1. Given that *m* varies directly as *p*, and that *m* is 8 when *p* is -6, find *m* when *p* is -3.

The form of the equation is $m = ap$ for direct variation. First we find the value of a by substituting in the given values for m and p that relate to each other.

Starting equation: $m = a \cdot p$

Substitute in values: $8 = a \cdot (-6)$

Divide by -6: $\frac{8}{-6} = \frac{a \cdot (-6)}{-6}$

Result: $-\frac{4}{3} = a$

Revised Starting Equation: $m = -\frac{4}{3}p$

Then, we find the desired value of m when $p = -3$.

Substitute in value of p: $m = -\frac{4}{3} \cdot (-3)$

Multiply to get m: $m = 4$

2. Given that *k* varies inversely as *q*, and that *k* is 2 when *q* is 7, find *k* when *q* is 42.

The form of the equation is $k = \frac{a}{q}$ for inverse variation. First we find the value of a by substituting in the given values for k and q that relate to each other.

Starting equation: $k = \frac{a}{q}$

Substitute in values: $2 = \frac{a}{7}$

Multiply by 7: $\cdot 7 \quad \cdot 7$

Result: $14 = a$

Revised Starting Equation: $k = \frac{14}{q}$

Then, we find the desired value of k when $q = 42$.

Substitute in value of q: $k = \frac{14}{42}$

Divide to get k: $k = \frac{1}{3}$

For #3 – 6, simplify completely:

3. $\sqrt{40b^2}$

$= \sqrt{40} \cdot \sqrt{b^2}$

$= \sqrt{2 \cdot 2 \cdot 2 \cdot 5} \cdot \sqrt{b^2}$

$= 2 \cdot \sqrt{2 \cdot 5} \cdot b$

$= 2b\sqrt{10}$

4. $\sqrt{-27}$

$= \sqrt{-1} \cdot \sqrt{27}$

$= i \cdot \sqrt{3 \cdot 3 \cdot 3}$

$= i \cdot 3\sqrt{3}$

$= 3i\sqrt{3}$

5. $7\sqrt{-48x^3}$

$= 7 \cdot \sqrt{-1} \cdot \sqrt{48} \cdot \sqrt{x^3}$

$= 7 \cdot i \cdot \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} \cdot \sqrt{x^3}$

$= 7 \cdot i \cdot 2 \cdot 2 \cdot \sqrt{3} \cdot x\sqrt{x}$

$= 28i\,x\sqrt{3x}$

6. $4x\sqrt{72x^2y^5}$

$= 4x \cdot \sqrt{72} \cdot \sqrt{x^2} \cdot \sqrt{y^5}$

$= 4x \cdot \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} \cdot \sqrt{x^2} \cdot \sqrt{y^5}$

$= 4x \cdot 2 \cdot 3 \cdot \sqrt{2} \cdot x \cdot y^2\sqrt{y}$

$= 24x^2y^2\sqrt{2y}$

7. *Y* varies jointly as the product of *x* and *p* squared. If *y* is 48 when *x* is 4 and *p* is 2, find *y* when *x* is 5 and *p* is 3.

The form of the equation is $y = a \cdot x \cdot p^2$ for the joint variation described. First we find the value of a by substituting in the given values for y, x and p that relate to each other.

Starting equation: $y = a \cdot x \cdot p^2$

Substitute in values: $48 = a \cdot 4 \cdot (2)^2$

Simplify: $48 = a \cdot 16$

Divide by 16: $\frac{48}{16} = \frac{a \cdot 16}{16}$

Result: $3 = a$

Revised Starting Equation: $y = 3 \cdot x \cdot p^2$

Then, we find the desired value of y when $x = 5$ and $p = 3$.

Substitute in values of x and p: $y = 3 \cdot 5 \cdot (3)^2$

Multiply to get y: $y = 135$

8. The volume V of a gas varies inversely as the pressure p on it. If the volume is 300 cm^3 under a pressure of 48 kg/cm^2, what is the volume under a pressure of 25 kg/cm^2?

The form of the equation is $V = \frac{a}{p}$ for inverse variation. First we find the value of a by substituting in the given values for V and p that relate to each other.

Starting equation: $V = \frac{a}{p}$

Substitute in values: $300 = \frac{a}{48}$

Multiply by 48: $\cdot 48 \quad \cdot 48$

Result: $14,400 = a$

Revised Starting Equation: $V = \frac{14{,}400}{p}$

Then, we find the desired value of V when $p = 25$.

