Algebra 2: Semester 2 Practice Final
"Unofficial" Worked-Out Solutions by Earl Whitney

1. The key to this problem is recognizing cubes as factors in the radicands.

$$\begin{aligned} & 8\sqrt[3]{27} + \sqrt[3]{40} - 6\sqrt[3]{135} \\ &= 8 \cdot \sqrt[3]{27} + \sqrt[3]{8} \cdot \sqrt[3]{5} - 6 \cdot \sqrt[3]{27} \cdot \sqrt[3]{5} \\ &= 8 \cdot 3 + 2 \cdot \sqrt[3]{5} - 6 \cdot 3 \cdot \sqrt[3]{5} \\ &= 24 + 2 \cdot \sqrt[3]{5} - 18 \cdot \sqrt[3]{5} \\ &= 24 - 16\sqrt[3]{5} \end{aligned}$$

Answer A

Cubes

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$

2. The key to this problem is recognizing that $64 = 4^3$.

$$\frac{4^{2/3} \cdot 64^{2/3}}{4^{4/3}} = \frac{4^{2/3} \cdot (4^3)^{2/3}}{4^{4/3}} = \frac{4^{2/3} \cdot 4^2}{4^{4/3}} = 4^{\left(\frac{2}{3} + 2 - \frac{4}{3}\right)} = 4^{1\frac{1}{3}} = 4^1 \cdot 4^{\frac{1}{3}} = 4 \cdot \sqrt[3]{4}$$

Answer D

3. Divide the exponents under the radical by 4 (the root) to get a new expression.

$$\begin{aligned} \sqrt[4]{625\, x^{48}\, y^{36}\, z^{72}} &= \sqrt[4]{625} \cdot x^{48/4} \cdot y^{36/4} \cdot z^{72/4} \\ &= 5\, x^{12}\, y^{9}\, z^{18} \end{aligned}$$

Answer C

4^{th} Powers

$1^4 = 1$
$2^4 = 16$
$3^4 = 81$
$4^4 = 256$
$5^4 = 625$

4. Make sure to rationalize the denominator.

$$\frac{\sqrt[3]{c^5} \cdot \sqrt[3]{c^4}}{\sqrt[3]{c^{10}}} = c^{5/3} \cdot c^{4/3} \cdot c^{-10/3} = c^{5/3 + 4/3 - 10/3} = c^{-1/3} = \frac{1}{c^{1/3}}$$

Now, we need an integer exponent in the denominator, so:

$$\frac{1}{c^{1/3}} \cdot \frac{c^{2/3}}{c^{2/3}} = \frac{c^{2/3}}{c} = \frac{\sqrt[3]{c^2}}{c}$$

Answer A

5. Addition and subtraction of polynomials can be performed in columns. To subtract, change the signs of the polynomial being subtracted and add.

Addition	**Subtraction**
$5x^2 + 6x - 4$ $3x^2 - 5x + 24$	$5x^2 + 6x - 4$ $-3x^2 + 5x - 24$
$8x^2 + x + 20$	$2x^2 + 11x - 28$

Answer A

6. When thinking about composite functions, consider each function as a mechanism that takes an input and does something to it. In this problem:

   - $g(x) = 5x$, so if you give something to g, it will multiply it by 5 and give you the result.
   - $h(x) = 3x + 8$, so if you give something to h, it will multiply it by 3, add 8, and give you the result.

   Given this, we get:

$$g\big(h(x)\big) = 5 \cdot (3x + 8) = 15x + 40$$

$$h\big(g(x)\big) = (3 \cdot (5x) + 8) = 15x + 8$$

Answer B

7. To find an inverse of a function, switch the x and y variables and solve for the new y.

Starting function: $y = -7x + 6$

Switch x and y: $x = -7y + 6$

Subtract 6 from each side:

$$\begin{array}{rcl} x & = & -7y + 6 \\ -6 & & -6 \\ \hline x - 6 & = & -7y \end{array}$$

Divide each side by -7:

$$\begin{array}{rcl} x - 6 & = & -7y \\ \div -7 & & \div -7 \\ \hline \frac{-x+6}{7} & = & y \end{array}$$

