Algebra 1: Chapter 8 Practice Test
"Unofficial" Worked-Out Solutions by Earl Whitney

1-14 Simplify each of the following expressions. Be sure that all exponents are positive.

1. $\boldsymbol{a^9 \cdot a^5 = a^{9+5} = a^{14}}$

Rule: When **multiplying** two terms with the **same base** ($\boldsymbol{a}$ in this problem) and **different exponents**, ***ADD the exponents***.

2. $\boldsymbol{(-10)(-10)^7 = (-10)^1(-10)^7 = (-10)^{1+7} = (-10)^8}$ *(not done yet; see below)*

Rule: When **multiplying** two terms with the **same base** ($\boldsymbol{-10}$ in this problem) and **different exponents**, ***ADD the exponents***.

Remember: if no exponent is shown, then the exponent is 1.

Hint: Consider the sign, any numbers and each variable as separate items. If a "–" sign is inside the parentheses, it gets taken to the power shown; if it is not inside the parentheses, it is applied at the end (see problem #4 below).

Remember: a "–" sign to an odd power gives a "–"
a "–" sign to an even power gives a "+"

Continue:

$$\boldsymbol{(-10)^8}$$
$$= (-)^{\boldsymbol{8}} \cdot (\boldsymbol{10})^{\boldsymbol{8}}$$
$$= + \quad \boldsymbol{100,000,000}$$
$$= \boldsymbol{100,000,000}$$

Note: $\boldsymbol{10^8}$ is a 1, followed by 8 zeros.

3. $\boldsymbol{(-4rs^2)^3 = (-)^3 \cdot (4)^3 \cdot (r)^3 \cdot (s^2)^3}$

$$\boldsymbol{= - \quad 64 \quad r^3 \quad s^6 \quad = -64\, r^3 s^6}$$

Rule: When taking a term to a **power**, ***take every item in the term to that power***.

Rule: When taking a variable that has an **exponent to another power**, ***MULTIPLY the exponents***.

4. $-(7z)^2 = -\ (7)^2 \cdot (z)^2$

$= -\ 49 \quad z^2 \quad = -49\,z^2$

Rule: When taking a term to a **power**, ***take every item in the term to that power***.

5. $(w^3)^2 = w^{3 \cdot 2} = w^6$

Rule: When taking a variable that has an **exponent to another power**, ***MULTIPLY the exponents***.

6. $x^{-4} = \frac{x^{-4}}{1} = \frac{1}{x^4}$ *The circled guy is moved across the fraction line.*

Rule: **A negative exponent** tells you to ***move the variable to the other side of the fraction line and make the exponent positive***.

Hint: If you don't have a fraction, make one like in the above solution.

7. $(2y^5)^2 = (2)^2 \cdot (y^5)^2$

$= \ 4 \ \cdot \ y^{5 \cdot 2} \ = 4\,y^{10}$

Rule: When taking a term to a **power**, ***take every item in the term to that power***.

Rule: When taking a variable that has an **exponent to another power**, ***MULTIPLY the exponents***.

8. $(3x^3)^2 \cdot x^4 = [\ (3^2) \cdot (x^3)^2\ ] \cdot x^4$

$= [\ 9 \cdot x^{3 \cdot 2}\ ] \cdot x^4$

$= [\ 9 \cdot x^6\ ] \cdot x^4$

$= \ 9\,x^6 \cdot x^4 \ = \ 9\,x^{6+4} \ = \ 9\,x^{10}$

Same rules as in #7.

Hint: Follow the rules of **PEMDAS**.

Parentheses, **E**xponents, **M**ultiplication and **D**ivision, **A**ddition and **S**ubtraction.

So, you should develop $(3x^3)^2$ first, then multiply by x^4. ***Yep; this is why you learned PEMDAS all those many years ago.***

9. $\frac{x^7}{x^5} = x^{7-5} = x^2$

Rule: When **dividing** two terms with the **same base** (x in this problem) and **different exponents**, ***SUBTRACT the exponents***.

