1) $3x^2 - x - 4$

Let's start by rewriting the equation with the implied 1 for the second term written out.

$$3x^2 - 1x - 4$$

Multiplying the lead coefficient and the constant we get: $3(-4) = -12$

Next, we need numbers that multiply to -12 and add to -1. Look at the box to the right. We try different combinations of values until we get one that works. We will use the box method, since the lead coefficient of the expression is not 1.

$\_\_ \cdot \_\_ = -12$

$\_\_ + \_\_ = -1$

Values are: $3, -4$

Step 1: Load our expression terms into the box.

Step 2: Find the greatest common factor (GCF) of each row.

Step 3: Divide the elements of each row by the GCF for the row (use either row to do this).

Step 1

$3x^2$	$+3x$
$-4x$	-4

Step 2

$3x$	$3x^2$	$+3x$
-4	$-4x$	-4

Step 3

	x	$+1$
$3x$	$3x^2$	$+3x$
-4	$-4x$	-4

Finally, pull the components from the box headers into a solution for the problem:

$$3x^2 - x - 4 = (3x - 4)(x + 1)$$

Looking over the possible answers to this problem, we notice **Answer B** is $(\boldsymbol{x+1})$, which is one of our factors.

Answer B

2) $x^2 + 11x + 24$

We will break our trinomial into two binomials.

$$x^2 + 11x + 24 = (x \quad\ )(x \quad\ )$$

We need numbers that multiply to 24 and add to 11. Look at the box to the right. We try different combinations of values until we get one that works.

$\_\_ \cdot \_\_ = 24$

$\_\_ + \_\_ = 11$

Values are: $3, 8$

Since we have only three terms and the lead coefficient is 1, we do not need to use the Box Method. We simply insert the values we found into the binomials in the solution.

$$x^2 + 11x + 24 = (x + 8)(x + 3)$$

Looking over the possible answers to this problem, we notice **Answer C** is $(\boldsymbol{x+8})$, which is one of our factors.

Answer C

3) I like to add vertically. But first, we need to rearrange the terms of the second expression so that the terms go from highest exponent to lowest exponents of x.

When working vertically, arrange like terms in columns:

$$\begin{array}{r} 5x^2 + 1x + 1 \\ 6x^3 \qquad\quad + 7x + 5 \\ \hline \mathbf{6x^3 + 5x^2 + 8x + 6} \end{array}$$

Melissa added unlike terms; it appears that she added terms in the order they were given, ignoring the exponents of x in the second expression.

4) A binomial has two terms. The leading coefficient (3) is the number in front of the variable. A degree of 5 means we need to have the power of the variable in the lead term be 5.

I will make my binomial: $\mathbf{3x^5 + 1}$ (others: $3z^5 + 147x^3,\ 3a^5 - 4y^3,$ etc.)

Note: Your second term could be either a constant or a term with a variable whose exponent is less than or equal to 5.

In problems 5 and 6, remember to re-order your terms and set up your columns properly.

5)
$$\begin{array}{r} 2a^2 \qquad + \ 4b^2 \qquad - 3 \\ 3a + \ 6b^2 + 4b \qquad \\ \hline \mathbf{2a^2 + 3a + 10b^2 + 4b - 3} \end{array}$$

6)
$$\begin{array}{r} 5x^6 + 5x^4 - 5 \\ 9x^6 - 3x^4 + 7 \\ \hline \mathbf{14x^6 + 2x^4 + 2} \end{array}$$

In problems 7 and 8, I change things so I do not have to subtract negatives, which I am not a fan of. To use this method, change any "−" in front of a parenthetical expression to a "+" and change all of the signs inside the parentheses. Then, add.

7)
$$\begin{array}{r} 3z^3 + 2z^2 + \ 7 \\ -2z^3 + 3z^2 + \ 6 \\ \hline \mathbf{z^3 + 5z^2 + 13} \end{array}$$

8)
$$\begin{array}{r} 2x^2 - 7x \qquad \\ +4x^2 \qquad - \ 8 \\ +5x^3 \qquad\qquad\qquad - \ 4 \\ \hline \mathbf{5x^3 + 6x^2 - 7x - 12} \end{array}$$

For Problems 9 and 10, use the distributive property of multiplication over addition.