Answer B

8. Starting function: $y = x^2 + 5$

Switch x and y: $x = y^2 + 5$

Subtract 5 from each side:

$$\begin{array}{rcl} x & = & y^2 + 5 \\ -5 & & -5 \\ \hline x - 5 & = & y^2 \end{array}$$

Take a square root of each side: $\sqrt{x - 5} = y$

Answer A

Notice that we do not get $y = \pm\sqrt{x - 5}$ as our result. The reason for this is that the question requires that the result be a function, and the " $\pm$ " means the result is not a function (it would have two y-values for each x-value).

9. Isolate the radical, then square both sides of the equation.

$$\begin{array}{lrcl}
\text{Starting equation:} & \sqrt{5x+9} - 10 & = & 12 \\
\text{Add 10 to each side:} & +10 & & +10 \\
\hline
 & \sqrt{5x+9} & = & 22 \\
\text{Square both sides:} & 5x+9 & = & 484 \\
\text{Subtract 9 from each side:} & -9 & & -9 \\
\hline
 & 5x & = & 475 \\
\text{Divide each side by 5:} & \div 5 & & \div 5 \\
\hline
 & x = 95 & &
\end{array}$$

Answer A

10. This problem has no solution because a square root cannot have a negative value.

Answer D

11. There are two parts to this problem. We must solve the inequality, but also we must make sure the expression under the radical is positive.

Part 1: Solve the radical

$$\begin{array}{lrcl}
\text{Original inequality:} & \sqrt{10x+14} & > & 22 \\
\text{Square both sides:} & 10x+14 & > & 484 \\
\text{Subtract 14 from each side:} & -14 & & -14 \\
\hline
 & 10x & > & 470 \\
\text{Divide each side by 10:} & \div 10 & & \div 10 \\
\hline
 & x & > & 47
\end{array}$$

Part 2: Check under the radical

Need radicand to be ≥ 0: $10x + 14 \geq 0$

Note above that $10x + 14 > 484$, so $10x + 14$ will also be > 0. Therefore, the solution is what we obtained in Part 1: $x > 47$.

Answer D

12. There are two parts to this problem. We must solve the inequality, but also we must make sure the expression under the radical is positive.

Part 1: Solve the radical

$$\begin{array}{lrcr}
\text{Original inequality:} & \sqrt{2x-3}\ +1 & \le & 7 \\
\text{Subtract 1 from each side:} & -1 & & -1 \\
\cline{2-4}
 & \sqrt{2x-3} & \le & 6 \\
\text{Square both sides:} & 2x-3 & \le & 36 \\
\text{Add 3 to each side:} & +3 & & +3 \\
\cline{2-4}
 & 2x & \le & 39 \\
\text{Divide each side by 2:} & \div 2 & & \div 2 \\
\cline{2-4}
\end{array}$$

$$x \le \frac{39}{2}$$

Part 2: Check under the radical

$$\begin{array}{lrcr}
\text{Need radicand to be} \ge 0\text{:} & 2x-3 & \ge & 0 \\
\text{Add 3 to each side:} & +3 & & +3 \\
\cline{2-4}
 & 2x & \ge & 3 \\
\text{Divide each side by 2:} & \div 2 & & \div 2 \\
\cline{2-4}
\end{array}$$

$$x \ge \frac{3}{2} \quad \Rightarrow \quad \frac{3}{2} \le x$$

We must combine the two results to get the final answer.

$$\frac{3}{2} \le x \le \frac{39}{2}$$

Answer D

13. The domain is all values of x that can produce a value of y. We must make sure the expression under the radical is positive or zero (≥ 0).

$$\begin{array}{lrcr}
\text{Need radicand to be} \ge 0\text{:} & x-3 & \ge & 0 \\
\text{Add 3 to each side:} & +3 & & +3 \\
\cline{2-4}
 & \boldsymbol{x} & \boldsymbol{\ge} & \boldsymbol{3}
\end{array}$$

is the domain

The range is all values of y that can be produced by values of x. The smallest value that $\sqrt{x-3}$ can have is zero, and it can have any value greater than that. Then:

