol{x}-\mathbf{8})$

$$\begin{array}{r} 5x^2 + x - 8 \\ x - 6 \\ \hline -30x^2 - 6x + 48 \\ 5x^3 + x^2 - 8x \qquad \\ \hline \mathbf{5x^3 - 29x^2 - 14x + 48} \end{array}$$

16) $(2x^3 - 11x^2 + 13x - 44) \div (x - 5)$

$$\mathbf{2x^2 - x + 8 - \frac{4}{x-5}}$$

$$x - 5 \;\big)\; 2x^3 - 11x^2 + 13x - 44$$

$$2x^3 - 10x^2$$

$$-x^2 + 13x - 44$$

$$-x^2 + 5x$$

$$8x - 44$$

$$8x - 40$$

$$-4$$

Note: The remainder is generally converted to a fraction whose denominator is the divisor.

17) $(x^4 - 10x^2 + 2x + 3) \div (x - 3)$

$$\mathbf{x^3 + 3x^2 - x - 1}$$

$$x - 3 \;\big)\; x^4 \qquad - 10x^2 + 2x + 3$$

$$x^4 - 3x^3$$

$$3x^3 - 10x^2 + 2x + 3$$

$$3x^3 - 9x^2$$

$$-x^2 + 2x + 3$$

$$-x^2 + 3x$$

$$-x + 3$$

$$-x + 3$$

$$0$$

Remember to leave room in the dividend for the missing x^3 term.

18) Find the other zeros of $f(x)$ given that $f(x) = x^3 - 4x^2 - 11x + 30$ and $x = -3$ is one zero.

First, divide $(x+3)$ out of $f(x)$.

$$\begin{array}{r|l} & x^2 - 7x + 10 \\ \hline x+3 & x^3 - 4x^2 - 11x + 30 \\ & \underline{x^3 + 3x^2} \\ & -7x^2 - 11x + 30 \\ & \underline{-7x^2 - 21x} \\ & 10x + 30 \\ & \underline{10x + 30} \\ & 0 \end{array}$$

Then, factor the remaining trinomial.

$$x^2 - 7x + 10 = (x-2)(x-5) = 0$$

Then, break the factored trinomial into separate equations and solve each:

$$(x-2) = 0 \qquad (x-5) = 0$$

$$\boldsymbol{x = \{2, 5\}}$$

Why do we divide $f(x)$ by $(x+3)$ if the zero is $x = -3$? Because if r is a zero of $f(x)$, then the corresponding factor of $f(x)$ is $(x - r)$.

19) Given the polynomial $f(x) = 9x^3 - 9x^2 - 4x + 4$ has a factor of $(x-1)$, factor $f(x)$ completely.

First, divide $(x-1)$ out of $f(x)$.

$$\begin{array}{r|l} & 9x^2 - 4 \\ \hline x-1 & 9x^3 - 9x^2 - 4x + 4 \\ & \underline{9x^3 - 9x^2} \\ & -4x + 4 \\ & \underline{-4x + 4} \\ & 0 \end{array}$$

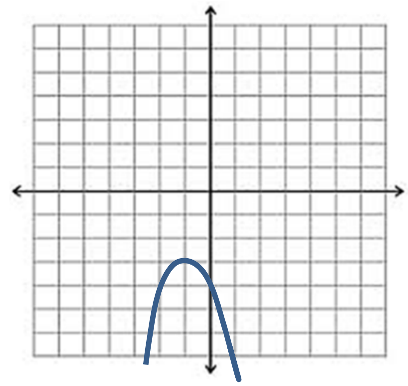
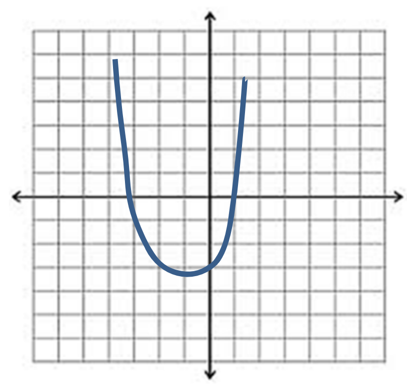
Then, factor the remaining polynomial. Note that it is a difference of squares:

$$9x^2 - 4 = (3x)^2 - 2^2$$

$$= (3x+2)(3x-2)$$

So, the fully factored polynomial is:

