Algebra 1: Semester 2 Practice Final
"Unofficial" Worked-Out Solutions by Earl Whitney

1. The situation described in this problem involves probability **without replacement**. Because of this, the probabilities change after the selection of the first sock.

   The drawer contains socks as follows:

Number of Socks	Before 1st Sock Selected	After 1st Sock Selected
Red socks	**2**	**1**
White socks	7	7
Blue socks	9	9
Total socks	**18**	**17**

1st sock → 2nd sock

$$P = \frac{2}{18} \cdot \frac{1}{17}$$

$$= \frac{1}{9} \cdot \frac{1}{17}$$

$$= \frac{1}{153}$$

Answer C

2. The situation described in this problem involves probability **with replacement**. The probabilities remain the same after the selection of the first marble.

   The jar contains marbles as follows:

	Number of Marbles
Red marbles	**6**
White marbles	5
Blue marbles	9
Total marbles	**20**

1st marble → 2nd marble

$$P = \frac{6}{20} \cdot \frac{6}{20}$$

$$= \frac{3}{10} \cdot \frac{3}{10}$$

$$= \frac{9}{100}$$

Answer C

3. Number of people who can come in 1st place: 9

   Number of people who can come in 2nd place: 8

   Number of ways the first two places can come out: $9 \cdot 8 = 72$

Answer B

Note: This can also be calculated as:

$${}_9P_2 = \frac{9!}{(9-2)!} = 9 \cdot 8 = 72$$

4. Choosing a 4-person team from a group of 9 runners requires selection **without regard to order**. Therefore, we are to calculate a **combination**.

$$ {}_9C_4 = \frac{9!}{4! \cdot (9-4)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126 $$

Answer B

5. The student has an average of 75 on 4 quizzes.

   If his next score is above 75, the mean increases. Consider A to D as follows:

   A. *His mode increases.* The mode is the most frequent score. So, this is possible, but we do not have any information about each of the other scores, so we do not know for sure.

   B. *His median score is 80.* The median is the middle score when the scores are ranked from low to high. So, this is possible, but we do not have any information about each of the other scores, so we do not know for sure.

   C. *His mean increases.* The mean is the average (add up the scores and divide by the number of scores). ✓ This must happen because you are averaging in a new score higher than the old mean. In fact, in this problem, the mean increases from 75 to 76.

   D. *His* 5^{th} *quiz grade is the highest score.* This is possible, but we do not have any information about each of the other scores, so we do not know for sure. The student could have had all 75's on his quizzes to have a mean of 75, or the student could have had two 85's and two 65's to have a mean of 75. There are also many other possibilities; we have insufficient information to know if his 5^{th} quiz grade is the highest score.

Answer C

6. The scores look like this:

   6, 8, 8, 8, 9, 9,   9, 9,   10, 10, 10, 10, 10, 10

   There are 14 total scores. The two middle numbers are both 9, so the median is 9.

Answer C

   Note: If the two middle numbers had been different values, we would have had to take their mean in order to get the median.

   Example: The median of a set of scores: {22, 24, 26, 28} is 25 because there are two middle scores that are different from each other. The mean of the two middle numbers, 24 and 26, is 25.

7. Consider each of the terms:
   A. *Bimodal* means there are two peaks in the distribution. This distribution has only one peak.
   B. *Skewed* means the distribution tails off to one side or the other. ✓ This distribution definitely tails off to the right and so, is "skewed to the right."
   C. A *normal* distribution has a center which is the mean, the median and the mode, and has equal tails on the left and right sides. This distribution is clearly not *normal*.
   D. A *uniform* distribution has the same frequency for all occurrences of the independent variable. The graph would contain a set of bars that all have the same height. This distribution has bars of varying heights.