$g(x) = 6 + \sqrt{x-3} \ge 6 + 0 = 6$ so **$g(x) \ge 6$ is the range**

Answer B

14. The function requires $x \geq -5$ in order to have a positive radicand, so the function starts on the left at $x = -5$. There is no "as $x \to -\infty$"**.**

    The end behavior will be on the right, as $x \to \infty$. Then, notice that as x increases without end, $\sqrt{x+5}$ also increases without end. So,

    as $x \to \infty$, $f(x) \to \infty$ **Answer D**

15. Any function of the form: $y = a \cdot \sqrt[n]{x-h} + k$ (with n being any **even** positive integer) will have a vertex at (h, k).

    For this equation, $h = 3$ and $k = 0$, so the vertex must be at $(3, 0)$. **Answer A**

16. Any function of the form: $y = a \cdot \sqrt[n]{x-h} + k$ (with n being any **odd** positive integer) will have an inflection point at (h, k). An *inflection point* is a point that looks like the center of the curve (in higher level mathematics, it is defined as a point where the slope of the curve changes from increasing to decreasing or from decreasing to increasing).

    For this equation, $h = 0$ and $k = 0$, so the inflection point is at $(0, 0)$. Translating the graph 2 units up and 5 units left resets the inflection point at $(-5, 2)$. **Answer A**

17. The domain is all values of x that can produce a value of y. In exponential functions, the **domain** is generally **all real numbers**.

    The exponential term, 2^x is always greater than zero, and can be any value above zero. So,

    $y = 2^x + 1 > 0 + 1 = 1$ so $\boldsymbol{y > 1}$ **is the range** **Answer A**

18. In logarithmic functions, the **range** is generally **all real numbers**.

    The argument of the logarithm, $(x - 2)$ must be greater than zero, and can be any value above zero (*you can only take logs of positive numbers*). So,

    Need argument to be $>$ 0: $x - 2 > 0$
    Add 3 to each side: $+2 \quad +2$
    $\boldsymbol{x > 2}$ **is the domain** **Answer D**

19. Each term in this problem can be simplified; then, the results can be added. You may need to calculate powers of 3 and 5 to see what values the logarithms have.

$$\log_3 243 + \ln(e^{10}) - \log_5 625$$
$$= \log_3(3^5) + \ln(e^{10}) - \log_5(5^4)$$
$$= 5 + 10 - 4 = 11$$

Answer C

exponent

20. $\log 36 - \textcircled{5}\log 4 + \log 20 \quad = \quad \log\left(\frac{36 \cdot 20}{4^5}\right) \quad = \quad \log\left(\frac{720}{1{,}024}\right) \quad = \quad \log\left(\frac{45}{64}\right)$

"−" indicates term goes in denominator

"+" indicates term goes in numerator

Answer C

exponents

21. $\textcircled{3} \cdot \ln a + \textcircled{2} \cdot \ln b - \textcircled{4} \cdot \ln c \quad = \quad \ln\left(\frac{a^3\, b^2}{c^4}\right)$

"+" indicates term goes In numerator

"−" indicates term goes in denominator

Answer A

22. When the base of the exponentiation and the base of the logarithm are the same, they cancel each other out.

Answer C

$$8^{\log_8 5} \; = \; 5$$

23. To solve exponential equations, take a logarithm of each side. The problem is solved below twice, once using *ln* and once using *log*. The answers are the same. You can use either.