Substitute in value of p: $V = \frac{14{,}400}{25}$

Divide to get V: $V = 576\ cm^3$

For # 9 - 22 , simplify each expression completely:

9. $5\sqrt{20} \cdot 3\sqrt{-2}$

$= 5 \cdot \sqrt{2 \cdot 2 \cdot 5} \cdot 3 \cdot \sqrt{-2}$

$= 5 \cdot 2 \cdot \sqrt{5} \cdot 3 \cdot \sqrt{-1} \cdot \sqrt{2}$

$= 30 \cdot i \cdot \sqrt{5} \cdot \sqrt{2}$

$= 30i\sqrt{10}$

10. $\sqrt{-9} \cdot \sqrt{-25}$

$= \sqrt{-1} \cdot \sqrt{9} \cdot \sqrt{-1} \cdot \sqrt{25}$

$= i \cdot 3 \cdot i \cdot 5$

$= 15\ i^2$

$= 15\ (-1)$

$= -15$

11. i^{43}

In determining the value of a power of i, divide by 4 and look at the remainder. The value desired, then, is the same as the power of i with the remainder as an exponent.

$43 \div 4 = 10\ rem\ 3$

So, $i^{43} = i^3 = i^2 \cdot i = (-1) \cdot i = -i$

12. $7i \cdot 9i$

$= 7 \cdot 9 \cdot i \cdot i$

$= 63\ i^2$

$= 63\ (-1)$

$= -63$

13. $4\sqrt{24x^3y}\cdot 3\sqrt{-2x^2}$

$= 4\cdot\sqrt{2\cdot 2\cdot 2\cdot 3}\cdot\sqrt{x^3}\cdot\sqrt{y}\cdot 3\cdot\sqrt{-1}\cdot\sqrt{2}\cdot\sqrt{x^2}$

$= 12\cdot\sqrt{2\cdot 2\cdot 2\cdot 3\cdot 2}\cdot\sqrt{x^5}\cdot\sqrt{y}\cdot i$

$= 12\cdot 2\cdot 2\cdot\sqrt{3}\cdot x^2\cdot\sqrt{x}\cdot\sqrt{y}\cdot i$

$= 48\,x^2\cdot i\cdot\sqrt{3}\cdot\sqrt{x}\cdot\sqrt{y}$

$= 48\,x^2\cdot i\cdot\sqrt{3xy}$

14. $-4i\cdot 7i$

$= -4\cdot 7\cdot i\cdot i$

$= -28\,i^2$

$= -28\,(-1)$

$= 28$

15. i^{30}

See the note on problem 11 on the previous page.

$30\div 4 = 7\ rem\ 2$

So, $i^{30} = i^2 = -1$

16. $3\sqrt{28}\cdot\sqrt{-21}$

$= 3\cdot\sqrt{28}\cdot\sqrt{-1}\cdot\sqrt{21}$

$= 3\cdot\sqrt{2\cdot 2\cdot 7}\cdot i\cdot\sqrt{3\cdot 7}$

$= 3i\cdot\sqrt{2\cdot 2\cdot 7\cdot 3\cdot 7}$

$= 3i\cdot 2\cdot 7\cdot\sqrt{3}$

$= 42i\,\sqrt{3}$

17. $\dfrac{\sqrt{64}}{\sqrt{25}}$

$= \dfrac{8}{5}$

18. $\dfrac{\sqrt{-22}}{\sqrt{2}}$

$= \dfrac{i\sqrt{22}}{\sqrt{2}} = i\sqrt{\dfrac{22}{2}}$

$= i\sqrt{11}$

19. $\dfrac{4}{\sqrt{7}}$

$= \dfrac{4}{\sqrt{7}}\cdot\dfrac{\sqrt{7}}{\sqrt{7}}$

$= \dfrac{4\sqrt{7}}{7}$

20. $\dfrac{\sqrt{10}}{\sqrt{-3}}$

$= \dfrac{\sqrt{10}}{\sqrt{-3}}\cdot\dfrac{\sqrt{-3}}{\sqrt{-3}}$

$= \dfrac{\sqrt{-30}}{-3}$

$= -\dfrac{i\,\sqrt{30}}{3}$

21. $\dfrac{\sqrt{24}}{\sqrt{18}}$

$= \sqrt{\dfrac{24}{18}} = \sqrt{\dfrac{4}{3}} = \dfrac{\sqrt{4}}{\sqrt{3}}$

$= \dfrac{2}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{2\sqrt{3}}{3}$

22. $\dfrac{6}{7i}$

$= \dfrac{6}{7i}\cdot\dfrac{i}{i}$

$= \dfrac{6i}{7i^2}$

$= \dfrac{6i}{7\cdot(-1)}$

$= -\dfrac{6i}{7}$

23. Suppose that y varies jointly with w and x and inversely with z, and that $y = 360$ when $w = 8, x = 25$, and $z = 5$. Find the constant of variation. Also, find the value of y when $w = 4, x = 4$, and $z = 3$.