Answer B

8. In looking at the graph, consider the following points:

   - The independent variable is on the x-axis. The dependent variable is on the y-axis. So, water consumption depends on the temperature; temperature does not depend on water consumption. This leaves out answer A.
   - Any answer that contains the word "all" in dealing with a distribution is suspect. It is not true in this graph that all of the people did anything. This leaves out answers B and D.
   - It is true that the graph has a generally upward slope. Therefore, answer C is reasonable: water consumption increases as the temperature increases.

Answer C

9. All of the answers are given in the form: "$\boldsymbol{y} = \cdots$"

   So, let's put the given equation in that form.

Step 1: Switch the terms on each side of the equal sign. $2y - 6 = 2x$

Step 2: Add 6 to each side: $+6 \quad + 6$

$$2y = 2x + 6$$

Step 3: Divide both sides by 2: $\div 2 \quad \div 2$

$$y = x + 3$$

Looking at the answers, we see that D is the exact same equation. Therefore the original equation and the one in D are a "system of equations with the same line and infinitely many solutions."

Answer D

10. The given line has a slope of 3. A line *perpendicular* to this would have a slope that is the reciprocal of this, but with the opposite sign. So, we want an equation with a slope of $-\frac{1}{3}$. Answer D is the only equation with this slope.

Answer D

Note one additional thing. When two lines are *perpendicular*, there is always exactly one solution to the set of simultaneous equations. In fact, any time two lines are *not parallel*, there is always exactly one solution to the set of simultaneous equations.

11. In order for lines in standard form (as the answers to this problem are) to be parallel, the coefficients of x and y must be proportional. This occurs in answers A and D.

$$A\colon\ \frac{3}{2}=\frac{-3}{-2} \qquad \text{and} \qquad D\colon\ \frac{5}{2}=\frac{-5}{-2}$$

If the two lines are the same line, with infinitely many solutions, the constants in the two equations are also proportional, as they are in D.

$$D\colon\ \frac{5}{2}=\frac{10}{4}$$

If the two lines are different, with no solution, the constants in the two equations are not proportional, as in A.

$$A\colon\ \frac{3}{2}\neq\frac{6}{-8}$$

Answer A

12. First, let's see which answers have (2, 1) as a solution:

A. $\mathbf{6(2)-2(1)}=12-2\mathbf{=10}$ ✓
B. $\mathbf{2(2)-3(1)}=4-3\mathbf{=1}$ ✓
C. $\mathbf{4(2)-7(1)}=8-7=1\neq\mathbf{-28}$ *no*!
D. $\mathbf{2(2)-3(1)}=4-3=1\neq\mathbf{28}$ *no*!

So, answers A and B work, and we must figure out which one provides only one solution. There are numerous ways to do this, but one easy way is to find another point on the original equation $y=3x-5$ and see if it is also on A or B.

An easy point to check occurs when $x=0$, so that $y=3(0)-5=-5$. Let's see if the point $(0,-5)$ exists on either A or B.

A. $\mathbf{6(0)-2(-5)}=0+10\mathbf{=10}$ ✓
B. $\mathbf{2(0)-3(-5)}=0+15=15\neq\mathbf{1}$ *no*!

Since we want the equation where the second point does not work, the answer is B.

Answer B

13. Since one of the equations is in the form "$y = \cdots$" this system of equations is tailor made for "substitution."

$$\mathbf{y = 2x + 7} \qquad 3x - 2\mathbf{y} = -12$$

Substituting what $\boldsymbol{y}$ is in the first equation into the second equation gives:

$$3x - 2(\mathbf{2x + 7}) = -12$$

Then solve for x:

Distribute the 2 over $(2x + 7)$: $3x - 4x - 14 = -12$

Combine x-terms: $-x - 14 = -12$

Add 14 to each side: $+14 \quad +14$

$-x = 2$

Step 4: Divide both sides by -1: $\div -1 \quad \div -1$

$x = -2$

Answer A

14. First, put the second equations in standard form (the first is already in standard form).

Starting Equation $\mathbf{-2y = -4x - 6}$

Add $\mathbf{4x}$ to each side: $\mathbf{+4x \quad + 4x}$

Standard Form $\mathbf{4x - 2y = \quad -6}$

Next, solve for $\boldsymbol{y}$ be eliminating $\boldsymbol{x}$.