Original equation:	$11^x \quad = \quad 247$	$11^x \quad = \quad 247$
Take ln or log of each side:	$x \cdot \ln 11 = \ln 247$	$x \cdot \log 11 = \log 247$
Divide by the multiplier of x:	$x = \frac{\ln 247}{\ln 11}$	$x = \frac{\log 247}{\log 11}$
Calculate:	$x = \frac{5.51}{2.40} = 2.30$	$x = \frac{2.39}{1.04} = 2.30$

Answer B

24. This problem can be solved in two different ways, and we will look at both.

Method 1: Exponents Only Method

Original problem: $4^{5x} = 64^{x+8}$

Convert 64 to 4^3: $4^{5x} = (4^3)^{x+8}$

Multiply exponents: $4^{5x} = 4^{3x+24}$

Exponents must be equal: $5x = 3x + 24$

Subtract $3x$ from each side: $-3x \quad -3x$

$2x = 24$

Divide each side by 2: $\div 2 \quad \div 2$

$x = 12$

Answer D

Method 2: Logarithm Method

Original problem: $4^{5x} = 64^{x+8}$

Take $\log_4$ of each side: $5x = (x+8) \cdot \log_4 64$

$\log_4 64 = 3$: $5x = (x+8) \cdot 3$

Multiply: $5x = 3x + 24$

Subtract $3x$ from each side: $-3x \quad -3x$

$2x = 24$

Divide each side by 2: $\div 2 \quad \div 2$

$x = 12$

25. Condense the expression on the right, then set the arguments equal.

Original equation: $\log_7(2x+9) = \log_7 x + \log_7(x+10)$

Condense expression on right: $\log_7(2x+9) = \log_7[x(x+10)]$

Arguments must be equal: $2x + 9 = x^2 + 10x$

Add $-2x - 9$ to each side: $-2x - 9 = \quad - \ 2x - 9$

$0 = x^2 + 8x - 9$

Factor: $0 = (x+9)(x-1)$

Interim Solutions: $x = \{-9, 1\}$

Test Solutions: -9 does not work because $\log_7(-9)$ does not exist.

1 works ✓ $\log_7(2 \cdot 1 + 9) = \log_7(1) + \log_7(1+10)$

Final Solution: $x = 1$

Answer C

26. Condense the expression on the left, then exponentiate both sides.

Original equation: $\log_8(x-12)+\log_8 x \quad = 2$

Condense expression on left: $\log_8[x(x-12)] \quad = \quad 2$

Take 8 to the power of each side: $x(x-12) \quad = \quad 8^2$

Multiply: $x^2-12x \quad = \quad 64$

Subtract 64 from each side: $-64 \quad -64$

$x^2-12x-64 \ = \quad 0$

Factor: $(x+4)(x-16) = \quad 0$

Interim Solutions: $x=\{-4,16\}$

Test Solutions: -4 does not work because $\log_8(-4)$ does not exist.

16 works ✓ $\log_8(16-12)+\log_8 16 \quad = 2$

Final Solution: $x=16$ **Answer C**

27. Values of the variables:

- Compounding is quarterly, so $n=4$
- $t=7$ years
- $r=7\%=.07$
- $P=\$7{,}500$

$$A=\$7{,}500\cdot\left(1+\frac{.07}{4}\right)^{4\cdot 7}=\$12{,}190.60$$

Answer B

28. Values of the variables:

- Compounding is annual, so $n=1$
- $A=\$8{,}750$
- $r=5\%=.05$
- $P=\$5{,}000$

Starting Equation: $\$8{,}750=\$5{,}000\cdot(1+.05)^t$

Divide both sides by \$5,000: $\div\,\$5{,}000 \quad \div\,\$5{,}000$

$1.75 \quad = \quad (1+.05)^t$

Take a log of both sides: $0.243 \quad = \quad t\cdot\log 1.05$

Calculate $\log 1.05=0.0211$: $0.243 \quad = \quad t\cdot 0.0211$

Divide by 0.0211: $\div\,0.0211 \quad \div\,0.0211$

$11.47 \quad = \quad t$

Answer B

29. Values of the variables:

- Compounding is continuous
- $r = 5\% = .05$
- $t = 5$ years
- $P = \$5{,}000$

$$A = \$5{,}000 \cdot e^{(.05)\cdot 5} = \$5{,}000 \cdot e^{0.25} = \$6{,}420.13$$

Answer B

30. Values of the variables:

- Compounding is continuous
- $r = 7\% = .07$
- $A = \$7{,}700$
- $P = \$7{,}000$

