1. Does $\boldsymbol{y = -x^2 - 5x - 4}$ open up or down and what is the max/min?

The sign of the lead coefficient (i.e., the coefficient of x^2) tells you whether the function opens up or down.

If the lead coefficient is positive: the curve opens up.

If the lead coefficient is negative: the curve opens down.

Since the lead coefficient is -1, we know that this quadratic function **opens down**.

To find the vertex and, therefore, the location of the function's maximum, we can either take the average of its roots (i.e., x-intercepts) or use the formula: $x = -\frac{b}{2a}$.

Method 1: The function factors into: $y = -(x+1)(x+4)$, so its roots are $x = \{-1, -4\}$. The average of these is $x = -\frac{5}{2}$. Method 2: If we choose, instead, to use the above orange formula, we get: $x = -\frac{b}{2a} = -\frac{-5}{2\cdot(-1)} = -\frac{5}{2}$.

The **maximum** is the y-value at $x = -\frac{5}{2}$. $\boldsymbol{y = -\left(-\frac{5}{2}\right)^2 - 5\cdot\left(-\frac{5}{2}\right) - 4 = \frac{9}{4}}$

2. Complete the square: $\boldsymbol{x^2 + 2x - 9 = 0}$

Starting Equation:	$x^2 + 2x - 9 = \quad 0$	
Add 9 to both sides:	$+9 \quad +9$	
Result:	$x^2 + 2x \quad = \quad 9$	
Add $\left(\frac{1}{2}\cdot 2\right)^2 = 1$:	$+1 \quad +1$	← To determine the value to add to both sides, divide the coefficient of x by 2 and square the result.
Result:	$x^2 + 2x + 1 = 10$	
Simplify the square:	$\boldsymbol{(x+1)^2 = 10}$	

3. Write in vertex form: $\boldsymbol{y = x^2 - 4x - 5}$

Starting Equation:	$y \quad = x^2 - 4x - 5$
Add 5 to both sides:	$+5 \qquad + 5$
Result:	$y + 5 = x^2 - 4x$
Add $\left(\frac{1}{2} \cdot 4\right)^2 = 4$:	$+4 \qquad + 4$
Result:	$y + 9 = x^2 - 4x + 4$
Simplify the square:	$y + 9 = (x - 2)^2$
Subtract 9 from both sides:	$-9 \qquad - 9$
Result:	$\boldsymbol{y \quad = (x - 2)^2 - 9}$

To determine the value to add to both sides, divide the coefficient of x by 2 and square the result.

What is the vertex?

The general case of the vertex form is: $y = (x - h)^2 + k$, where (h, k) is the vertex.

So, the vertex is: $(\mathbf{2}, -\mathbf{9})$

For #4-5, solve the system.

4. $\begin{cases} \boldsymbol{y = 2x^2 + 3x} \\ \boldsymbol{y = 3x^2 + 5x - 8} \end{cases}$

Since both equations are of the form "$y = \cdots$" we can set them equal to each other:

$$2x^2 + 3x = 3x^2 + 5x - 8$$
$$-2x^2 - 3x \quad - 2x^2 - 3x$$
$$0 = x^2 + 2x - 8$$
$$0 = (x + 4)(x - 2)$$

So, our solutions for x are: $x = \{-4, 2\}$

Next, find the y-values for each x-value.

In this step, you can use either of the original equations. I selected the one that is easier to use.

$$y = 2x^2 + 3x$$

$x = -4$: $\quad y = 2 \cdot (-4)^2 + 3 \cdot (-4) = 20$

$x = 2$: $\quad y = 2 \cdot (2)^2 + 3 \cdot (2) = 14$

So, the solution set is: $\{(-\mathbf{4}, \mathbf{20}), (\mathbf{2}, \mathbf{14})\}$

5. $\begin{cases} y = x \\ 2y^2 + 7x - 9 = 0 \end{cases}$

Since only one equation is of the form "$y = \cdots$" we can substitute it into the other:

$$2y^2 + 7x - 9 = 0$$

Then, since $y = x$

$$2x^2 + 7x - 9 = 0$$

$$(2x + 9)(x - 1) = 0$$

The $(x - 1)$ term gives the solution of $x = 1$ by simply changing the sign of the number in the factor.