$\mathbf{5x - 2y = -3}$ (multiply by 4) $\mathbf{20x - 8y = -12}$

$\mathbf{4x - 2y = -6}$ (multiply by -5) $\mathbf{-20x + 10y = 30}$

Add the two equations: $\mathbf{2y = 18}$

Divide both sides by $\mathbf{2}$: $\mathbf{\div 2 \quad \div 2}$

$\mathbf{y = 9}$

Answer D

15. I like to set up a table for this:

Large Candles		**Small Candles**		**Total Cost**
Number	**Cost**	**Number**	**Cost**	
3	t	5	s	\$6.40
4	t	6	s	\$7.50

Reading across each row gives:

$\mathbf{3t + 5s = 6.4}$

$\mathbf{4t + 6s = 7.5}$

Answer A

16. I will use elimination, but you could use substitution if you wanted to.
Solve for $\boldsymbol{y}$ by eliminating $\boldsymbol{x}$.

$2.5x + 1.25y = 1375$	(multiply by -1)	$-2.5x - 1.25y = -1375$
$1x + \quad 1y = \quad 700$	(multiply by 2.5)	$2.5x + 2.5\ y = \quad 1750$
Add:		$1.25y = \quad 375$
Divide both sides by 1.25:		$\div 1.25 \quad \div 1.25$
		$y = \quad 300$

This gives the answer containing 300 children (variable $\boldsymbol{y}$), which is sufficient to select the correct answer from those provided.

Answer C

17. I would set up the following table to generate the equations needed. Values in **bold blue** are provided in the question. Values in *black italics* are generated to help solve the problem.

	Quarters	**Dimes**	**Total**
Number of Coins	*x*	*12-x*	**12**
Value of Coins	*.25x*	*.10(12-x)*	**2.25**

Next, solve for $\boldsymbol{x}$ using the equation formed in the bottom row.

Bottom row equation:	$.25x + .10(12 - x) = 2.25$
Distribute $.10$ over $(12 - x)$:	$.25x + 1.20 - .10x = 2.25$
Combine *x*-terms:	$.15x + 1.20 \quad = 2.25$
Subtract 1.20 from each side:	$-1.20 \quad -1.20$
	$.15x = 1.05$
Divide both sides by $.15$:	$\div .15 \quad \div .15$
	$x = 7$

This gives the answer containing 7 quarters, which is sufficient to select the correct answer from those provided.

Answer B

18. The sum of the number of white marbles, $\boldsymbol{x}$, and the number of red marbles, $\boldsymbol{y}$, must equal the total, $\mathbf{15}$. So one equation is:

$$\boldsymbol{x+y=15}$$

This narrows the choices to answers A and B.

Then, $\boldsymbol{y}$ is $\mathbf{3}$ more than six times $\boldsymbol{x}$. So the other equation is:

$$\boldsymbol{y=6x+3}$$

Answer A

19. The equation given has a slope of $\mathbf{2}$. So the parallel line must have a slope of $\mathbf{2}$. Since the answers are in slope-intercept form, we can substitute $\boldsymbol{m=2}$ and the point values, $\boldsymbol{x=6}$ and $\boldsymbol{y=-8}$, into the generalized slope-intercept equation:

Generalized slope-intercept equation:	$\boldsymbol{y=mx+b}$
Substitute known values for variables:	$\boldsymbol{-8=2\cdot 6+b}$
Multiply:	$\boldsymbol{-8=\ \ 12+b}$
Subtract $\mathbf{12}$ from each side:	$\boldsymbol{-12\quad -12}$
	$\boldsymbol{-20=\qquad b}$

Then, since $\boldsymbol{m=2}$ and $\boldsymbol{b=-20}$, the equation becomes: $\boldsymbol{y=2x-20}$

Answer A

20. The equation given has a slope of $-\frac{1}{3}$. So the parallel line must have a slope of the opposite reciprocal of this, which is $\mathbf{3}$.