Starting Equation: $\$7{,}700 = \$7{,}000 \cdot e^{(.07)\cdot t}$

Divide both sides by \$7,000: $\div \$7{,}000 \quad \div \$7{,}000$

$$\underline{\hphantom{xxxxxxxxxxxxxxxx}}$$

$$1.10 = e^{(.07)\cdot t}$$

Take the natural log of both sides: $0.0953 = .07\,t$

Divide by $.07$: $\div .07 \quad \div .07$

$$\underline{\hphantom{xxxxxxxxxx}}$$

$$1.36 = t$$

Answer C

31. To get values omitted from the domain and range, consider the asymptotes.

A vertical asymptote exists at the value of x where the denominator is zero.

Want denominator not equal to zero: $2x - 8 \neq 0$

Add 8 to each side: $+8 \quad +8$

$$\underline{\hphantom{xxxxxxxx}}$$

$$2x \neq 8$$

Divide by 2: $\div 2 \quad \div 2$

$$\underline{\hphantom{xxxxxxxx}}$$

$\boldsymbol{x \neq 4}$ **is the domain**

A horizontal asymptote exists at the value that y approaches as $x \to \infty$. Since the numerator and denominator have the same degree, the value of y that we seek is the ratio of the lead coefficients of the expressions in the numerator and denominator.

Since: $y = \dfrac{\mathbf{1}x-6}{\mathbf{2}x-8}$ $\qquad$ $\boldsymbol{y \neq \frac{1}{2}}$ **is the range**

Answer A

32. A horizontal asymptote exists at the value that y approaches as $x \to \infty$. Since the numerator and denominator have the same degree, the value of y that we seek is the ratio of the lead coefficients of the expressions in the numerator and denominator.

Since: $y = \frac{\mathbf{3}x+4}{\mathbf{1}x-5}$ $\boldsymbol{y \to \frac{3}{1} = 3}$ as $\boldsymbol{x \to \infty}$ and as $\boldsymbol{x \to -\infty}$ **Answer C**

33. Translation converts to changes in the equation as follows:

- Translation $\boldsymbol{h}$ **units left changes** $\boldsymbol{x}$ **to** $\boldsymbol{x + h}$.
- Translation $\boldsymbol{h}$ **units right changes** $\boldsymbol{x}$ **to** $\boldsymbol{x - h}$.
- Translation $\boldsymbol{k}$ **units down adds** $\boldsymbol{-k}$ **as a constant** to the expression.
- Translation $\boldsymbol{k}$ **units up adds** $\boldsymbol{k}$ **as a constant** to the expression.

In this problem, we **change** $\boldsymbol{x}$ **to** $\boldsymbol{x - 1}$ and **add** $\boldsymbol{2}$ **as a constant** to the expression. So,

$$f(x) = \frac{1}{x-1} + 2 = \frac{2(x-1)+1}{x-1} = \frac{2x-1}{x-1}$$

Answer B

34. Identify the graph by finding the asymptotes.

A vertical asymptote exists at the value of x where the denominator is zero.

Want denominator equal to zero: $x + 3 = \quad 0$

Subtract 3 from each side: $\quad -3 \quad -3$

$\boldsymbol{x \quad = -3}$ **is a vertical asymptote**

A horizontal asymptote exists at the value that y approaches as $x \to \infty$. Since the numerator and denominator have the same degree, the value of y that we seek is the ratio of the lead coefficients of the expressions in the numerator and denominator.

Since: $y = \frac{\mathbf{3}x+1}{\mathbf{1}x+3}$ $\boldsymbol{y = \frac{3}{1} = 3}$ **is a horizontal asymptote** **Answer A**

35. Factor first and then simplify. You may look at the answers to help you do your factoring.

$$\frac{x^2-x-30}{2x^2-11x-6} = \frac{(x+5)(x-6)}{(2x+1)(x-6)} = \frac{(x+5)}{(2x+1)}$$