$$\boldsymbol{f(x) = (x-1)(3x+2)(3x-2)}$$

20) Which function is represented by the graph? A) $f(x) = (x-1)^2 + 3$ B) $f(x) = -(x+1)^2 - 3$ C) $f(x) = (x+1)^2 - 3$ D) $f(x) = -(x-1)^2 + 3$ The vertex is $(-1, -3)$, and the curve opens downward (so there is a negative lead coefficient). The equation is: $\boldsymbol{f(x) = -(x+1)^2 - 3}$ **Answer B** Note: **Vertex Form** is: $f(x) = a(x-h)^2 + k$, where (h, k) is the vertex of the curve, and a determines the direction of the curve and the magnitude of its stretch or compression.	
21) Which function is represented by the graph? A) $(x-3)(x-1)$ B) $(x-3)(x+1)$ C) $(x+3)(x-1)$ D) $(x+3)(x+1)$ The zeros are $x = -3, 1$, so the equation is: $\boldsymbol{f(x) = (x+3)(x-1)}$ **Answer C** Note: **Intercept Form** is: $f(x) = a(x-r_1)(x-r_2)$, where r_1 and r_2 are the zeros of the quadratic equation and a determines the direction of the curve and the magnitude of its stretch or compression.	

For #22 – 27, factor completely:

22) $10x^4 - 40$

Starting Expression: $10x^4 - 40$

Factor out the greatest common factor: $10\,(x^4 - 4) = 10\,[(x^2)^2 - 2^2]$

Factor the difference of squares: $\boldsymbol{10\,(x^2+2)\,(x^2-2)}$

To solve this problem, you need to recall how to factor a difference of squares:

$a^2 - b^2 = (a+b)\,(a-b)$ Memorize this!

23) $2x^3 + 3x^2 - 8x - 12$

Starting Expression:	$2x^3 + 3x^2 - 8x - 12$
Group the terms into pairs:	$(2x^3 + 3x^2) - (8x + 12)$
Factor out the GCF from each pair:	$x^2\,(2x + 3) - 4\,(2x + 3)$
Collect terms:	$(x^2 - 4)\,(2x + 3)$
Factor the difference of squares:	$(x - 2)\,(x + 2)\,(2x + 3)$
Final factored form:	$\mathbf{(x - 2)\,(x + 2)\,(2x + 3)}$

Be careful about your signs when grouping terms.

24) $x^3 + 27$

This is a sum of cubes. Here are the formulas for the sum and difference of cubes. Memorize them!

$$a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$$

Then,

$x^3 + 27 = x^3 + 3^3$ So, we will let $a = x$ and $b = 3$.

$$= (x + 3)\,(x^2 - 3 \cdot x + 3^2)$$
$$= \mathbf{(x + 3)\,(x^2 - 3x + 9)}$$

25) $6x^3 + 4x^2 - 16x$

Starting Expression:	$6x^3 + 4x^2 - 16x$
Factor out the GCF from each term:	$2x\,(3x^2 + 2x - 8)$ note: $A \cdot C = 3 \cdot (-8) = -24$

Next: use the AC method to find two values whose sum is $+2$ and product is -24. Those values are 6 and -4. The AC method is explained in detail in the Chapter 5 Companion.

Replace $+2x$ with $+6x - 4x$:	$2x\,(3x^2 + 6x - 4x - 8)$
Group the four terms into pairs:	$2x\,[(3x^2 + 6x) - (4x + 8)]$
Factor out the GCFs from each pair:	$2x\,[3x\,(x + 2) - 4\,(x + 2)]$
Collect Terms:	$2x\,[(3x - 4)\,(x + 2)]$
Final factored form:	$2x\,(3x - 4)\,(x + 2)$

26) $x^4 - 10x^2 + 9$

Similar to Problem 3, above, this equation has higher powers of x than we are used to, so we may need a trick here. If you see this equation as simply a quadratic in terms of x^2-terms instead of x-terms, then you do not need the trick. If you do not see that, consider this:

Let $u = x^2$. Then, we can re-write our equation as: $u^2 - 10u + 9 = 0$. Then:

Starting Equation:	$u^2 - 10u + 9 = 0$
Factor the trinomial:	$(u - 1)\,(u - 9) = 0$
Substitute x^2 back in for u:	$(x^2 - 1)\,(x^2 - 9) = 0$
Factor each difference of squares:	$(x + 1)\,(x - 1)\,(x + 3)\,(x - 3)$
Final factored form:	$\mathbf{(x + 1)\,(x - 1)\,(x + 3)\,(x - 3)}$

27) $2x^4 - 16x$

Starting Expression:	$2x^4 - 16x$
Factor out the GCF from each term:	$2x\,(x^3 - 8)$

Recognize the second term as a difference of cubes and recall the equation above that helps us deal with this situation:

$$a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$$

Then,

$2x\,(x^3 - 8) = 2x\,(x^3 - 2^3)$ So, we will let $a = x$ and $b = 2$.

$= 2x\,(x - 2)\,(x^2 + 2 \cdot x + 2^2)$

$= \mathbf{2x\,(x - 2)\,(x^2 + 2x + 4)}$