Since the answers are in slope-intercept form, we can substitute $\boldsymbol{m=3}$ and the point values, $\boldsymbol{x=-3}$ and $\boldsymbol{y=4}$, into the generalized slope-intercept equation:

Generalized slope-intercept equation:	$\boldsymbol{y=mx+b}$
Substitute known values for variables:	$\boldsymbol{4=3\cdot(-3)+b}$
Multiply:	$\boldsymbol{4=\ \ -9+b}$
Add $\mathbf{9}$ to each side:	$\boldsymbol{+9\quad +9}$
	$\boldsymbol{13=\qquad b}$

Then, since $\boldsymbol{m=3}$ and $\boldsymbol{b=13}$, the equation becomes: $\boldsymbol{y=3x+13}$

Answer D

21. If you have not memorized your squares and square roots yet, now would be a good time to do so.

Notice from the table of squares, that 68 is between 64 and 81.

Then, $\sqrt{\mathbf{68}}$ is between $\sqrt{\mathbf{64}}$ and $\sqrt{\mathbf{81}}$.

So, $\sqrt{\mathbf{68}}$ is between $\mathbf{8}$ and $\mathbf{9}$.

Remember the negative sign. $-\sqrt{\mathbf{68}}$ is between $-\mathbf{8}$ and $-\mathbf{9}$.

Answer A

Squares

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$

$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$

$11^2 = 121$
$12^2 = 144$
$13^2 = 169$
$14^2 = 196$
$15^2 = 225$

22. For a similar problem with cube roots, you follow the same process.

Cubes

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$

Notice from the table of cubes, that 90 is between 64 and 125.

Then, $\sqrt[3]{\mathbf{90}}$ is between $\sqrt[3]{\mathbf{64}}$ and $\sqrt[3]{\mathbf{125}}$.

So, $\sqrt[3]{\mathbf{90}}$ is between $\mathbf{4}$ and $\mathbf{5}$.

Answer A

23. Use PEMDAS for the *order of operations*.

- P Anything in Parentheses is evaluated first.
- E Items with Exponents are evaluated next.
- M Multiplication and …
  D Division are performed next.
- A Addition and …
  S Subtraction are performed last.

Note: When there are multiple operations in the same category, for example, a division and two multiplications, the operations are performed from left to right.

Problem:	$2^2 - 9 + 7^0 + 6 \cdot \sqrt{9^2 + 19}$
Treat the radical like parentheses:	$2^2 - 9 + 7^0 + 6 \cdot \sqrt{81 + 19}$
Work under the radical:	$2^2 - 9 + 7^0 + 6 \cdot \sqrt{100}$
	$2^2 - 9 + 7^0 + 6 \cdot 10$
Work exponents:	$4 - 9 + 1 + 6 \cdot 10$
Multiply (and divide, if necessary):	$4 - 9 + 1 + 60$
Add and subtract:	56

Answer D

24. Remember PEMDAS; exponents first, negative sign last. Add parentheses if it helps you.

Also, remember that a *negative exponent* means "*move the term over the fraction sign and change the exponent to positive*."