To solve the other term:

$$(2x + 9) = 0$$
$$-9 \quad -9$$
$$2x = -9$$
$$\div 2 \quad \div 2$$
$$x = -\frac{9}{2}$$

So, our solutions for x are: $x = \left\{-\frac{9}{2}, 1\right\}$

Next, find the y-values for each x-value.

$x = -\frac{9}{2}$: $\quad y = x = -\frac{9}{2}$

$x = 1$: $\quad y = x = 1$

So, the solution set is: $\left\{\left(-\frac{9}{2}, -\frac{9}{2}\right), (1, 1)\right\}$

For #6-7, a) solve and graph the inequality and b) write the solution:

6. $x^2 - 3x \geq 28$

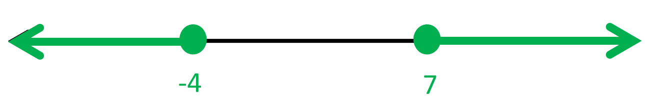
Starting Equation:	$x^2 - 3x \geq 28$
Subtract 28 from both sides:	$-28 \quad -28$
Result:	$x^2 - 3x - 28 \geq 0$
Factor:	$(x - 7)(x + 4) \geq 0$

Create an interval table to determine the sign of $(x - 7)(x + 4)$ in each interval. Break points are the roots of $(x - 7)(x + 4) = 0$, which are $x = \{-4, 7\}$.

Interval:	$(-\infty, -4)$	$(-4, 7)$	$(7, \infty)$
Sign of: $(x - 7)(x + 4)$	+	−	+

b. So, $(x - 7)(x + 4) \geq 0$ when $x \leq -4$ **or** $x \geq 7$.

a.



7. $x^2 - 16 \le 0$

Starting Equation: $x^2 - 16 \le 0$

Factor: $(x-4)(x+4) \le 0$

Create an interval table to determine the sign of $(x-4)(x+4)$ in each interval. Break points are the roots of $(x-4)(x+4) = 0$, which are $x = \{-4, 4\}$.

Interval:	$(-\infty, -4)$	$(-4, 4)$	$(4, \infty)$
Sign of: $(x-4)(x+4)$	$+$	$-$	$+$

b. So, $(x-4)(x+4) \le 0$ when $\mathbf{-4 \le x \le 4}$.

a.

For #8-9, solve using the quadratic formula. If needed, write in terms of *i* and simplify all radicals. No Decimals!

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

8. $x^2 + 4x + 3 = 0$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4 \cdot (1) \cdot (3)}}{2 \cdot (1)} = \frac{-4 \pm \sqrt{4}}{2} = \frac{-4 \pm 2}{2}$$

The two solutions, then, are:

$$x = \frac{-4-2}{2} = \frac{-6}{2} = \mathbf{-3} \qquad \text{and} \qquad x = \frac{-4+2}{2} = \frac{-2}{2} = \mathbf{-1}$$

9. $2x^2 - 7x + 11 = -1$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot (2) \cdot (11)}}{2 \cdot (2)} = \frac{7 \pm \sqrt{-39}}{4} = \frac{\mathbf{-7 \pm i\sqrt{39}}}{\mathbf{4}}$$

For #10 – 13, solve by factoring.

10. $x^2 + 6x - 27 = 0$

$(x+9)(x-3) = 0$

$\mathbf{x = \{-9, 3\}}$

When the factored form is this simple, you can simply change the signs of the numbers in the factors to get the solutions for x.

11. $2x^2 - 9x + 4 = 0$

$(2x-1)(x-4) = 0$

$2x - 1 = 0$

$+1 \quad +1$

$2x = 1$

$\div 2 \quad \div 2$

$\boldsymbol{x = \frac{1}{2}}$

$x - 4 = 0$

$+4 \quad +4$

$\boldsymbol{x = 4}$

Or … simply flip the sign of the " -4" in the factor to get $x = +4$.