10. $\frac{4^6 \cdot 4^2}{4^5} = \frac{4^{6+2}}{4^5} = \frac{4^8}{4^5} = 4^{8-5} = 4^3 = 64$

Rule: When **multiplying** two terms with the **same base** (4 in this problem) and **different exponents**, ***ADD the exponents***.

Rule: When **dividing** two terms with the **same base** (4 in this problem) and **different exponents**, ***SUBTRACT the exponents***.

Remember: Since 4 is a number, you can calculate: $4^3 = 64$.

SHORTCUT: $\frac{4^6 \cdot 4^2}{4^5} = 4^{6+2-5} = 4^3 = 64$

11. $\left(-\frac{x}{2}\right)^3 = (-)^3 \cdot \frac{(x)^3}{(2)^3}$

$= - \quad \frac{x^3}{8} = -\frac{x^3}{8}$

Rule: When taking a term to a **power**, ***take every item in the term to that power***.

Remember: Consider the sign, any numbers and each variable as separate items.

12. $\frac{1}{x^6} \cdot x^9 = \frac{1}{x^6} \cdot \frac{x^9}{1} = \frac{x^9}{x^6} = x^{9-6} = x^3$

Rule: When **dividing** two terms with the **same base** (x in this problem) and **different exponents**, ***SUBTRACT the exponents***.

Note: Feel free to skip the middle steps and proceed directly to $x^{9-6} = x^3$ if you are comfortable doing so.

13. $x^5 \cdot x^{-5} = x^{5+(-5)} = 5^0 = 1$

Rule: When **multiplying** two terms with the **same base** (x in this problem) and **different exponents**, ***ADD the exponents***.

Note: Anything (except zero) to the zero power is 1.

$$\mathbf{5^0 = 1} \qquad \mathbf{42^0 = 1} \qquad \mathbf{3.14159^0 = 1}$$

$$\mathbf{(your\ dog)^0 = 1}$$

$$\mathbf{0^0\ is\ undefined.}$$

14. $\frac{36n^2m^7}{6n^4m^3} = \frac{36}{6} \cdot \frac{n^2}{n^4} \cdot \frac{m^7}{m^3}$

$= \frac{6}{1} \cdot \frac{1}{n^{4-2}} \cdot \frac{m^{7-3}}{1}$

$= \frac{6}{1} \cdot \frac{1}{n^2} \cdot \frac{m^4}{1} = \frac{6m^4}{n^2}$

Notice the technique used here.
Each variable remains on the top or bottom depending on where it has a higher exponent in the original problem (m's on the top; n's on the bottom). Then, just subtract exponents to get the remaining exponent for each variable.

Rule: When **dividing** two terms with the **same base** (n and m in this problem) and **different exponents**, ***SUBTRACT the exponents***.

15. Write a rule for the exponential function.

x	-2	-1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

"Write a Rule" problems boil down to finding the two parameters, a and b, where the function is of the form: $y = a \cdot b^x$.

Step 1: Find b. Notice that the ratio of each term to the preceding term is 3. **The base, b, of an exponential function is always the common ratio of the terms**. So, $b = 3$.

Step 2: Next, find a. Notice that when $x = 0$, the equation $y = a \cdot b^x$ becomes $y = a \cdot b^0 = a \cdot 1 = a$. Awesome! So, **$a$ is the value of y when $x = 0$**. Look in the chart to see that when $x = 0$, we get $y = 1$. Therefore, $a = 1$.

Step 3: The resulting function, then, is $y = 1 \cdot 3^x$, or more succinctly, $y = 3^x$.

Step 4: Test your result: $y = 3^x$ gives the following table of values:

x	-2	-1	0	1	2
y	$3^{-2} = \frac{1}{9}$	$3^{-1} = \frac{1}{3}$	$3^0 = 1$	$3^1 = 3$	$3^2 = 9$

Looks like a match! **Answer:** $y = 3^x$

16. Write a rule for the exponential function:

x	-2	-1	0	1	2
y	**2**	4	8	16	32

Step 1: Find b. Notice that the ratio of each term to the preceding term is 2. Remember that **the base, b, of an exponential function is always the common ratio of the terms**. So, $\boldsymbol{b = 2}$.