9) $20x(3 - 2x) = 20x(3) + 20x(-2x) = 60x - 40x^2 = \mathbf{-40x^2 + 60x}$

10) $3x^2(2x^2 - 5x - 3) = 3x^2(2x^2) + 3x^2(-5x) + 3x^2(-3) = \mathbf{6x^4 - 15x^3 - 9x^2}$

For problems 11-14, use either the box method or the FOIL method. I will use FOIL.

11) $(x+3)(x+4) =$

$F:\ x \cdot x = x^2$
$O:\ +x \cdot 4 = +4x$
$I:\ +3 \cdot x = +3x$
$L:\ +3 \cdot 4 = +12$ $\quad = \mathbf{x^2 + 7x + 12}$

12) $(x+9)(x-9) =$

$F:\ x \cdot x = x^2$
$O:\ +x \cdot (-9) = -9x$
$I:\ +9 \cdot x = +9x$
$L:\ +9 \cdot (-9) = -81$ $\quad = \mathbf{x^2 - 81}$

13) $(x+7)(x+7) =$

$F:\ x \cdot x = x^2$
$O:\ +x \cdot 7 = +7x$
$I:\ +7 \cdot x = +7x$
$L:\ +7 \cdot 7 = +49$ $\quad = \mathbf{x^2 + 14x + 49}$

14) $(2x+4)(2x-4) =$

$F:\ 2x \cdot 2x = 4x^2$
$O:\ +2x \cdot (-4) = -8x$
$I:\ +4 \cdot 2x = +8x$
$L:\ +4 \cdot (-4) = -16$ $\quad = \mathbf{4x^2 - 16}$

For Problems 15 to 26, factor using the easiest method that works well. First factor out a GCF, then (if necessary) use either the two-binomial method or the box method.

15) $7x^2 - 35x$

Factor out the GCF ($7x$) and this one is done.

$7x^2 - 35x = \mathbf{7x(x-5)}$

16) With 4 terms, we will factor out the GCF (-1), then use the Box Method.

$$-x^3 - 5x^2 + 8x + 40 = -(x^3 + 5x^2 - 8x - 40)$$

Step 1: Load our expression terms ($x^3 + 5x^2 - 8x - 40$) into the box.

Step 2: Find the greatest common factor (GCF) of each row.

Step 3: Divide the elements of each row by the GCF for the row (use either row to do this).

Step 1

x^3	$+5x^2$
$-8x$	-40

Step 2

x^2	x^3	$+5x^2$
-8	$-8x$	-40

Step 3

	x	$+5$
x^2	x^3	$+5x^2$
-8	$-8x$	-40

Finally, pull the components from the box headers into a solution for the problem:

$$-x^3 - 5x^2 + 8x + 40 = -(x^3 + 5x^2 - 8x - 40) = -(\boldsymbol{x^2 - 8})(\boldsymbol{x + 5})$$

17) With 4 terms, we will factor out the GCF (2), then use the Box Method.

$$2x^3 + 4x^2 - 6x - 12 = 2(x^3 + 2x^2 - 3x - 6)$$

Step 1: Load our expression terms ($x^3 + 2x^2 - 3x - 6$) into the box.

Step 2: Find the greatest common factor (GCF) of each row.

Step 3: Divide the elements of each row by the GCF for the row (use either row to do this).

Step 1

x^3	$+2x^2$
$-3x$	-6

Step 2

x^2	x^3	$+2x^2$
-3	$-3x$	-6

Step 3

	x	$+2$
x^2	x^3	$+2x^2$
-3	$-3x$	-6

Finally, pull the components from the box headers into a solution for the problem:

$$2x^3 + 4x^2 - 6x - 12 = 2(x^3 + 2x^2 - 3x - 6) = \boldsymbol{2(x^2 - 3)(x + 2)}$$

18) $x^2 - x - 12$

We will break our trinomial into two binomials.

$$x^2 - 1x - 12 = (x \quad\;)(x \quad\;)$$

We need numbers that multiply to -12 and add to -1. Look at the box to the right. We try different combinations of values until we get one that works.