Answer C

36. Regarding the second fraction, “*flip that guy and multiply*.”

$$\frac{x^2-3x-10}{x^2+2x-3} \div \frac{x+5}{x+3} = \frac{x^2-3x-10}{x^2+2x-3} \cdot \frac{x+3}{x+5}$$

$$= \frac{(x-5)(x+2)}{(x-1)\mathbf{(x+3)}} \cdot \frac{\mathbf{(x+3)}}{(x+5)}$$

$$= \frac{(x-5)(x+2)}{(x-1)(x+5)}$$

Answer A

37. One of the easiest problems on the test. Don’t expect one like this on the real final.

$$\frac{7}{x-4} - \frac{11}{x-4} = \frac{7-11}{x-4} = \frac{-4}{x-4}$$

Answer D

38. Get a common denominator and add.

$$\frac{4x+5}{x^2-25} + \frac{7}{x-5} = \frac{4x+5}{(x-5)(x+5)} + \frac{7}{(x-5)}$$

$$= \frac{4x+5}{(x-5)(x+5)} + \frac{7\,\mathbf{(x+5)}}{(x-5)\mathbf{(x+5)}}$$

$$= \frac{(4x+5)+(7x+35)}{(x-5)(x+5)}$$

$$= \frac{11x+40}{x^2-25}$$

Answer A

39. Cross multiply, but remember that denominators cannot be zero, so $x \neq \{5, -6\}$.

$$\frac{x+4}{x-5} = \frac{x-3}{x+6} \qquad \Rightarrow \qquad (x+4)(x+6) = (x-5)(x-3)$$

$$\begin{array}{rcr} x^2 + 10x + 24 & = & x^2 - 8x + 15 \\ -x^2 + \ \ 8x - 24 & = & -x^2 + 8x - 24 \\ \hline 18x & = & -9 \\ \div 18 & & \div 18 \\ \hline \end{array}$$

Add and subtract items so that the result has the x’s on one side and the constants on the other.

$$x = -\tfrac{1}{2}$$

Answer B

40. Get a common denominator and then work with the numerators. Remember that denominators cannot be zero, so $x \neq \{1, 6\}$.

$$\frac{3x}{x-1} + \frac{2x}{x-6} = \frac{5x^2 - 15x + 20}{x^2 - 7x + 6}$$

$$\frac{\mathbf{(x-6)}}{\mathbf{(x-6)}} \cdot \frac{3x}{(x-1)} + \frac{2x}{(x-6)} \cdot \frac{\mathbf{(x-1)}}{\mathbf{(x-1)}} = \frac{5x^2 - 15x + 20}{(x-6)(x-1)}$$

Now, work with the numerators only. The denominator can be discarded.

$$\mathbf{(x-6)} \cdot \mathbf{(3x)} + \mathbf{(2x)} \cdot \mathbf{(x-1)} = 5x^2 - 15x + 20$$

$$\mathbf{(3x^2 - 18x)} + \mathbf{(2x^2 - 2x)} = 5x^2 - 15x + 20$$

$$\begin{array}{rr} 5x^2 - 20x = & 5x^2 - 15x + 20 \\ -\,5x^2 + 15x & -\,5x^2 + 15x \\ \hline -\,5x = & 20 \\ \div(-5) & \div(-5) \\ \hline \end{array}$$

$$x = -4$$

Answer D

41. This inequality generates 3 regions to test, as follows.

$$\frac{\mathbf{x-1}}{\mathbf{x+2}} > \mathbf{0}$$

A regional break is created where the numerator is zero:

$x - 1 = 0 \quad \rightarrow \quad \mathbf{x = 1}$

A regional break is created where the denominator is zero:

$x + 2 = 0 \quad \rightarrow \quad \mathbf{x = -2}$

Test $x = -3$

$$\frac{-3-1}{-3+2}$$

$$= \frac{-4}{-1} = 4 > 0$$

✓

Test $x = 0$

$$\frac{0-1}{0+2}$$

$$= -\frac{1}{2} > 0$$

X

Test $x = 2$

$$\frac{2-1}{2+2}$$

$$= \frac{1}{4} > 0$$

✓

Regions A and B meet the requirements of the inequality, so, $x < -2 \quad \text{or} \quad x > 1$