Step 2: Next, find a. Remember that when $\boldsymbol{x = 0}$, the equation $\boldsymbol{y = a \cdot b^x}$ becomes $\boldsymbol{y = a \cdot b^0 = a \cdot 1 = a}$. Still awesome! So, **$a$ is the value of y when $x = 0$**. Look in the chart to see that when $\boldsymbol{x = 0}$, we get $\boldsymbol{y = 8}$. Therefore, $\boldsymbol{a = 8}$.

Step 3: The resulting function is $\boldsymbol{y = 8 \cdot 2^x}$.

Step 4: Test your result: $\boldsymbol{y = 8 \cdot 2^x}$ gives the following table of values:

x	-2	-1	0	1	2
y	$\boldsymbol{8 \cdot 2^{-2} = \frac{8}{4} = 2}$	$\boldsymbol{8 \cdot 2^{-1} = \frac{8}{2} = 4}$	$\boldsymbol{8 \cdot 2^0 = 8 \cdot 1 = 8}$	$\boldsymbol{8 \cdot 2^1 = 8 \cdot 2 = 16}$	$\boldsymbol{8 \cdot 2^2 = 8 \cdot 4 = 32}$

Looks like a match! **Answer:** $\boldsymbol{y = 8 \cdot 2^x}$

If you are content to have an answer at this point, do NOT read the alternative below.

Alternative Answer: Notice that $\boldsymbol{8 = 2^3}$. Then:

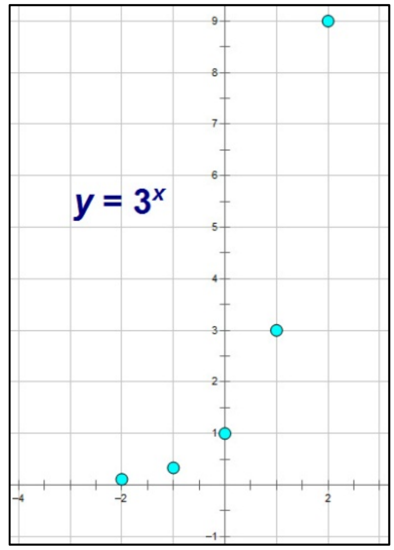
$$\boldsymbol{y = 8 \cdot 2^x = 2^3 \cdot 2^x = 2^{3+x} = 2^{x+3}}$$

Cool! So an alternative expression for the above function is: $\boldsymbol{y = 2^{x+3}}$. Importantly, this is the same function. There are just multiple ways to express it!

17. Graph the function $y = 3^x$ and state the domain and range of the function.

Step 1: For a simple exponential function like this, select the typical x-values and calculate the corresponding y-values.

x	-2	-1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9



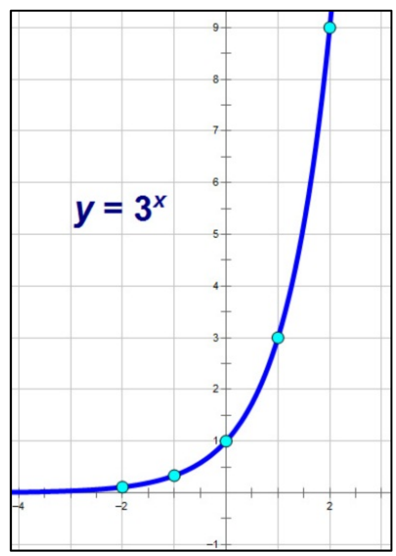
Step 2: Plot the points, like on the graph to the right.

Notice that, for negative values of x, the points appear to approach the x-axis but do not cross it. This makes the x-axis an ***asymptote*** for our function. As long as the function does not have a constant added to the exponential expression (e.g., your curve is $y = 3^x$ and not $y = 3^x + 4$), the asymptote will always be: $y = 0$ (which is the equation of the x-axis).