$\_\_ \cdot \_\_ = -12$
$\_\_ + \_\_ = -1$
Values are: $3, -4$

Since we have only three terms and the lead coefficient is 1, we do not need to use the Box Method. We simply insert the values we found into the binomials in the solution.

$$x^2 - x - 12 = (\boldsymbol{x + 3})(\boldsymbol{x - 4})$$

Squares
$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$
$13^2 = 169$
$14^2 = 196$
$15^2 = 225$

19) $x^2 - 144$

This form is a difference of squares because: $144 = 12^2$.

The formula for a difference of squares is: $\boldsymbol{a^2 - b^2 = (a - b)(a + b)}$

We notice that the two items squared in our problem are x and 12. Therefore,

$$x^2 - 144 = (\boldsymbol{x - 12})(\boldsymbol{x + 12})$$

20) $x^2 - 64y^{10}$

This form is a difference of squares because: $64y^{10} = (8y^5)^2$.

The formula for a difference of squares is: $\boldsymbol{a^2 - b^2 = (a - b)(a + b)}$

We notice that the two items squared in our problem are x and $8y^5$. Therefore,

$$x^2 - 64y^{10} = (\boldsymbol{x - 8y^5})(\boldsymbol{x + 8y^5})$$

21) $2x^2 + 16x + 32$ First, we need to factor out the GCF (2).

$$2x^2 + 16x + 32 = 2(x^2 + 8x + 16)$$

We will break our trinomial into two binomials.

$$2(x^2 + 8x + 16) = 2(x \quad)(x \quad)$$

$\_\_ \cdot \_\_ = 16$
$\_\_ + \_\_ = 8$
Values are: $4, 4$

We need numbers that multiply to 16 and add to 8. Look at the box to the right. We try different combinations of values until we get one that works.

Since we have only three terms and the lead coefficient is 1, we do not need to use the Box Method. We simply insert the values we found into the binomials in the solution.

$$2x^2 + 16x + 32 = 2(x^2 + 8x + 16) = 2(x + 4)(x + 4) = \boldsymbol{2(x + 4)^2}$$

22) $-x^2 - 7x + 8$ First, we need to factor out the GCF (-1).

$$-x^2 - 7x + 8 = -(x^2 + 7x - 8)$$

We will break our trinomial into two binomials.

$$-(x^2 + 7x - 8) = 2(x \quad)(x \quad)$$

$\_\_ \cdot \_\_ = -8$
$\_\_ + \_\_ = 7$
Values are: $8, -1$

We need numbers that multiply to -8 and add to 7. Look at the box to the right. We try different combinations of values until we get one that works.

Since we have only three terms and the lead coefficient is 1, we do not need to use the Box Method. We simply insert the values we found into the binomials in the solution.

$$-x^2 - 7x + 8 = -(x^2 + 7x - 8) = -(\boldsymbol{x + 8})(\boldsymbol{x - 1})$$

23) $4x^2 - 3x - 10$ With a lead coefficient of 4, we will use the Box Method.

$4x^2 - 3x - 10$ Multiplying the lead coefficient and the constant we get: $4(-10) = -40$

Next, we need numbers that multiply to -40 and add to -3. Look at the box to the right. We try different combinations of values until we get one that works.

$\_\_ \cdot \_\_ = -40$

$\_\_ + \_\_ = -3$

Values are: -8, 5

Step 1: Load our expression terms into the box.

Step 2: Find the greatest common factor (GCF) of each row.

Step 3: Divide the elements of each row by the GCF for the row (use either row to do this).

Step 1

$4x^2$	$-8x$
$5x$	-10

Step 2

$4x$	$4x^2$	$-8x$
$+5$	$5x$	-10

Step 3

	x	-2
$4x$	$4x^2$	$-8x$
$+5$	$5x$	-10

Finally, pull the components from the box headers into a solution for the problem:

$$4x^2 - 3x - 10 = (\mathbf{4x+5})(\mathbf{x-2})$$

24) $x^2 + 12xy + 36y^2$

$\_\_ \cdot \_\_ = 36$

$\_\_ + \_\_ = 12$

Values are: 6, 6

We will break our trinomial into two binomials.

$$x^2 + 12xy + 36y^2 = (x \quad y)(x \quad y)$$

We need numbers that multiply to 36 and add to 12. Look at the box to the right. We try different combinations of values until we get one that works.