12. $3x^2 + 15x = 0$

$3x(x+5) = 0$

$\boldsymbol{x = \{0, -5\}}$

13. $4x^2 - 25 = 0$

$(2x-5)(2x+5) = 0$

$\boldsymbol{x = \left\{\frac{5}{2}, -\frac{5}{2}\right\}}$

For #s 14 – 19, solve by square rooting. If needed, write in terms of *i* and simplify all radicals. No Decimals!

14. $2x^2 + 5 = 41$ $-5 \quad -5$ $2x^2 = 36$ $\div 2 \quad \div 2$ $x^2 = 18$ $x = \pm\sqrt{18}$ by square rooting $\boldsymbol{x = \pm 3\sqrt{2}}$	**15.** $(x-3)^2 = 4$ $x - 3 = \pm 2$ by square rooting $+3 \quad +3$ $x = 3 \pm 2$ $\boldsymbol{x = \{1, 5\}}$
16. $2(x-6)^2 - 45 = 53$ $+45 \quad +45$ $2(x-6)^2 = 98$ $\div 2 \quad \div 2$ $(x-6)^2 = 49$ $x - 6 = \pm 7$ by square rooting $+6 \quad +6$ $x = 6 \pm 7$ $\boldsymbol{x = \{-1, 13\}}$	**17.** $8(x+4)^2 - 18 = -2$ $+18 \quad +18$ $8(x+4)^2 = 16$ $\div 8 \quad \div 8$ $(x+4)^2 = 2$ $x + 4 = \pm\sqrt{2}$ by square rooting $-4 \quad -4$ $\boldsymbol{x = -4 \pm \sqrt{2}}$

18. $3(x+4)^2+40=-20$ $-40 \quad -40$ $3(x+4)^2=-60$ $\div 3 \quad \div 3$ $(x+4)^2=-20$ $x+4=\pm\sqrt{-20}$ by square rooting $x+4=\pm 2i\sqrt{5}$ $-4 \quad -4$ $\boldsymbol{x=-4\pm 2i\sqrt{5}}$	**19.** $2(x-1)^2+65=-95$ $-65 \quad -65$ $2(x-1)^2=-160$ $\div 2 \quad \div 2$ $(x-1)^2=-80$ $x-1=\pm\sqrt{-80}$ by square rooting $x-1=\pm 4i\sqrt{5}$ $+1 \quad +1$ $\boldsymbol{x=1\pm 4i\sqrt{5}}$

For #20-23, find the discriminant (Δ) and give the number of solutions. Note: $\Delta = b^2-4ac$

20. $x^2+6x+2=0$ $\Delta=(6)^2-4(1)(2)=\mathbf{28}$ A positive discriminant means there are **two real solutions**.	**21.** $2x^2-8x+21=9$ $-9 \quad -9$ $2x^2-8x+12=0$ $\Delta=(-8)^2-4(2)(12)=\mathbf{-32}$ A negative discriminant means there are **zero real solutions**. Note: there are two complex solutions (i.e., solutions containing i).
22. $2x^2+2=-4x$ $+4x \quad +4x$ $2x^2+4x+2=0$ $\Delta=(4)^2-4(2)(2)=\mathbf{0}$ A zero discriminant means there is **one real solution**.	**23.** $x^2-4x-7=0$ $\Delta=(-4)^2-4(1)(-7)=\mathbf{44}$ A positive discriminant means there are **two real solutions**.

24. When Harry the human cannonball is projected into the air his height is modeled by the equation $f(x)=-12(x-2)^2+122$ where x is the time, in seconds, after Harry is launched. What is the maximum height that Harry attains?

The general case of vertex form is: $y=(x-h)^2+k$, where (h,k) is the vertex.

So, the vertex is: $(2, 122)$. The vertex occurs at either the minimum or the maximum of the equation. Since this equation opens down (lead coefficient is -12), it has a maximum.

The maximum height is the y-value of the vertex $=$ 122 units.

For #25-28, graph. Include the vertex, intercepts, and maximum or minimum.

25. $y = -(x-2)^2 + 4$

This function is in vertex form, so we can read the vertex directly from the equation: **vertex =** $(2, 4)$.

The function **opens down** because the lead coefficient is negative. Therefore, the function has a maximum at the vertex.