Definition

Asymptote: a line that becomes increasingly nearer to a curve as x or y increases or decreases, but which is never reached.

Step 3: Connect the points with a smooth curve that does not cross the asymptote (see below right).



Step 4: The **domain** is always **"all real values of x"** in simple exponential equations.

Step 5: The **range** is always either "$y > asymptote$" or "$y < asymptote$". Since the asymptote for this problem is $y = 0$ and the curve is *above* the asymptote, our range is: "$y > 0$".

Note: If the curve had been *below* the x-axis, the range would have been "$y < 0$".

For 18-20 Solve for x:

18. $\mathbf{4^x = 64}$

Let's try some powers of $\mathbf{4}$:

$\mathbf{4^1 = 4}$ $\mathbf{4^2 = 4 \cdot 4 = 16}$ $\mathbf{4^3 = 16 \cdot 4 = 64}$ **Got it!**

Since $\boldsymbol{x}$ is the exponent on $\mathbf{4}$ that results in $\mathbf{64}$, then $\boldsymbol{x} = \mathbf{3}$.

19. $\mathbf{5^x = 25}$

Let's try some powers of $\mathbf{5}$:

$\mathbf{5^1 = 5}$ $\mathbf{5^2 = 5 \cdot 5 = 25}$ **Got it! Easy peezy!**

Since $\boldsymbol{x}$ is the exponent on $\mathbf{5}$ that results in $\mathbf{25}$, then $\boldsymbol{x} = \mathbf{2}$.

20. $\mathbf{7^{3x} = 7^{x+8}}$

This one is a little different. The key here is to know that if the two powers of 7 are equal, then the exponents must also be equal. So, we get:

Step 1: Set exponents equal: $\mathbf{3x = x + 8}$
Step 2: Subtract $\boldsymbol{x}$: $\mathbf{-x \quad -x}$

$\mathbf{2x = 8}$
Step 3: Divide by 2: $\mathbf{\div 2 \quad \div 2}$

$\boldsymbol{x} = 4$

21. What is the solution for x in $3^x = 27$?
A. $x = 27$
B. $x = 9$
C. $x = 3$
D. $x = 2$

Let's try some powers of $\mathbf{3}$:

$\mathbf{3^1 = 3}$ $\mathbf{3^2 = 3 \cdot 3 = 9}$ $\mathbf{3^3 = 9 \cdot 3 = 27}$ **Got it! Easy peezy!**

Since $\boldsymbol{x}$ is the exponent on $\mathbf{3}$ that results in $\mathbf{27}$, then $\boldsymbol{x} = \mathbf{3}$. **Answer C**

22. What is the solution for y in $16 = 2^y$?

A. $y = 16$ C. $y = 3$

B. $y = 5$ D. $y = 0.5$

Let's try some powers of $\mathbf{2}$:

$\mathbf{2^1 = 2}$ $\mathbf{2^2 = 2 \cdot 2 = 4}$ $\mathbf{2^3 = 4 \cdot 2 = 8}$ $\mathbf{2^4 = 8 \cdot 2 = 16}$ **Got it!**

Maybe not easy peezy, but still not too hard. Had to go to a 4th power this time.

Since $\boldsymbol{y}$ is the exponent on $\mathbf{2}$ that results in $\mathbf{16}$, then $\boldsymbol{y} = \mathbf{4}$. **Answer Missing**

23. What is the simplified form of $(2g^3h^4)^3$?

A. $12g^6h^7$ C. $64g^6h^7$

B. $8g^9h^{12}$ D. $12g^6h^{12}$

$(\mathbf{2}\boldsymbol{g}^{\mathbf{3}}\boldsymbol{h}^{\mathbf{4}})^{\mathbf{3}} = (\mathbf{2})^{\mathbf{3}} \cdot (\boldsymbol{g}^{\mathbf{3}})^{\mathbf{3}} \cdot (\boldsymbol{h}^{\mathbf{4}})^{\mathbf{3}}$

$= \mathbf{8} \quad \boldsymbol{g}^{\mathbf{9}} \quad \boldsymbol{h}^{\mathbf{12}} \quad = \quad \mathbf{8}\, \boldsymbol{g}^{\mathbf{9}}\, \boldsymbol{h}^{\mathbf{12}}$ **Answer B**

Rule: When taking a term to a **power**, ***take every item in the term to that power***.