Since we have only three terms and the lead coefficient is 1, we do not need to use the Box Method. We simply insert the values we found into the binomials in the solution.

$$x^2 + 12xy + 36y^2 = (x+6y)(x+6y) = (\mathbf{x+6y})^2$$

25) $y^3 + 12y^2 - 28y$ First, we need to factor out the GCF (y).

$\_\_ \cdot \_\_ = -28$

$\_\_ + \_\_ = 12$

Values are: $-2, 14$

$$y^3 + 12y^2 - 28y = y(y^2 + 12y - 28)$$

We will break our trinomial into two binomials.

$$y(y^2 + 12y - 28) = y(y \quad)(y \quad)$$

We need numbers that multiply to -28 and add to 12. Look at the box to the right. We try different combinations of values until we get one that works.

Since we have only three terms and the lead coefficient is 1, we do not need to use the Box Method. We simply insert the values we found into the binomials in the solution.

$$y^3 + 12y^2 - 28y = y(y^2 + 12y - 28) = \mathbf{y(y-2)(y+14)}$$

26) $6a^2 - 24$ First, we need to factor out the GCF (6).

$$6a^2 - 24 = 6(a^2 - 4)$$

The resulting binomial is a difference of squares because: $4 = 2^2$.

The formula for a difference of squares is: $\boldsymbol{a^2 - b^2} = (\boldsymbol{a} - \boldsymbol{b})(\boldsymbol{a} + \boldsymbol{b})$

We notice that the two items squared in our problem are a and 2. Therefore,

$$6a^2 - 24 = 6(a^2 - 4) = \mathbf{6}(\boldsymbol{a} - \mathbf{2})(\boldsymbol{a} + \mathbf{2})$$

Problems 27 to 34 require us to factor first and then solve the factored equation.

27) $x^2 + 11x = -30$ First, we need to get all of the terms on one side:

$$\begin{aligned} x^2 + 11x \quad &= -30 \\ +30 \quad &+ 30 \\ \hline x^2 + 11x + 30 &= \quad 0 \end{aligned}$$

$\_\_ \cdot \_\_ = 30$

$\_\_ + \_\_ = 11$

Values are: $5, 6$

We will break our trinomial into two binomials.

$$x^2 + 11x + 30 = (x \qquad)(x \qquad)$$

We need numbers that multiply to 30 and add to 11. Look at the box to the right. We try different combinations of values until we get one that works.

Since we have only three terms and the lead coefficient is 1, we do not need to use the Box Method. We simply insert the values we found into the binomials in the solution.

$$x^2 + 11x + 30 = 0$$

$$(x + 5)(x + 6) = 0$$

$$\boldsymbol{x} = \{\mathbf{-5}, \mathbf{-6}\}$$

When the factored form is this simple, you can simply change the signs of the numbers in the factors to get the solutions for x.

If you do not like the shortcut, here is the long way to get the solutions. You should use whatever works for you. The goals are accuracy first, then speed.

Recall that when the product of two values is zero, one of the values must be zero. Then,

$$(x + 5)(x + 6) = 0$$

$$\begin{aligned} x + 5 &= \quad 0 \\ -5 &\quad - 5 \\ \hline \boldsymbol{x} \quad &= \mathbf{-5} \end{aligned} \qquad\qquad \begin{aligned} x + 6 &= \quad 0 \\ -6 &\quad - 6 \\ \hline \boldsymbol{x} \quad &= \mathbf{-6} \end{aligned}$$

So, $\boldsymbol{x} = \{\mathbf{-5}, \mathbf{-6}\}$

28) $4x^3 + 14x^2 + 12x = 0$ First, we need to factor out the GCF $(2x)$.

$$4x^3 + 14x^2 + 12x = 0$$

$$2x(2x^2 + 7x + 6) = 0$$

With a lead coefficient of 2, we will use the Box Method.

$$2x(2x^2 + 7x + 6) = 0$$ Multiplying the lead coefficient and the constant we get: $2(6) = 12$

Next, we need numbers that multiply to 12 and add to 7. Look at the box to the right. We try different combinations of values until we get one that works.

$\_\_ \cdot \_\_ = 12$
$\_\_ + \_\_ = 7$
Values are: $3, 4$

Step 1: Load our expression terms into the box.