$$-2^{-2} = -(2^{-2}) = -\left(\frac{1}{2^2}\right) = -\frac{1}{4}$$

Answer B

25. There are multiple ways to do this, two of which are shown here.

Method 1: Prime Factor Tree Method

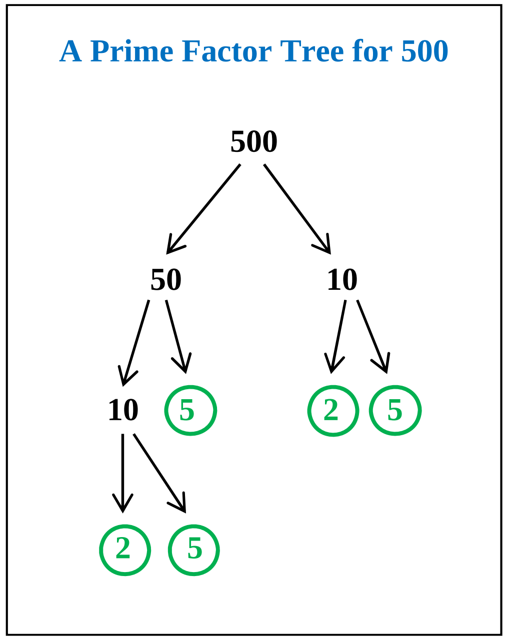
Create a prime factor tree for 500 and place all of the factors under the radical (preferably in numerical order).

$$\sqrt{500} = \sqrt{\underline{2 \cdot 2} \cdot 5 \cdot \underline{5 \cdot 5}}$$

Note the underlined pairs in bold blue. Each pair represents a square that can be pulled outside the radical as its root.

$$\sqrt{\underline{2 \cdot 2} \cdot 5 \cdot \underline{5 \cdot 5}} = 2 \cdot 5 \cdot \sqrt{5} = 10\sqrt{5}$$

Answer B



Method 2: Square Recognition Method

If you are able to recognize a square that is a factor of the radicand, your work can be shortened.

$$\sqrt{500} = \sqrt{100 \cdot 5} = \sqrt{100} \cdot \sqrt{5} = 10\sqrt{5}$$

26. Use the distributive property.

$$4 \cdot \left(8\sqrt{5} - \sqrt{3}\right) = 32\sqrt{5} - 4\sqrt{3}$$

No further simplification can be performed.

Answer A

27. Cube each item in parentheses. Remember that when taking a power to a power, you multiply the exponents.

$$(4g^3h^4)^3 = 4^3 \cdot g^{3 \cdot 3} \cdot h^{4 \cdot 3} = 64g^9h^{12}$$

Answer B

28. Combine like elements. Remember that when multiplying elements with the same base and different exponents, you add the exponents.

Original Problem: $(-3c^8)\,(2c^6d^8)$

Rearrange elements: $-\,(3\cdot 2)\,(c^8\cdot c^6)\,(d^8)$

Perform operations: $-6\,c^{14}\,d^8$

Answer A

29. First take care of the negative exponents by moving them to the other side of the fraction line and making the exponents positive. Then simplify.

$$\frac{36m^{-4}n^6}{4mn^{-2}p^{-4}} = \frac{36\cdot n^6\cdot n^2\cdot p^4}{4\cdot m\cdot m^4}$$

← **movement indicated by color**

$$= \frac{36}{4}\cdot\frac{(n^6\cdot n^2)\cdot p^4}{(m^1\cdot m^4)}$$

$$= \frac{9}{1}\cdot\frac{n^8\cdot p^4}{m^5} = \frac{9\,n^8\,p^4}{m^5}$$

Answer C

30. To simplify the product of two radicals, pull everything under a single radical and then sort things out.

$$\sqrt{xy^3}\cdot\sqrt{x^3y} = \sqrt{xy^3x^3y} = \sqrt{x^4y^4}$$

When taking a root of a variable to a power, divide the power by the root. In this case:

$$\sqrt{x^4y^4} = x^{4/2}\cdot y^{4/2} = x^2y^2$$

Answer D

31. Let's look at powers of 4:

$4^0 = 1 \quad 4^1 = 4 \quad 4^2 = 16 \quad 4^3 = 64$ ✓

So, $x = 3$.

Answer C

32. Let's look at powers of 2:

$2^0 = 1 \quad 2^1 = 2 \quad 2^2 = 4 \quad 2^3 = 8 \quad 2^4 = 16 \quad 2^5 = 32$ ✓

So, $y = 5$.