Rule: When taking a variable that has an **exponent to another power**, ***MULTIPLY the exponents***.

24. What is the simplified form of $(-4c^8)(2c^6d^8)$?

A. $-8c^{14}d^8$ C. $-8c^{48}d^{14}$

B. $-8c^{48}d^{14}$ D. $-8c^{14}d^{14}$

Hint: When multiplying a couple of complicated looking terms together, handle the parts of the term one at a time. That is, handle the sign first, then the numbers, then each of the variables. Here goes:

$$(-4c^8)(2c^6d^8)$$
$$= (-) \cdot (4 \cdot 2) \cdot (c^8 \cdot c^6) \cdot (d^8)$$
$$= (-) \cdot (8) \cdot (c^{8+6}) \cdot (d^8)$$
$$= -8\, c^{14}\, d^8$$

Answer A

Rule: When **multiplying** two terms with the **same base** ($\boldsymbol{c}$ in this problem) and **different exponents**, ***ADD the exponents***.

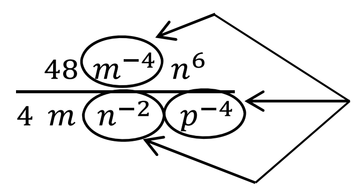
25. What is the simplified form of $\dfrac{48m^{-4}n^{6}}{4mn^{-2}p^{-4}}$? Assume that $m \neq 0$ and $p \neq 0$.

A. $\dfrac{12n^4p^4}{m^3}$ C. $\dfrac{12n^8p^4}{m^5}$

B. $\dfrac{12n^8}{m^5p^4}$ D. $\dfrac{12m^5}{n^8p^4}$

There is a new wrinkle in this problem – a fraction that contains variables with negative exponents. **No need to panic!** Just ask yourself what a negative exponent means. *A negative exponent shouts to you to move it across the fraction line and make it positive.* Listen and you will hear it shout – that panic in your brain is actually the negative exponents shouting to you. Let's give it a shot.

> *Step 1 involves creating a new fraction from the one in the problem we have been given.*
>
> *Handle the parts of the fraction one at a time. Start with the 48, then the* $\boldsymbol{m^{-4}}$*, then the* $\boldsymbol{n^{6}}$ *and so on.*
>
> *Move items with negative exponents across the line and make the exponents positive. Keep the rest of the items where they are.*
>
> *The result will be a fraction with no negative exponents! Very cool!*



The circled guys will be moving across the fraction line.

Step 1: Get rid of negative exponents: $\dfrac{48\,n^6\,n^2\,p^4}{4\,m^4\,m}$

Remember that this m has an exponent of 1.

Step 2: Multiply like variables: $\dfrac{\mathbf{48}\,\boldsymbol{n^{6+2}}\;\boldsymbol{p^4}}{\mathbf{4}\,\boldsymbol{m^{4+1}}}$

Step 3: Simplify the variables: $\dfrac{\mathbf{48}\,\boldsymbol{n^{8}}\,\boldsymbol{p^4}}{\mathbf{4}\,\boldsymbol{m^{5}}}$

Rule: When **multiplying** two terms with the **same base** ($\boldsymbol{n}$ and $\boldsymbol{m}$ in this problem) and **different exponents**, ***ADD the exponents***.

Step 4: Divide the number values: $\dfrac{\mathbf{12}\,\boldsymbol{n^{8}}\,\boldsymbol{p^4}}{\boldsymbol{m^{5}}}$ **Answer C**

Note: If any of the variables in the fraction had remained in both the numerator and denominator, we would have had to simplify further. See problem #14.