Step 2: Find the greatest common factor (GCF) of each row.

Step 3: Divide the elements of each row by the GCF for the row (use either row to do this).

Step 1

$2x^2$	$3x$
$4x$	6

Step 2

x	$2x^2$	$3x$
$+2$	$4x$	6

Step 3

	$2x$	$+3$
x	$2x^2$	$3x$
$+2$	$4x$	6

Finally, pull the components from the box headers into a solution for the problem:

$$2x(2x^2 + 7x + 6) = 2x(x + 2)(2x + 3) = 0$$

$$\boldsymbol{x = 0}$$

$$\begin{aligned} x + 2 &= 0 \\ -2 \quad & -2 \\ \boldsymbol{x} &= \boldsymbol{-2} \end{aligned}$$

$$\begin{aligned} 2x + 3 &= 0 \\ -3 \quad & -3 \\ 2x &= -3 \\ \div 2 \quad & \div 2 \\ \boldsymbol{x} &= -\frac{\boldsymbol{3}}{\boldsymbol{2}} \end{aligned}$$

So, $\boldsymbol{x = \left\{0, -2, -\frac{3}{2}\right\}}$

If you like shortcuts, here's a good one: To get the solution for x for each factor, change the sign of the constant in the factor and divide by the lead coefficient of the factor. For example, if the factored form of your equation were:

$$(2x + 3)\ (3x - 5)\ (4x + 1)\ (2x - 9)\ (5x + 2) = 0$$ Then, your solutions would be:

$$-\frac{3}{2} \qquad \frac{5}{3} \qquad -\frac{1}{4} \qquad \frac{9}{2} \qquad -\frac{2}{5}$$

29) $6x^2 - 42x = 0$ First, we need to factor out the GCF $(6x)$.

$6x^2 - 42x = 6x(x - 7) = 0$

$\boldsymbol{x = \{0,\ 7\}}$ by changing the signs in the factors (note when x is a factor, the solution is $x = 0$)

30) $4x^2 - 29x + 7 = 0$

With a lead coefficient of 4, we will use the Box Method.

$(4x^2 - 29x + 7) = 0$ Multiplying the lead coefficient and the constant we get: $4(7) = 28$

Next, we need numbers that multiply to 28 and add to -29. Look at the box to the right. We try different combinations of values until we get one that works.

$\_\_ \cdot \_\_ = 28$

$\_\_ + \_\_ = -29$

Values are: $-1, -28$

Step 1: Load our expression terms into the box.

Step 2: Find the greatest common factor (GCF) of each row.

Step 3: Divide the elements of each row by the GCF for the row (use either row to do this).

Step 1

$4x^2$	$-1x$
$-28x$	7

Step 2

x	$4x^2$	$-1x$
-7	$-28x$	7

Step 3

	$4x$	-1
x	$4x^2$	$-1x$
-7	$-28x$	7

Finally, pull the components from the box headers into a solution for the problem:

$4x^2 - 29x + 7 = (x - 7)(4x - 1) = 0$

Using the shortcut from problem 28, we get: $\boldsymbol{x = \left\{7,\ \frac{1}{4}\right\}}$

31) $x^2 + 8x + 16 = 0$

We will break our trinomial into two binomials.

$$x^2 + 8x + 16 = (x \quad\quad)(x \quad\quad) = 0$$

We need numbers that multiply to 16 and add to 8. Look at the box to the right. We try different combinations of values until we get one that works.

$\_\_ \cdot \_\_ = 16$
$\_\_ + \_\_ = 8$
Values are: $4, 4$

Since we have only three terms and the lead coefficient is 1, we do not need to use the Box Method. We simply insert the values we found into the binomials in the solution.

$$x^2 + 8x + 16 = 0$$

$$(x + 4)(x + 4) = (x + 4)^2 = 0$$

$\boldsymbol{x = \{-4\}}$ by changing the sign in the factor

32) $9x^2 = 25$ First, we need to get all of the terms on one side:

$$\begin{array}{rcl} 9x^2 & = & 25 \\ -25 & & -25 \\ \hline 9x^2 - 25 & = & 0 \end{array}$$

This form is a difference of squares because: $9x^2 = (3x)^2$ and $25 = 5^2$.