Answer B

33. To solve a radical equation, square both sides:

Original problem: $\sqrt{x} = 6$

Square both sides: $(\sqrt{x})^2 = 6^2$

$x = 36$ **Answer D**

34. This one is easy if you calculate $\sqrt{64}$ first.

Original problem: $2\sqrt{64} = x$

Simplify $\sqrt{64}$: $2 \cdot 8 = x$

Multiply: $16 = x$ **Answer C**

35. Group like items together to make this easier. Standard form is to put higher powers first; then, for terms of the same power, put the terms in alphabetical order.

Original problem: $(6a - 2b^2 - a) + (b - 3 + 9a^2)$

Rearrange terms: $9a^2 - 2b^2 + (6a - a) + b - 3$

Combine like terms: $9a^2 - 2b^2 + 5a + b - 3$ **Answer C**

36. Use FOIL or the box method: $(r - 8)\,(r + 5)$

F	First	$r \cdot r = r^2$	
O	Outside	$r \cdot 5 = 5r$	$5r - 8r = -3r$
I	Inside	$-8 \cdot r = -8r$	
L	Last	$-8 \cdot 5 = -40$	

So, $(r - 8)\,(r + 5) = r^2 - 3r - 40$ **Answer D**

37. Use FOIL or the box method: $(b + 7)\,(b + 7)$

F	First	$b \cdot b = b^2$	
O	Outside	$b \cdot 7 = 7b$	$7b + 7b = 14b$
I	Inside	$7 \cdot b = 7b$	
L	Last	$7 \cdot 7 = 49$	

So, $(b + 7)\,(b + 7) = b^2 + 14b + 49$ **Answer A**

38. Use FOIL or the box method: $(6c+9)\,(6c-9)$

F	First	$6c \cdot 6c = 36c^2$	
O	Outside	$6c \cdot (-9) = -54c$	$-54c + 54c = 0c$
I	Inside	$9 \cdot 6c = 54c$	
L	Last	$9 \cdot (-9) = -81$	

So, $(6c+9)\,(6c-9) = 36c^2 - 81$

Answer B

39. Ugly expressions like this one can be made prettier by adding or subtracting in columns. Subtractions can be changed into additions by changing the sign of each term being subtracted.

$$\begin{array}{l} (5q^5+4) \\ -(2q^3+9) \\ +(6q^5-q^3) \end{array}$$

Line up each term in columns

$$\begin{array}{rrr} 5q^5 & & +4 \\ & -2q^3 & -9 \\ +6q^5 & -1q^3 & \\ \hline 11q^5 & -3q^3 & -5 \end{array}$$

Answer A

40. Multiplication of polynomials can be performed in columns.

$$\begin{array}{rrrr} & x^2 & -2x+ & 3 \\ & & x+ & 5 \\ \hline & 5x^2 & -10x & +15 \\ x^3 & -2x^2 & +3x & \\ \hline x^3 & +3x^2 & -7x & +15 \end{array}$$

Product of 5 and $(x^2 - 2x + 3)$: $5x^2 - 10x + 15$

Product of x and $(x^2 - 2x + 3)$: $x^3 - 2x^2 + 3x$

Answer C

41. Factor $16x^2 - 49$

Note that this is a difference of squares. $16x^2 = (4x)^2$ and $49 = 7^2$.

The general equation for factoring a difference of squares is:

$$a^2 - b^2 = (a-b)\,(a+b)$$

Let $a = 4x$ and $b = 7$ to get:

$$16x^2 - 49 = (4x-7)\,(4x+7)$$

Answer A

42. Factor $x^2 + 15x + 14$

We want two numbers so that:

$\_\_\_ + \_\_\_ = 15$

$\_\_\_ \cdot \_\_\_ = 14$

Thinking through the possibilities, we get the values 1 and 14. So,

$x^2 + 15x + 14 = (x+1)\,(x+14)$

Answer D

43. Factor $x^2 + x - 30$

We want two numbers so that:

$$\_\_\_ + \_\_\_ = 1$$
$$\_\_\_ \cdot \_\_\_ = -30$$

Thinking through the possibilities, we get the values 6 and -5. So,

$$x^2 + x - 30 = (x + 6)\,(x - 5)$$

Answer B

44. Factor $x^2 + 11x + 30$

We want two numbers so that:

$$\_\_\_ + 6 = 11$$
$$\_\_\_ \cdot 6 = 30$$

The missing value must be 5. So,

$$x^2 + 11x + 30 = (x + 6)\,(x + 5)$$

Answer D

45. Factor $2x^2 + 7x - 30$

multiply the values of a and c

-60

We want two numbers so that:

$$\_\_\_ + \_\_\_ = 7$$
$$\_\_\_ \cdot \_\_\_ = -60$$

Thinking through the possibilities, we get the values 12 and -5. So,

$$\begin{aligned} &2x^2 + 7x - 30 \\ &= 2x^2 + 12x - 5x - 30 \\ &= (2x^2 + 12x) - (5x + 30) \\ &= 2x(x + 6) - 5(x + 6) \\ &= (2x - 5)(x + 6) \end{aligned}$$

notice the sign change

Answer B

46. Factor $2x^2 - 2x - 4 = 2(x^2 - x - 2)$

We want two numbers so that:

$$\_\_\_ + \_\_\_ = -1$$
$$\_\_\_ \cdot \_\_\_ = -2$$

Thinking through the possibilities, we get the values 1 and -2. So,

$$\begin{aligned} &2x^2 - 2x - 4 \\ &= 2(x - 2)(x + 1) \end{aligned}$$

Since neither $(x - 2)$ nor $(x + 1)$ is an answer, let's try multiplying them by 2.

$$2\,(x - 2) = 2x - 4$$
$$2\,(x + 1) = 2x + 2 \quad ✓$$

Answer B

47. First put the equation in general form, then factor it.

Original Equation: $x^2 + 8x = 84$

Subtract 84 from each side: $-84 \quad -84$

$x^2 + 8x - 84 = 0$

Now, factor the quadratic equation:

We want two numbers so that:

$\_\_\_ + \_\_\_ = 8$

$\_\_\_ \cdot \_\_\_ = -84$

Thinking through the possibilities, we get the values 14 and -6. So,

$$x^2 + 8x - 84 = (x + 14)\,(x - 6) = 0$$

Then either $(x + 14) = 0$ or $(x - 6) = 0$.
Solving each of these gives solutions of $x = \{-14, 6\}$

Answer C

48. First put the equation in general form, then factor it.

Original Equation: $x^2 = -4x + 96$

Add $4x - 96$ to each side: $+4x - 96 \quad +4x - 96$

$x^2 + 4x - 96 = 0$

Now, factor the quadratic equation:

We want two numbers so that:

$\_\_\_ + \_\_\_ = 4$

$\_\_\_ \cdot \_\_\_ = -96$

Thinking through the possibilities, we get the values 12 and -8. So,

$$x^2 + 4x - 96 = (x + 12)\,(x - 8) = 0$$

Then either $(x + 12) = 0$ or $(x - 8) = 0$.
Solving each of these gives solutions of $x = \{-12, 8\}$

Answer A

49. In $h^2 + 13h - 4 = 0$ we have: $a = 1$, $b = 13$, and $c = -4$.

The quadratic equation is: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So, we have: $x = \frac{-13 \pm \sqrt{(13)^2 - 4(1)(-4)}}{2(1)} = \frac{-13 \pm \sqrt{169 + 16}}{2} = \frac{-13 \pm \sqrt{185}}{2}$

Answer D

50. First put the equation in general form, then use the Quadratic Equation.

Original Equation:	x^2	$=$	$15x - 34$
Add $-15x + 34$ to each side:	$-15x + 34$		$-15x + 34$
	$x^2 - 15x + 34$	$=$	0

So, we have: $a = 1$, $b = -15$, and $c = 34$.