Answer A

42. This inequality generates 3 regions to test. But first, you must get everything on one side of the inequality.

$$\begin{array}{rcl} \frac{x-1}{x+2} & \le & 5 \\ -5 & & -5 \\ \hline \frac{x-1}{x+2} - 5 & \le & 0 \end{array}$$

$$\frac{x-1}{x+2} - \frac{5(x+2)}{x+2} \le 0$$

$$\frac{x-1-5x-10}{x+2} \le 0$$

$$\frac{x-1-5x-10}{x+2} \le 0$$

$$\frac{-4x-11}{x+2} \le 0$$

A regional break is created where the numerator is zero:

$$-4x - 11 = 0 \quad \rightarrow \quad x = \frac{-11}{4}$$

A regional break is created where the denominator is zero:

$$x + 2 = 0 \quad \rightarrow \quad x = -2$$

At this point you can identify the correct answer without doing the regional work illustrated in problem 41 above. The breakpoints are: $x = \frac{-11}{4}$ and $x = -2$. Note that answers A and B have these breakpoints and the difference between them is whether $x = -2$ is a solution. Note that when $x = -2$ the denominator of the inequality is zero, so $x = -2$ is *not* a solution.

Answer B

43. The distance between the vertex and the focus is 6, so $p = 6$.

Then, since the path from the vertex to the focus is horizontal (in the x-direction), the form of the equation is:

$x = \frac{1}{4p}(y-k)^2 + h$ and with a vertex of $(0, 0)$, this becomes: $x = \frac{1}{4p}y^2$

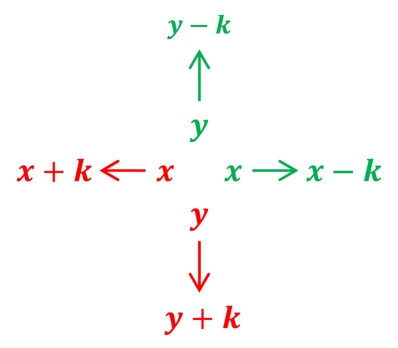
Then, with $p = 6$, this becomes: $x = \frac{1}{24}y^2$ or $24x = y^2$

Answer A

44. Translation converts to changes in the equation as follows:

- Translation h **units left changes** x **to** $x + h$.
- Translation h **units right changes** x **to** $x - h$.
- Translation k **units down changes** y **to** $y + k$.
- Translation k **units up changes** y **to** $y - k$.

Translation Illustration



In this problem, we **change** x **to** $x + 2$ and y **to** $y - 3$. So,

$$y^2 = \frac{1}{5}x \quad \Rightarrow \quad (y-3)^2 = \frac{1}{5}(x+2)$$

Answer B

45. Since the coefficients of the square terms are the same, this is the equation of a circle.

$$\begin{array}{ll}
\text{Starting equation:} & 3x^2+3y^2+24y+18x-27 = \quad 0 \\
\text{Divide by 3 to simplify:} & \qquad\quad \div 3 \qquad\qquad\qquad\quad \div 3 \\
\hline
 & x^2+y^2+8y+6x-9 = \quad 0 \\
\text{Add 9 to each side:} & \qquad\qquad\qquad\quad +9 \quad +9 \\
\hline
 & x^2+y^2+8y+6x \quad = \quad 9 \\
\text{Rearrange terms:} & \mathbf{(x^2+6x+\_\_)}+\mathbf{(y^2+8y+\_\_)}=9 \\
\text{Complete the squares:} & \mathbf{(x^2+6x+9)}+\mathbf{(y^2+8y+16)}=9\mathbf{+9+16} \\
\text{Simplify:} & \mathbf{(x+3)^2} \quad + \quad \mathbf{(y+4)^2} \quad = 34
\end{array}$$

Answer A

46. Translation converts to changes in the equation as follows:

- Translation $\boldsymbol{h}$ **units left changes** $\boldsymbol{x}$ **to** $\boldsymbol{x+h}$.
- Translation $\boldsymbol{h}$ **units right changes** $\boldsymbol{x}$ **to** $\boldsymbol{x-h}$.
- Translation $\boldsymbol{k}$ **units down changes** $\boldsymbol{y}$ **to** $\boldsymbol{y+k}$.
- Translation $\boldsymbol{k}$ **units up changes** $\boldsymbol{y}$ **to** $\boldsymbol{y-k}$.