The formula for a difference of squares is: $\boldsymbol{a^2 - b^2 = (a - b)(a + b)}$

We notice that the two items squared in our problem are $3x$ and 5. Therefore,

$$9x^2 - 25 = (3x - 5)(3x + 5) = 0$$

Using the shortcut from problem 28, we get: $\boldsymbol{x = \left\{\frac{5}{3}, -\frac{5}{3}\right\}}$

33) $x^2 - 20 = x$ First, we need to get all of the terms on one side:

$$\begin{array}{rcl} x^2 \quad - 20 &=& x \\ -x \qquad &&-x \\ \hline x^2 - 1x - 20 &=& 0 \end{array}$$

$\_\_ \cdot \_\_ = -20$

$\_\_ + \_\_ = -1$

Values are: $-5, 4$

We will break our trinomial into two binomials.

$x^2 - 1x - 20 = (x \quad)(x \quad) = 0$

We need numbers that multiply to -20 and add to -1. Look at the box to the right. We try different combinations of values until we get one that works.

Since we have only three terms and the lead coefficient is 1, we do not need to use the Box Method. We simply insert the values we found into the binomials in the solution.

$x^2 - x - 20 = 0$

$(x - 5)(x + 4) = 0$

$\boldsymbol{x = \{5, -4\}}$ by changing the signs in the factors

34) $3x^2 + 2x - 8 = 0$

With a lead coefficient of 3, we will use the Box Method.

$3x^2 + 2x - 8 = 0$ Multiplying the lead coefficient and the constant we get: $3(-8) = -24$

Next, we need numbers that multiply to -24 and add to 2. Look at the box to the right. We try different combinations of values until we get one that works.

$\_\_ \cdot \_\_ = -24$

$\_\_ + \_\_ = 2$

Values are: $6, -4$

Step 1: Load our expression terms into the box.

Step 2: Find the greatest common factor (GCF) of each row.

Step 3: Divide the elements of each row by the GCF for the row (use either row to do this).

Step 1

$3x^2$	$6x$
$-4x$	-8

Step 2

$3x$	$3x^2$	$6x$
-4	$-4x$	-8

Step 3

	x	$+2$
$3x$	$3x^2$	$6x$
-4	$-4x$	-8

Finally, pull the components from the box headers into a solution for the problem:

$3x^2 + 2x - 8 = (3x - 4)(x + 2) = 0$

Using the shortcut from problem 28, we get: $\boldsymbol{x = \left\{\frac{4}{3}, -2\right\}}$

Problems 35 and 36 require factoring only.

35) $x^2 + 11x + 30 = 0$

We will break our trinomial into two binomials.

$$x^2 + 11x + 30 = (x \quad)(x \quad)$$

We need numbers that multiply to 30 and add to 11. Look at the box to the right. We try different combinations of values until we get one that works.

$\_\_ \cdot \_\_ = 30$
$\_\_ + \_\_ = 11$
Values are: 5, 6

Since we have only three terms and the lead coefficient is 1, we do not need to use the Box Method. We simply insert the values we found into the binomials in the solution.

$$x^2 + 11x + 30 = (x + 5)(x + 6)$$

The missing factor, then is: $\boldsymbol{(x + 5)}$ **Answer D**

36) With 4 terms, we will factor out the GCF (-2), then use the Box Method.

$$-2x^3 + 6x^2 - 10x + 30 = -2(x^3 - 3x^2 + 5x - 15)$$

Step 1: Load our expression terms $(x^3 - 3x^2 + 5x - 15)$ into the box.

Step 2: Find the greatest common factor (GCF) of each row.

Step 3: Divide the elements of each row by the GCF for the row (use either row to do this).

Step 1

x^3	$-3x^2$
$5x$	-15

Step 2

x^2	x^3	$-3x^2$
$+5$	$5x$	-15

Step 3

	x	-3
x^2	x^3	$-3x^2$
$+5$	$5x$	-15

Finally, pull the components from the box headers into a solution for the problem:

$$-2x^3 + 6x^2 - 10x + 30 = -2(x^3 - 3x^2 + 5x - 15) = \boldsymbol{-2(x^2 + 5)(x - 3)}$$