The quadratic equation is: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So, we have: $x = \dfrac{+15 \pm \sqrt{(-15)^2 - 4(1)(34)}}{2(1)} = \dfrac{15 \pm \sqrt{225 - 136}}{2} = \dfrac{15 \pm \sqrt{89}}{2}$

Answer A

51. Factor $x^2 + 6x - 7$

We want two numbers so that:

$\_\_\_ + \_\_\_ = 6$

$\_\_\_ \cdot \_\_\_ = -7$

Thinking through the possibilities, we get the values 7 and -1. So,

$x^2 + 6x - 7 = (x + 7)\,(x - 1) = 0$

Then either $(x + 7) = 0$ or $(x - 1) = 0$.
Solving each of these gives solutions of $x = \{-7, 1\}$

Answer D

52. Factor $2x^2 + 2x - 24 = 2(x^2 + x - 12)$

We want two numbers so that:

$\_\_\_ + \_\_\_ = 1$

$\_\_\_ \cdot \_\_\_ = -12$

Thinking through the possibilities, we get the values 4 and -3. So,

$2x^2 + 2x - 24 = 2\,(x + 4)\,(x - 3) = 0$

Then either $(x + 4) = 0$ or $(x - 3) = 0$.
Solving each of these gives solutions of $x = \{-4, 3\}$

Answer B

53. The domain is *All Real Numbers* because any real value of x can be input into the equation and get a resulting y-value.

The range is $y \geq -3$ because the smallest $|x|$ can be is zero, and it can be any positive number. Therefore:

$$g(x) = |x| - 3 \geq 0 - 3 = -3$$

Answer B

54. The domain is *All Real Numbers* because any real value of x can be input into the equation and get a resulting y-value.

The range is $y \leq -1$ because the largest $-x^2$ can be is zero, and it can be any negative number. Therefore:

$$y = -x^2 - 1 \leq 0 - 1 = -1$$

Answer B

Note also that the range can be seen on the graph as y-values -1 and below.

55. Because the function has a negative coefficient of x^2, the graph has an inverted U-shape, and the range is all values equal to or below the y-value of the vertex.

To calculate the y-value of the vertex, first find the x-value of the vertex at $x = -\frac{b}{2a}$, then insert this value into the function to calculate the y-value.

$$x_{vertex} = -\frac{b}{2a} = \frac{-10}{-2} = 5$$

$$y_{vertex} = -x^2 + 10x - 16 = -(5^2) + 10(5) - 16 = -25 + 50 - 16 = 9$$

The range is $y \leq y_{vertex}$ or $h(x) \leq 9$.

Answer C

56. Since the independent variable is $t = time$, the domain represents *the time after the punt*.

Answer A

57. The function $y = x^2 - 4$ has the following characteristics:

- It has a vertex at $x = 0$ (since $x_{vertex} = -\frac{b}{2a}$ and $b = 0$, $x_{vertex} = -\frac{0}{2} = 0$)
- It crosses the y-axis at $y = -4$ (when $x = 0$, $y = 0^2 - 4 = -4$).

Answer A

58. Translating a function 3 units up is simply adding 3 to all values of y. The resulting function is:

$$f(x) = -x^2 + 3$$

Answer C

59. The graph of $\boldsymbol{y = |x| - 2}$ has the following characteristics.

- It is in the shape of an upward-V because the coefficient of $\boldsymbol{|x|}$ is positive.
- It has a vertex at $(\boldsymbol{0, -2})$.

Answer D

Note: the vertex of any absolute value function of the form $\boldsymbol{y = a \cdot |x - h| + k}$ is always at the point $(\boldsymbol{h, k})$. In the function above, $\boldsymbol{a = 1}$, $\boldsymbol{h = 0}$ and $\boldsymbol{k = -2}$.

60. Translation of the function should be done one step at a time:

Starting equation:	$\boldsymbol{y = \|x\|}$
Translate over the x-axis (make an inverted V):	$\boldsymbol{y = -\|x\|}$
Translate up 1 unit (add 1 to all $\boldsymbol{y}$-values):	$\boldsymbol{y = -\|x\| + 1}$

Answer D