The form of the equation is $\boldsymbol{y = \frac{a \cdot w \cdot x}{z}}$. First we find the value of $\boldsymbol{a}$ by substituting in the given values for $\boldsymbol{w}, \boldsymbol{x}$ and $\boldsymbol{z}$ that relate to each other.

Starting equation: $\boldsymbol{y = \frac{a \cdot w \cdot x}{z}}$

Substitute in values: $\boldsymbol{360 = \frac{a \cdot 8 \cdot 25}{5}}$

Simplify: $\boldsymbol{360 = a \cdot 40}$

Divide by 40: $\boldsymbol{\frac{360}{40} = \frac{a \cdot 40}{40}}$

Result: $\boldsymbol{9 = a}$

Revised the Starting Equation: $\boldsymbol{y = \frac{9 \cdot w \cdot x}{z}}$

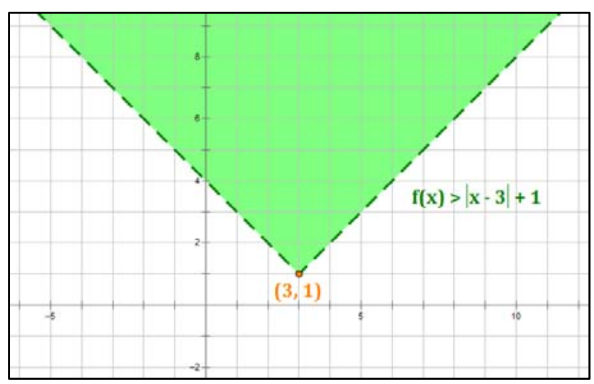
Then, we find the desired value of $\boldsymbol{y}$ when $\boldsymbol{w = 4, x = 4, z = 3}$.

Substitute in values of $\boldsymbol{w}$, $\boldsymbol{x}$, $\boldsymbol{z}$: $\boldsymbol{y = \frac{9 \cdot 4 \cdot 4}{3}}$

Simplify to get $\boldsymbol{y}$: $\boldsymbol{y = 48}$

24. Graph the inequality $y > |x - 3| + 1$

Here's what we know about the inequality:



- $y > |x - 3| + 1$
- The vertex is $\boldsymbol{(3, 1)}$.
- The graph opens up because the coefficient of the absolute value term is positive ($= +\boldsymbol{1}$).
- The slope of the curve is $+\boldsymbol{1}$ on the right and $-\boldsymbol{1}$ on the left, because the coefficient of the absolute value term is $+\boldsymbol{1}$.
- The line is dashed because the sign in the inequality is $>$, not $\geq$.
- The shaded portion is above the curve because of the $>$ (greater than) sign.

For more on absolute value problems, download the Algebra Handbook and/or the Algebra(Main) app from www.mathguy.us.

For # 25 - 30 , perform the indicated operation:

25. $(5+4i)^2$

$= (5+4i)\cdot(5+4i)$

F: $5\cdot 5 = 25$

O: $5\cdot 4i = 20i$

I: $4i\cdot 5 = 20i$

L: $4i\cdot 4i = 16i^2 = -16$

Result: $(25-16)+(20i+20i)$

$= 9+40i$

26. $\left(3+2\sqrt{5}\right)+\left(-5+6\sqrt{5}\right)$

For addition of multiple "weird" terms, it is sometimes best to line things up vertically.

$$\begin{array}{rcr} 3 & + & 2\sqrt{5} \\ -5 & + & 6\sqrt{5} \\ \hline -2 & + & 8\sqrt{5} \end{array}$$