Maximum = 4

To find the roots, multiply out the terms of the equation and factor it:

$$y = -(x-2)^2 + 4$$

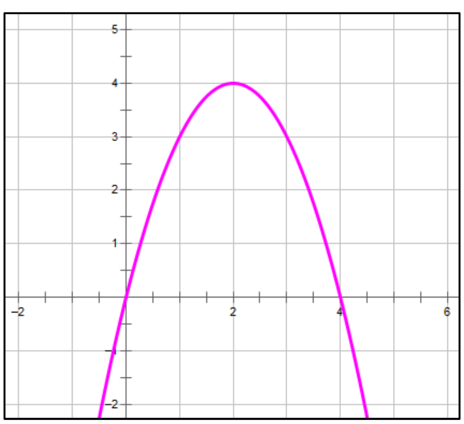
$$y = -(x^2 - 4x + 4) + 4$$

$$y = -(x^2 - 4x)$$

$$y = -x(x-4)$$

So the x-intercepts are $x = \{0, 4\}$

Vertex: $(2, 4)$
x-intercepts: $x = \{0, 4\}$
max/min: max $= 4$



26. $y = 2(x-1)(x+3)$

This function is in intercept form, so we can read the **x-intercepts** directly from the equation, $x = \{1, -3\}$

The x-coordinate of the vertex is halfway between the x-intercepts, at $x = -1$.

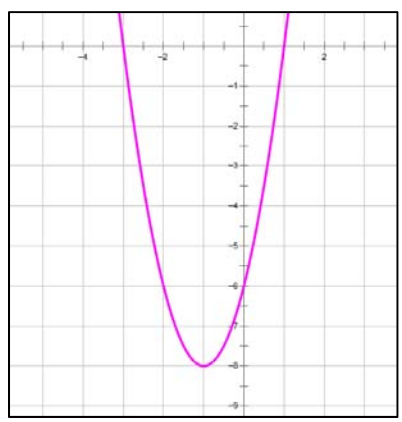
The y-value of the vertex is obtained by substituting $x = -1$ into the original equation.

$$y = 2(-1-1)(-1+3) = -8$$

The function **opens up** and has a minimum because the lead coefficient is positive. The minimum is equal to the y-coordinate of the vertex.

Minimum = -8

Vertex: $(-1, -8)$
x-intercepts: $x = \{1, -3\}$
max/min: min $= -8$



27. $y = x^2 - 4x - 12$

$y = (x-6)(x+2)$
so, **x-intercepts are at** $x = \{6, -2\}$

The x-coordinate of the vertex is halfway between the roots, at $x = 2$.

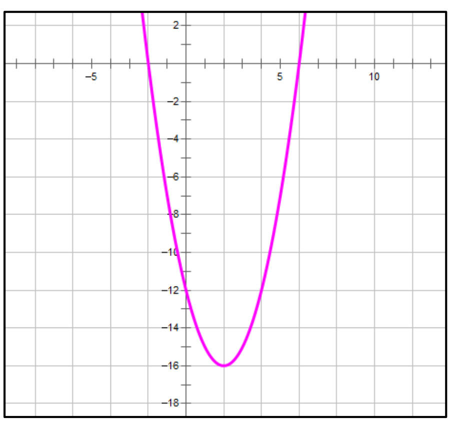
The y-value of the vertex is obtained by substituting $x = 2$ into the original equation.

$$y = (2)^2 - 4 \cdot (2) - 12 = -16$$

The function **opens up** and has a minimum because the lead coefficient is positive. The minimum is equal to the y-coordinate of the vertex.

Minimum = -16

Vertex: $(2, -16)$
x-intercepts: $x = \{6, -2\}$
max/min: min $= -16$



28. $y = -(x-3)^2$

This function is in vertex form and is also in intercept form (with $k = 0$). Yes, this happens sometimes.

Because this function has only one factor, it has only one x-intercept, which can be read from the equation as $x = 3$.

The vertex occurs at that x-intercept. So, **the vertex is** $(3, 0)$.

Since the lead coefficient is negative, the function **opens down**. So, it has a maximum at the vertex.

Maximum = 0

Vertex: $(3, 0)$
x-intercept: $x = 3$
max/min: max $= 0$