In this problem, we **change** $\boldsymbol{x}$ **to** $\boldsymbol{x-2}$ and $\boldsymbol{y}$ **to** $\boldsymbol{y+6}$. So,

$$x^2+y^2=8 \qquad \Rightarrow \qquad (x-2)^2+(y+6)^2=8$$

Answer B

47. Conditional probability. Use numbers for only the populations identified.

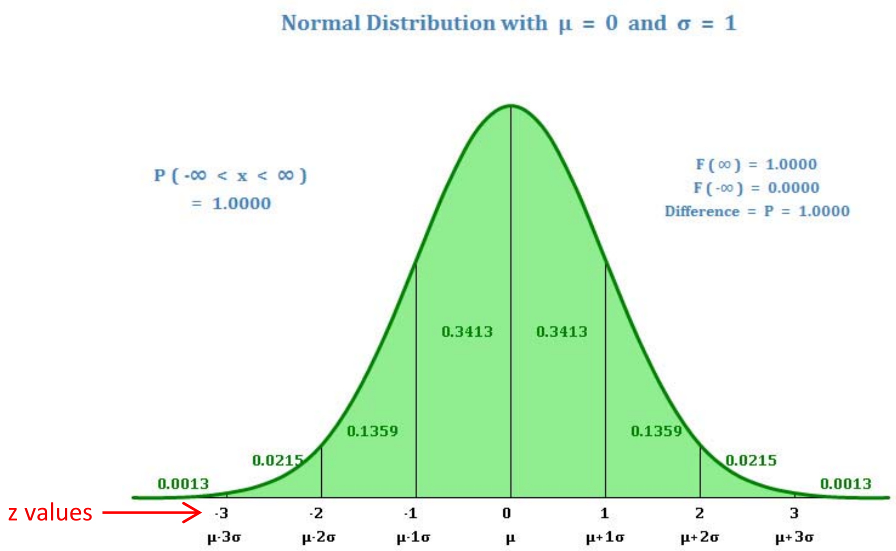
$$P \;=\; \frac{females\ with\ part\ time\ jobs}{all\ people\ with\ part\ time\ jobs} \;=\; \frac{225}{225+200} \;=\; \frac{225}{425} \;=\; 0.53$$

Answer B

48. 10 tails out of 30 coin tosses:

$$P \;=\; {}_{30}C_{10}\cdot \underset{\nearrow \atop \textbf{tails}}{\left(\frac{1}{2}\right)^{10}} \cdot \underset{\nwarrow \atop \textbf{heads}}{\left(\frac{1}{2}\right)^{20}} \;=\; {}_{30}C_{10}\cdot\left(\frac{1}{2}\right)^{30} \;=\; \frac{30!}{10!\cdot 20!\cdot 2^{30}} = 0.028$$

Answer D



49. Calculate the z-statistic and locate it on the above graph.

$$z = \frac{x-\mu}{\sigma} = \frac{59-64}{2.5} = -2$$

We want the probability to the left of $z = -2$ because we want women with heights below (i.e., shorter than) 59 inches.

The cumulative probability to the left of $z = -2$ is $0.0013 + 0.0215 = 0.0228$. The closest answer to this on the sample test is 0.025.

Answer B

50. Calculate the z-statistics and locate them on the above graph.

$$z_1 = \frac{x-\mu}{\sigma} = \frac{67-70}{3} = -1 \qquad z_2 = \frac{x-\mu}{\sigma} = \frac{76-70}{3} = 2$$

We want the probability between $z = -1$ and $z = 2$ because we want men with heights between 67 and 76 inches.

The probability between $z = -1$ and $z = 2$ is roughly $0.34 + .034 + 0.14 = 0.82$.

Answer D