27. $(6-3i)(6+3i)$

$= (6-3i)\cdot(6+3i)$

F: $6\cdot 6 = 36$

O: $6\cdot 3i = 18i$

I: $-3i\cdot 6 = -18i$

L: $-3i\cdot 3i = -9i^2 = 9$

Result: $(36+9)+(18i-18i)$

$= 45$

28. $(12+5i)-(7-9i)$

For subtraction of multiple "weird" terms, you can change the signs of the items being subtracted and add.

$$\begin{array}{rcr} 12 & + & 5i \\ -7 & + & 9i \\ \hline 5 & + & 14i \end{array}$$

29. $\left(8-2\sqrt{3}\right)\left(1+6\sqrt{3}\right)$

$= \left(8-2\sqrt{3}\right)\cdot\left(1+6\sqrt{3}\right)$

F: $8\cdot 1 = 8$

O: $8\cdot 6\sqrt{3} = 48\sqrt{3}$

I: $-2\sqrt{3}\cdot 1 = -2\sqrt{3}$

L: $-2\sqrt{3}\cdot 6\sqrt{3} = -12\cdot 3 = -36$

Result: $(8-36)+(48\sqrt{3}-2\sqrt{3})$

$= -28+46\sqrt{3}$

30. $5i(2+3i)+4(7-i)$

$= (10i+15i^2)+(28-4i)$

$= (-15+10i)+(28-4i)$

$$\begin{array}{rcr} -15 & + & 10i \\ 28 & - & 4i \\ \hline 13 & + & 6i \end{array}$$

For # 31 - 34 , simplify completely:

For fractions with ugly denominators, we want to "rationalize the denominator." This is accomplished by multiplying the fraction by another fraction, equal to 1, that gets rid of the radical ($\sqrt{\ }$) or the i-term in the denominator.

- For fractions with a root in the denominator, let both the numerator and denominator of the multiplier be equal to the denominator of the original problem with the sign in front of the root changed.
- For fractions with an i-term in the denominator, let both the numerator and denominator of the multiplier be equal to the denominator of the original problem with the sign in front of the i-term changed. Note, a complex number with the sign in front of the i-term changed is called the conjugate of the original complex number.

31. $\frac{1}{5-\sqrt{3}}$

$$= \frac{1}{5-\sqrt{3}} \cdot \frac{5+\sqrt{3}}{5+\sqrt{3}}$$

$$= \frac{5+\sqrt{3}}{5 \cdot 5 + 5 \cdot \sqrt{3} - \sqrt{3} \cdot 5 - \sqrt{3} \cdot \sqrt{3}}$$

$$= \frac{5+\sqrt{3}}{25 + 5\sqrt{3} - 5\sqrt{3} - 3}$$

$$= \frac{5+\sqrt{3}}{22+0}$$

$$= \frac{5+\sqrt{3}}{22}$$

32. $\frac{7}{4+3i}$

$$= \frac{7}{4+3i} \cdot \frac{4-3i}{4-3i}$$

$$= \frac{28-21i}{4 \cdot 4 + 4 \cdot (-3i) + 3i \cdot 4 + 3i \cdot (-3i)}$$

$$= \frac{28-21i}{16 + 12i - 12i - 9i^2}$$

$$= \frac{28-21i}{16+9+0}$$

$$= \frac{28-21i}{25}$$

33. $\frac{3i}{11-10i}$

$= \frac{3i}{11-10i} \cdot \frac{11+10i}{11+10i}$

$= \frac{33i+30i^2}{11 \cdot 11 + 11 \cdot 10i - 10i \cdot 11 - 10i \cdot 10i}$

$= \frac{33i + 30 \cdot (-1)}{121 + 110i - 110i - 100i^2}$

$= \frac{-30+33i}{121+100+0}$

$= \frac{-30+33i}{221}$

34. $\frac{2}{3+\sqrt{7}}$

$= \frac{2}{3+\sqrt{7}} \cdot \frac{3-\sqrt{7}}{3-\sqrt{7}}$

$= \frac{6-2\sqrt{7}}{3 \cdot 3 + 3 \cdot (-\sqrt{7}) + \sqrt{7} \cdot 3 + \sqrt{7} \cdot (-\sqrt{7})}$

$= \frac{6-2\sqrt{7}}{9 - 3\sqrt{7} + 3\sqrt{7} - 7}$

$= \frac{6-2\sqrt{7}}{2+0}$

$= \frac{6-2\sqrt{7}}{2}$

$= 3 - \sqrt{7}$

Don't forget to check your result to see if it can be reduced. In this problem, a factor of **2** can be factored out of both the numerator and denominator.